Manchester Public Schools

Algebra 1, Geometry, and Algebra 2 Curricula

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Manchester Public Schools
Algebra 1, Geometry & Algebra 2 Curricula
2012

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The Philosophy and Vision for Mathematics in Manchester

MANCHESTER PUBLIC SCHOOLS

PHILOSOPHY

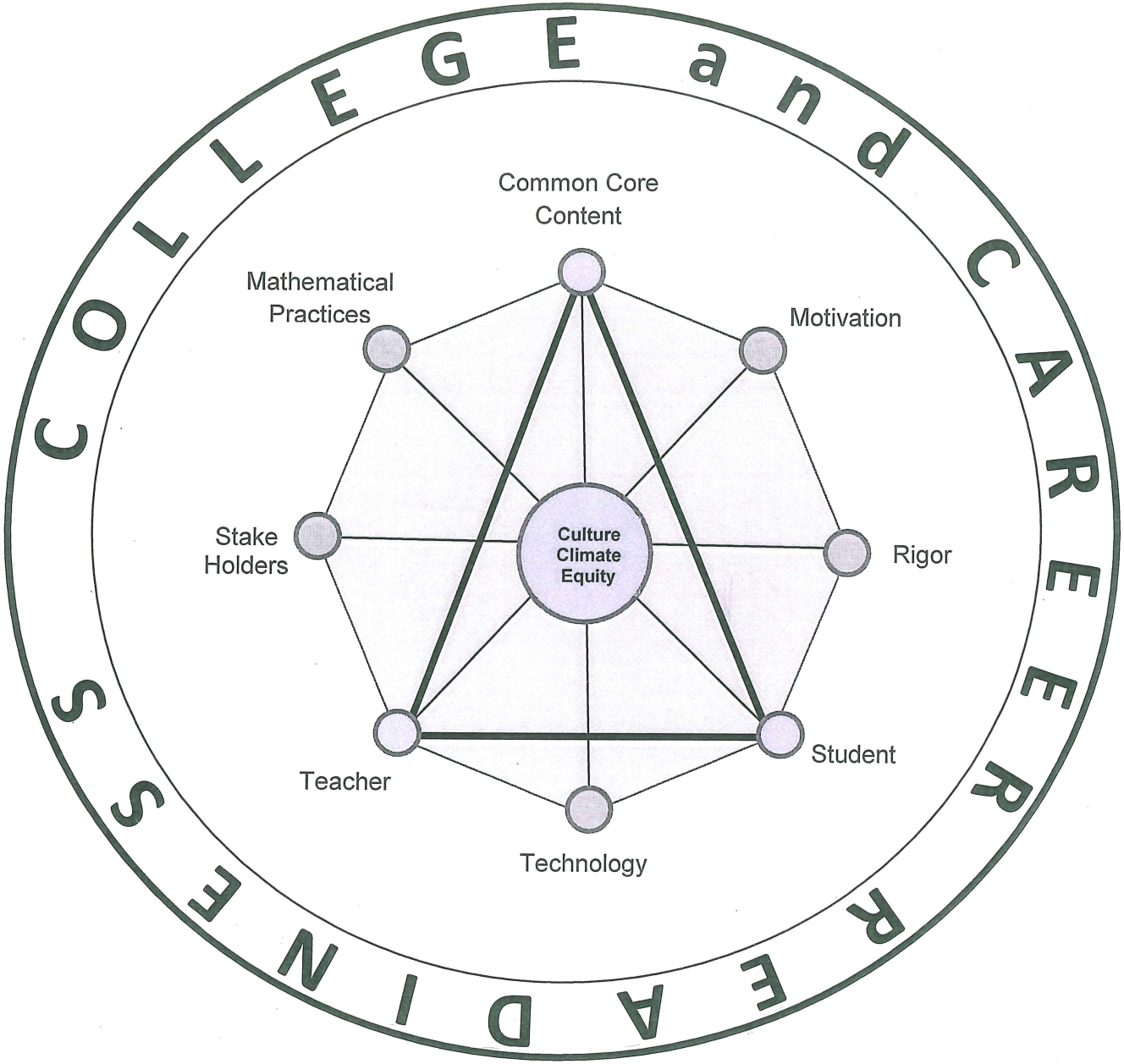
Our goal is to have each student realize his/her full potential and develop an understanding of the ever-changing math skills needed for the 21st century. We recognize each individual's unique style of learning and diverse needs, as we teach the new COMMON CORE mathematics in Manchester Public Schools. Our program is designed to empower each learner in achieving a thorough understanding through the lens of mathematical practices. The important processes and proficiencies of this COMMON CORE document are to: utilize math to reason abstractly, instill habitual practices of precision and perseverance in solving problems strategically, develop viable arguments using math to explain logic and infuse a command of technology as an appropriate tool of support. These are the new understandings of mathematics education, designed for the future of all students to be successful in this century.

VISION

The vision of Manchester Public Schools is to empower all students to become successful lifelong strategic mathematical problem solvers.

21st Century Mathematics Teaching

"Our vision is to rigorously challenge ALL students to empower for math success"



Developed by: Dr. Ann Richardson, Kristin Smith and Heather Elsinger; 2012

Manchester Public Schools

Algebra 1, Geometry & Algebra 2 Curricula

2012

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Overview of Mathematics Curriculum Design

Overview of the Mathematics Curriculum Design

The Mathematics Curriculum purposefully connects the Common Core State Standards (CCSS), pedagogical teaching and learning practices while using performance expectations to create a rigorous curriculum design (Ainsworth, 2010). The Mathematics Curriculum that is structured by grade level offers:

- Specific learning outcomes that define what students should understand and be able to do
- Emphasis on standards-based skills
- Academic Vocabulary specific to the area of math
- 21st Century learning skills
- Higher-level thinking skills
- Research-based, effective teaching strategies
- A curriculum glossary to promote consistency of understanding
- Unwrapped Standards to promote consistency of understanding
- Personally meaningful and socio culturally relevant learning opportunities

The curriculum format includes units of study, essential questions and unit goals, performance standards, performance assessments, and suggested instructional strategies. To ensure the curriculum becomes an integral part of lesson planning, several appendices offer supportive additional resources to enrich the teaching and learning of mathematics in the classroom. The Mathematics Curriculum is designed to promote clarity and specificity, to stress conceptual understanding of key ideas, and to balance a combination of procedure and understanding, so that students in Manchester are college and/or career ready successfully meeting the challenges of a global society.

Mathematics Curriculum K-12 for the 21st Century

Mathematics Content

These Common Core State Standards for Mathematics define what students should understand and be able to do in their study of mathematics. Asking a student to understand something means asking a teacher to assess whether the student has understood it. But what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, *why* a particular mathematical statement is true or where a mathematical rule comes from. There is a world of difference between a student who can summon a mnemonic device to expand a product such as $(a + b)(x + y)$ and a student who can explain where the mnemonic comes from. The student who can explain the rule understands the mathematics, and may have a better chance to succeed at a less familiar task such as expanding $(a + b + c)(x + y)$. Mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness.

The Standards set grade-specific standards but do not define the intervention methods or materials necessary to support students who are well below or well above grade-level expectations. It is also beyond the scope of the Standards to define the full range of supports appropriate for English language learners and for students with special needs. At the same time, all students must have the opportunity to learn and meet the same high standards if they are to access the knowledge and skills necessary in their post-school lives. The Standards should be read as allowing for the widest possible range of students to participate fully from the outset, along with appropriate accommodations to ensure maximum participation of students with special education needs. For example, for students with disabilities reading should allow for use of Braille, screen reader technology, or other assistive devices, while writing should include the use of a scribe, computer, or speech-to-text technology. In a similar vein, speaking and listening should be interpreted broadly to include sign language. No set of grade-specific standards can fully reflect the great variety in abilities, needs, learning rates, and achievement levels of students in any given classroom. However, the Standards do provide clear signposts along the way to the goal of college and career readiness for all students.

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8 Mathematical Practices

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

8 Mathematical Practices	
1. Make sense of problems and persevere in solving them.	5. Use appropriate tools strategically.
2. Reason abstractly and quantitatively.	6. Attend to precision.
3. Construct viable arguments and critique the reasoning of others.	7. Look for and make use of structure.
4. Model with Mathematics.	8. Look for and express regularity in repeated reasoning.

http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf, pages 3,4 and 8

A Balanced Combination

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction. **The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.** In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf, pages 3,4 and 8

Algebra 1 Mathematics Curriculum

Manchester High School
Common Core State Standards
Mathematics Curriculum

Description of Standards

The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in fourth credit courses or advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+). All standards without a (+) symbol should be in the common mathematics curriculum for all college and career ready students. Standards with a (+) symbol may also appear in courses intended for all students.

The high school standards are listed in conceptual categories including Number and Quantity, Algebra, Functions, Modeling, Geometry, and Statistics and Probability, and Contemporary Mathematics.
Conceptual categories portray a coherent view of high school mathematics; a student's work with functions, for example, crosses a number of traditional course boundaries, potentially up through and including calculus. Modeling is best interpreted not as a collection of isolated topics but in relation to other standards.

Number and Quantity • The Real Number System (N-RN) • Quantities (N-Q) • The Complex Number System (N-CN) • Vector and Matrix Quantities (N-VM) **Algebra** • Seeing Structure in Expressions (A-SSE) • Arithmetic with Polynomials and Rational Expressions (A-APR) • Creating Equations (A-CED) • Reasoning with Equations and Inequalities (A-REI) **Functions** • Interpreting Functions (F-IF) • Building Functions (F-BF) • Linear, Quadratic, and Exponential Models (F-LE) • Trigonometric Functions (F-TF)	**Geometry** • Congruence (G-CO) • Similarity, Right Triangles, and Trigonometry (G-SRT) • Circles (G-C) • Expressing Geometric Properties with Equations (G-GPE) • Geometric Measurement and Dimension (G-GMD) • Modeling with Geometry (G-MG) **Modeling** **Statistics and Probability** • Interpreting Categorical and Quantitative Data (S-ID) • Making Inferences and Justifying Conclusions (S-IC) • Conditional Probability and the Rules of Probability (S-CP) • Using Probability to Make Decisions (S-MD) **Contemporary Mathematics** • Discrete Mathematics (CM-DM)

NEASC CURRICULUM RESPONSIBILITY CHECKLIST

	ART	BUSINESS & FINANCE	CAREER SERVICES	ENGLISH	FAMILY & CONSUMER SCIENCE	GUIDANCE	LIBRARY	HF AI TH	MATHEMATICS	PERFORMING ARTS	PHYSICAL EDUCATION	SCIENCE	SOCIAL STUDIES	TECHNOLOGY	WORLD LANGUAGES
Academic Expectations All students will:															
1. Reads and understand literary, informational, and persuasive texts in order to analyze, interpret, and evaluate them.				1	1			1	2			2	1		2
2. produce written materials which develop thoughts, share information, influence, persuade, and entertain.				1	2			1	2			2	1	2	1
3. comprehend verbal and nonverbal presentations at the literal, inferential and evaluative levels, and speak using appropriate conventions, forms of expression, and tools.	2			1	1		2	2	2		1		2	2	1
4. comprehend and demonstrate number sense, probability and statistics, geometry, and algebra.				2	2				1			2		2	
5. understand and apply basic concepts, principles, and theories of biology and physical science, and their interrelationships.				2	1			1			1	1		2	
6. identify and apply appropriate technologies to conduct research, communicate information and ideas, produce original works, organize data, and solve problems.	2	1		1	2		1		2			2	2	1	2
7. demonstrate abstract thinking, curiosity, imagination, and creativity in the arts or through academic problem solving.	1	2		1	1				2	1	1	2	2	2	

NEASC CURRICULUM RESPONSIBILITY CHECKLIST

	ART	BUSINESS & FINANCE	CAREER SERVICES	ENGLISH	FAMILY & CONSUMER SCIENCE	GUIDANCE	HF AI TH	LIBRARY	MATHEMATICS	PERFORMING ARTS	PHYSICAL EDUCATION	SCIENCE	SOCIAL STUDIES	TECHNOLOGY	WORLD LANGUAGES
Civic and Social Expectations All students will:															
1. treat others with respect regardless of race, ethnicity, class, or orientation.	1	1				2	1		2	2	1		1		
2. demonstrate ethical behavior through honesty and integrity.	1	1				2	1	2	2	2	1		1		
3. be responsible for their words, actions, and commitments.	1	1				2	1		2	2	1		1		
4. contribute positively to the community.	1	1				1	2			1	2		2		
5. contribute to the school by participating in co-curricular or service activities.	2	2				1	2			1	1		2		

MHS Math Department
Curriculum Syllabus
Common Core State Standards 2012
Algebra 1

MONTH	CURRICULUM TOPICS
September - Quarter 1 **Unit 1** **Relationship Between Quantities and Reasoning with Equations**	• **Reason quantitatively and use units to solve problems -** (N.Q.1, N.Q.2, N.Q.3) • **Create equations that describe numbers or relationships -** (A.CED.1, A.CED.2, A.CED.3, A.CED.4) • **Interpret the structure of expressions -** (A.SSE.1.a, A.SSE.2.b)
October **Unit 2** **Linear and Exponential Relationships**	• **Solve equations and inequalities in one variable -** (A.REI.3) • **Understand solving equations as a process of reasoning and explain the reasoning -** (A.REI.1) • **Represent and solve equations and inequalities graphically -** (A.REI.10, A.REI.11, A.REI.12) • **Solve systems of equations -** (A.REI.5, A.REI.6)
November (Quarter 2)	• **Understand the concept of a function and use function notation -** (F.IF.1, F.IF.2, F.IF.3) • **Interpret functions that arise in applications in terms of a context -** (F.IF.4, F.IF.5, F.IF.6) • **Build a function that models a relationship between two quantities -** (F.BF.1, F.BF.2) • **Analyze functions using different representations -** (F.IF.7.a, F.IF.7.e, F.IF.9)
December	• **Build new functions from existing functions -** (F.BF.3) • **Construct and compare linear, quadratic, and exponential models and solve problems** (F.LE.1.a, F.LE.1.b, F.LE.1.c, F.LE.2, F.LE.3) • **Extend the properties of exponents to rational exponents -** (N.RN.1, N.RN.2)
January (Quarter 3) **Unit 4** **Expressions and Equations**	• **Interpret expressions for functions in terms of the situation the model -** (F.LE.5) ***MID-YEAR EXAM*** • **Write expressions in equivalent forms to solve problems -** (A.SSE.3.a, A.SSE.3.b, A.SSE.3.c) • **Interpret the structure of expressions -** (A.SSE.1.a, A.SSE.2.b, A.SSE.2) • **Perform arithmetic operations on polynomials -** (A.APR.1)

MHS Math Department
Curriculum Syllabus
Common Core State Standards 2012
Algebra 1

MONTH	CURRICULUM TOPICS
February	• **Create equations that describe numbers or relationships -** (A.CED.1, A.CED.2, A.CED.4) • **Solve equations and inequalities in one variable -** (A.REI.4.a, A.REI.4.b) • **Solve systems of equations -** (A.REI.7)
March **Unit 5** **Quadratic Functions and Modeling**	• **Use properties of rational and irrational numbers -** (N.RN.3) • **Interpret functions that arise in applications in terms of a context -** (F.IF.4, F.IF.5, F.IF.6) • **Analyze functions using different representations -** (F.IF.7.a, F.IF.7.b, F.IF.8.a, F.IF.8.b, F.IF.9)
April (Quarter 4)	• **Build a function that models a relationship between two quantities -** (F.BF.1.a, F.BF.1.b) • **Build new functions from existing functions -** (F.BF.3, F.BF.4.a) • **Construct and compare linear, quadratic, and exponential models and solve problems -** (F.LE.3)
May **Unit 3** **Descriptive Statistics**	• **Summarize, represent, and interpret data on a single count or measurement variable -** (S.ID.1, S.ID.2, S.ID.3) • **Summarize, represent, and interpret data on two categorical and quantitative variables -** (S.ID.5, S.ID.6.a, S.ID.6.b, S.ID.6.c)
June	• **Interpret linear models -** (S.ID.7, S.ID.8, S.ID.9)

MHS Math Department
Curriculum Syllabus
Common Core State Standards 2012
Algebra 1

UNIT 1 Pacing Guide – 15 days

- Summer packet review through solving multi-step equations (2 days)
- Unit conversion, forms of equations, and Jaclyn's Garden activity (3 days)
- Linear, Quadratic and Exponential comparisons and Investigation activity (4 days)
- Linear equations, inequalities in one variable and systems of equations (4 days)
- Review and unit assessment (2 days)

UNIT 2 Pacing Guide – 17 days

- Model situations with systems of equations and inequalities, Inequalities Exploration and Linear Programming (3 days)
- Pattern interpretation and development of equations and functions, Patterns to Functions Activity, Model Comparison Activity, Building Functions Activities (4 days)
- Function characteristics – notation, domain, range, graphs, Function Stations (3 days)
- Parameters and translations of functions, Graph Comparisons Activity, Function Investigation (2 days)
- Review and unit assessment (2 days)
- Rational exponents exploration (3 days)

UNIT 4 Pacing Guide – 21 days

- Introduction to polynomials, Polynomial Powers activity (4 days)
- Factor and solve polynomial expressions and equations, factor, complete the square, quadratic formula, Parking Lot problem (5 days)
- Solving quadratic equations, completing the square, quadratic formula, applications, Domino activity, Graphing Stations activity, Roots check-in, Method Analysis activity (8 days)

- Systems of linear and quadratic equations, Highway Problem (2 days)
- Review and unit assessment (2 days)

UNIT 5 Pacing Guide – 18 days

- Model and interpret situations with quadratic functions, Group Activity (3 days)
- Properties of rational and irrational numbers (2 days)
- Create and compare function characteristics, graphs, and parameters (5 days)
- Comparison of families of functions symbolically and graphically (3 days)
- Comparison of functions and their inverses symbolically and graphically (3 days)
- Review and unit assessment (2 days)

UNIT 3 Pacing Guide – 15 days

- Data representations, histograms, box plots, line plots (2 days)
- Interpretation of data from various representations (2 days)
- Representation and Interpretation of categorical data (2 days)
- Representation and interpretation of quantitative data, scatterplot, correlation, regression, College Tuition Performance Task (5 days)
- Comparison of correlation and causation (2 days)
- Review and unit assessment (2 days)

MHS Algebra 1 Mathematics Curriculum

Unit 1 – Relationships Between Quantities and Reasoning with Equations

Domain: Number and Quantity (N)

Essential Question: How do expressions and equations help us model and solve problems?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Translate quantitative relationships into equations to model situations and use algebraic reasoning to solve.** **N-Q(***)** Reason quantitatively and use units to solve problems	**How…** **N.Q.1:** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data display **N.Q.2:** Define appropriate quantities for the purpose of descriptive modeling **N.Q.3:** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities	**Students will:** **Fastest Runner Worksheet** Fastest runner problem (CCSS).docx **System of Equations with Graphing** System problem (CCSS).docx	**N.Q.1:** Include word problems where quantities are given in different units, which must be converted to make sense of the problem. For example, a problem might have an object moving 12 foot per second and another at 5 miles per hour. To compare speeds, students convert 12 feet per second to miles per hour: $24000\ \text{sec} \bullet \frac{1\text{min}}{60\text{sec}} \bullet \frac{1\text{hr}}{60\text{min}} \bullet \frac{1\text{day}}{24\text{hr}}$ which is more than 8 miles per hour. Graphical representations and data displays include, but are not limited to: line graphs, circle graphs, histograms, multi-line graphs, scatterplots, and multi-bar graphs.

Vocabulary: absolute value, coefficients, constant, equations, exponents, factor, inequalities, linear, parts of expressions, problems, properties of exponents, quadratic, radical expressions, rational exponents, solution method, terms, variable

Name: _____ Date: _____ Period: _____

Numerical Reasoning Open Ended #1

Manchester Road Race

Three friends decided to run the Manchester Road Race together.

The table shows the average speed for each of them.

Runner	Average Speed
Katelyn	228 inches per second
Tatyana	10.0 miles per hour
Danasia	17 feet per second

Which friend crossed the finish line first?

_____

_____

_____

_____

_____

Name: _____ Date: _____ Period: _____

Numerical Reasoning Open Ended #2

NFL Combine

The top three touchdown scoring wide receivers in the NFL for 2011 are listed below, along with their 40yd dash time from the NFL Combine and TD receptions from 2011.

Player Name	2011 Receiving TD's	40 yard dash time
Calvin Johnson	16	1655 feet per min
Jordy Nelson	15	8.8 yards per sec
Laurent Robinson	11	18.7 miles per hour

Based on the data above, who is the faster wide receiver? Do you think speed had anything to do with the number of touchdowns they scored?

_____

_____

_____

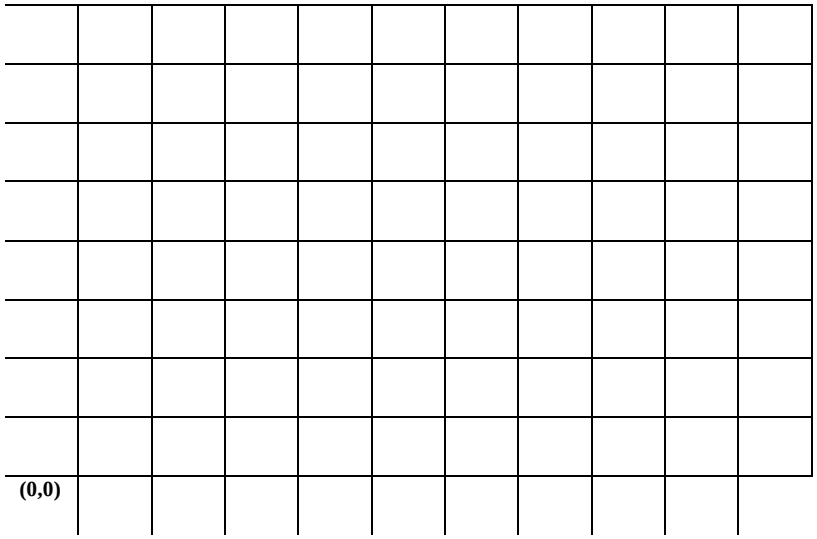
_____

_____

Students at Manchester High School want to have a soda machine. The principal agrees to install one provided the students maintain it. The students found a used machine on E-bay for $120 and a supplier that charges 45 cents a can. The students will then decide to sell soda for 75 cents a can.

A. Write an equation to represent the cost of the machine and the soda.
B. Write an equation to represent the money the machine will collect from selling cans of soda.
C. Create a graph to compare the cost (money spent) with the revenue (money made) from selling soda.
D. From your graph, estimate how many cans of soda must be sold before the students begin to make a profit.
E. Determine, algebraically, how many cans of soda must be sold before the students begin to make a profit.

If the students expect to sell 35 cans per day, should they invest in the soda machine? Explain your thoughts.



MHS Algebra 1 Mathematics Curriculum

Unit 1 - Relationships Between Quantities and Reasoning with Equations

Domain: <u>Algebra</u> (A)

Essential Question: How do expressions and equations help us model and solve problems?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

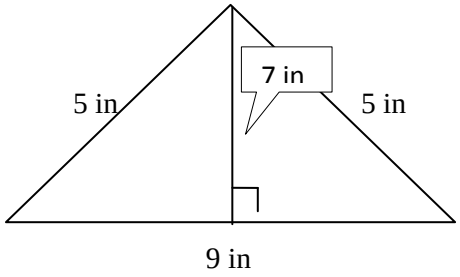
Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Translate quantitative relationships into equations to model situations and use algebraic reasoning to solve.** **A-SSE(***)** Interpret the structure of expressions	**How…** **A.SSE.1:** Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity.	**Students will:** **Equations and Variables sheet** Understanding Equations and Varialb **Formula manipulation activity** Common Core MR A.SSE.1 + A.CED.4.c	**A.SSE.1a** Identify the different parts of the expression and explain their meaning within the context of a problem. **A.SSE.1b** Decompose expressions and make sense of the multiple factors and terms by explaining the meaning of the individual parts. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*

Vocabulary: absolute value, coefficients, constant, equations, exponents, factor, inequalities, linear, parts of expressions, problems, properties of exponents, quadratic, radical expressions, rational exponents, solution method, terms, variable

Interpreting variables in context and rearranging formulas to highlight a quantity of interest.

Example One:

The formula of a triangle is $A = \frac{1}{2}bh$.



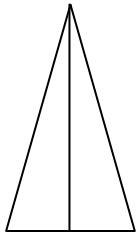
What variable represents the height? _____ base?_____

What is the height? _____ base? _____

Use the formula to solve for the area:

Example Two:

The area of the following triangle measures $45\,in^2$. $A = \frac{1}{2}bh$



What operations are being performed on the variable *h*? _____

What operations would be needed to insolate and solve for height?

Example Three:

The following is the perimeter formula for a rectangle: $P = 2L + 2W$.

What operations are being performed in the equation? _____ and _____

What operations would we perform in order to solve for *w*? _____ and _____

Solve for *w* (Isolate the variable):

Example Four:

If I need to convert Celsius to Fahrenheit, I know I have to use the formula: $F = \frac{9}{5}C + 32$.

What operations are being performed in this equation?

_____ and _____

If we were to solve for C, which operations would we have to perform in order to isolate C?

Solve for C:

Example Five:

The following formula is used to find the volume of a cylinder: $V = \pi r^2 h$

What operations are used in this formula?

_____ and _____

What are their inverses (opposite)?

_____ and _____

If we were to solve for the variable *h*, what would the rearranged equation look like?

If we were to solve for the variable r, what would the rearranged equation look like?

Instructional Activity Course and Unit	**Teaching the Standard** **Example #1: Algebra One Unit 1 A.CED.4 + A.SSE.1**
Standard(s):	A.SSE.1: Interpret expressions that represent a quantity in terms of its context a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing on or more of their parts as a single entity. A.CED.4: Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations
Objective(s):	Students will be able to: Interpret and rearrange, expression and formulas to highlight a quantity of interest to analyze part of the expressions and formulas.
Prior knowledge:	Students should have a basic understand of a single variable equation and how to solve.
Procedure: **Teacher Demonstrates** **"I Do"**	Introduce the following equation: A persons weight in lbs. (w) can be predicted based by the persons height in inches (h) by using the equation w = 1.7h +15. Class will discuss the coefficient, variable, constant, and factor. Class will discuss the difference between an equation and expression. Teacher will isolate the variable one step problem: for example isolate r for $C = 2\pi r$ Teacher will isolate the variable two step problem: for example isolate r for $T = 8.25h + 25$ Teacher will isolate the variable factoring problems: for example isolate r for $P = 2(L + W)$
Guided Practice: **"We Do"**	Have students solve for specific variables from the CAPT formula chart.
Closure	

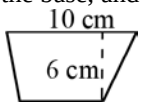
Unit 1 - Relationships Between Quantities and Reasoning with Equations

Domain: Algebra (A)

Essential Question: How do expressions and equations help us model and solve problems?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Translate quantitative relationships into equations to model situations and use algebraic reasoning to solve.** **A-CED(***)** Create equations that describe numbers or relationship	**How…** **A.CED.1:** Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* **A.CED.2:** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	**Students will:** **Equations and Modeling** ***Algebra 1 Station Activities (Reasoning with Equations-set 2 pg.119)*** **Equation Investigation** Investigation Pre-Notes.docx Investigation Unit 1.docx	**A.CED.1:** Equations can represent real world and mathematical problems. Include equations and inequalities that arise when comparing the values of two different functions, such as one describing linear growth and one describing exponential growth. Examples: • Given that the following trapezoid has area 54 cm^2, set up an equation to find the length of the base, and solve the equation.  Lava coming from the eruption of a volcano follows a parabolic path. The height h in feet of a piece of lava t seconds after it is ejected from the volcano is given by $h(t) = -t^2 + 16t + 936$. After how many seconds does the lava reach its maximum height of 1000 ft?

Vocabulary: absolute value, coefficients, constant, equations, exponents, factor, inequalities, linear, parts of expressions, problems, properties of exponents, quadratic, radical expressions, rational exponents, solution method, terms, variable

CCSS Algebra 1 Unit 1 – Equation Investigation PRE-NOTES
(A.CED.1. A.CED.2)

Preliminary Instruction Needed:

Equations and descriptions of linear, quadratic, and exponential functions:

- Discussion of modeling and application of each type of equation
- Problem solving with linear equations (graph, table and equation)
- Problem solving with quadratic and exponential equations (table of values)

Teacher Guidelines:

Students will compare linear, quadratic, and exponential equations developed from situations.

Students will use graphs and tables to determine solutions to equations.

Students will determine appropriate equations to model situations and use various methods to solve.

TYPES OF SITUATIONS TO CONSIDER:

<u>LINEAR ($y=mx+b$):</u>

- Constant rate of change
- Cost equations with set parameters

<u>QUADRATIC ($y=ax^2+bx+c$):</u>

- Area of rectangles or triangles with set length and width relationships

<u>EXPONENTIAL ($y=ab^x$):</u>

- Value of investment doubles, triples, or increases by a percent
- Half-life of radioactive element
- Compound interest formula

a.) The length of a photograph is 3 cm less than twice the width. The area is $54cm^2$. Find the dimensions of the photograph.

Classification:

Equation:

Solution:

b.) Peter purchases a baseball card for $150 as an investment. His research tells him that the value of the card should double every 3 years. When will the card reach a value of $4000?

Classification:

Equation:

Solution:

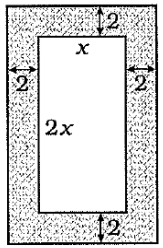
c.) Josiah collects donation for a walk-a-thon at school. He asks his aunt to donate $5 to start and then contribute $1.25 for each mile that Josiah walks. How many miles will he walk if his aunt's total contribution is $10?

Classification:

Equation:

Solution:

d.) Suppose you are designing a rectangular garden as shown in figure. The length of the garden is twice the width. You also want a 2 foot walkway surrounding the garden. If the area of the walkway is 136 square feet, find the width of the garden.



Classification:

Equation:

Solution:

e.) Cesium-137 has a half-life of 30 years. Suppose a lab stored a 100 gram sample in 1970. How many grams will remain in the year 2012?

Classification:

Equation:

Solution:

f.) Sarita earns $250 weekly in her job as a salesperson. She also earns 40% of the totals sales for the week. How much will she need to sell if she wants to earn a total of $600 weekly?

Classification:

Equation:

Solution:

CCSS Algebra 1 Unit 1 – Equation Investigation
(A.CED.1. A.CED.2)

Preliminary Instruction Needed:

Equations and descriptions of linear, quadratic, and exponential functions:

- Discussion of modeling and application of each type of equation
- Problem solving with linear equations (graph, table and equation)
- Problem solving with quadratic and exponential equations (table of values)

Description of Activity:

Students will determine appropriate equations to model situations described and use various methods to solve.

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b.) Peter purchases a baseball card for $150 as an investment. His research tells him that the value of the card should double every 3 years. When will the card reach a value of $4000?

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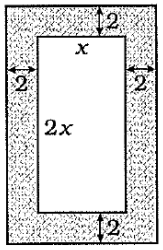

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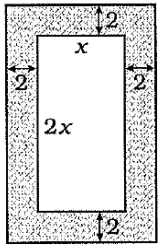

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Equation:

Solution:

MHS Algebra 1 Mathematics Curriculum

Unit 1 - Relationships Between Quantities and Reasoning with Equations

Domain: <u>Algebra</u> (A)

Essential Question: How do expressions and equations help us model and solve problems?

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Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Translate quantitative relationships into equations to model situations and use algebraic reasoning to solve.** **A-CED(***)** Create equations that describe numbers or relationships	**How…** **A.CED.3:** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*	**Students will:** ***Utah uen.org website resource*** **Inequalities Practice Activity** Inequalities Sheet.docx **Linear Programming Groups** A.CED.3 Review.docx A.CED.3 lin prog apps.docx	**A.CED.3**: Example: • A club is selling hats and jackets as a fundraiser. Their budget is $1500 and they want to order at least 250 items. They must buy at least as many hats as they buy jackets. Each hat costs $5 and each jacket costs $8. • Write a system of inequalities to represent the situation. • Graph the inequalities. • If the club buys 150 hats and 100 jackets, will the conditions be satisfied? • What is the maximum number of jackets they can buy and still meet the conditions?

Vocabulary: absolute value, coefficients, constant, equations, exponents, factor, inequalities, linear, parts of expressions, problems, properties of exponents, quadratic, radical expressions, rational exponents, solution method, terms, variable

Situation #2. Rub-a-dub-dub.

Kara is filling her bathtub. The cold water flows at a rate of 4 gal/min. The hot water flows at a rate of 3 gal/min. Kara wants no more than 60 gal of water in the tub.

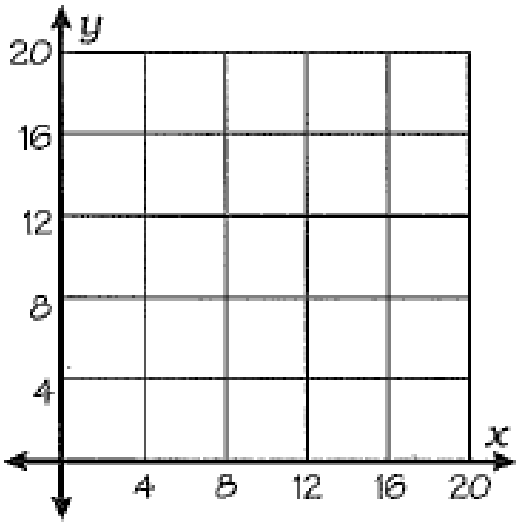
Let x = time that cold water is turned on
Let y = time that hot water is turned on

inequality: _____

4. Which of the following is a solution of the inequality:
a. (5,16) **b.** (10,4) **c.** (12,5)

5. How many minutes will it take to get 60 gal of water if only cold water is turned on?

6. If x = 3 min, what are all possible values of y?



The Jolt Bakery

Jack and Jill Jolt own a small bakery that makes fresh cookies daily. They bake two kinds of cookies – plain cookies and cookies with icing. The cookies are sold by the box, and Jack and Jill are confident they can sell all the cookies they make. Buit there are three constraints that limit their production today.

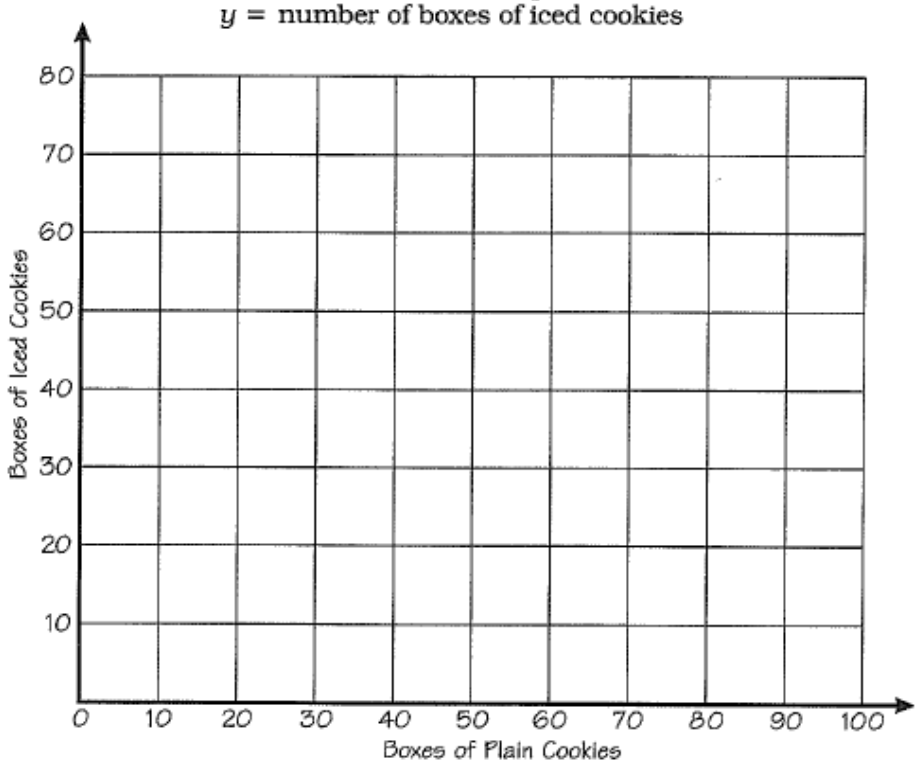
DOUGH: One box of plain cookies requires 1.2 pounds of cookie dough.
One box of iced cookies requires 0.9 pound of cookie dough.
Jack and Jill have 72 pounds of cookie dough.

ICING: Plain cookies require no icing.
One box of iced cookies requires 0.4 pound of icing.
Jack and Jill have 20 pounds of icing.

TIME: One box of plain cookies requires about 0.10 hour to prepare.
One box of iced cookies requires about 0.15 hour to prepare
Jack and Jill together have 9 hours for preparation.

Write three inequalities. Then find the intersection of these inequalities to show all combinations of cookies that Jack and Jill can make with the constraints given.

Let x = number of boxes of plain cookies
y = number of boxes of iced cookies



EXTRA: Suppose the profit on each box of plain cookies is $3.00, and the profit on each box of iced cookies is $4.00. How many boxes of each kind of cookies should Jack and Jill make to maximize profit?

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What Do You Call a New Movie That Is Just Like an Old Movie?

Write and graph a system of inequalities that models the situation. Circle the number-letter pair for each ordered pair that is a solution. Write the letter in the matching numbered box at the bottom.

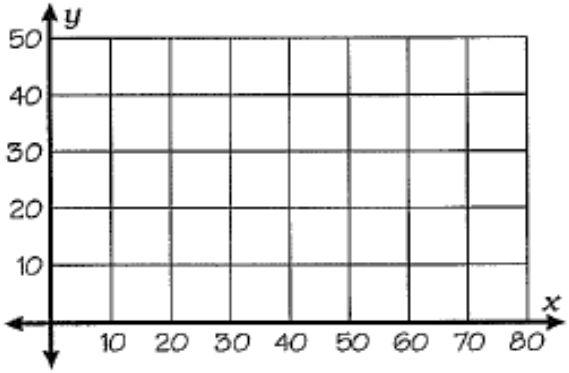
Situation 1. SOMETHING FISHY.

The owner of Fred's Fish Market orders cod and salmon. He wants to buy at least 50 pounds of fish but cannot spend more than $300. Cod is $4 per pound and salmon is $7 per pound.

Let x = number of pounds of cod
Let y = number of pounds of salmon

inequality #1: _____

inequality #2: _____



Which of the following are solutions?

8•E (40, 15) **11•P** (50, 18) **4•S** (30, 20) **10•U** (55, 8) **7•R** (20, 35)

Situation 2. FLOWER POWER.

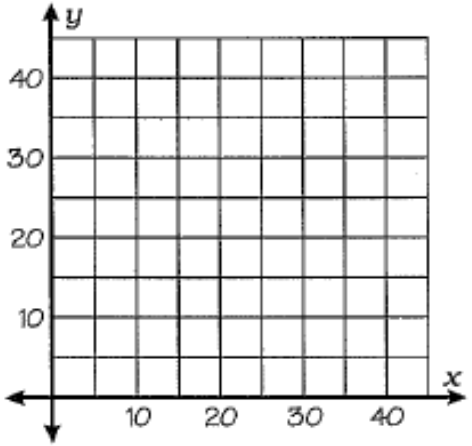
Mr. Bloom is designing a rectangular flower garden with a fence around it. He can use no more than 80 ft of fencing. He wants the width to be at least 5 ft and the length to be at least 20 ft.

Let x = width of the garden (ft)
Let y = length of the garden (ft)

inequality #1: _____

inequality #2: _____

inequality #3: _____



Which of the following are solutions?

7•S (10, 23) **11•E** (7, 30) **9•T** (18, 25) **3•A** (8, 35) **2•I** (20, 20)

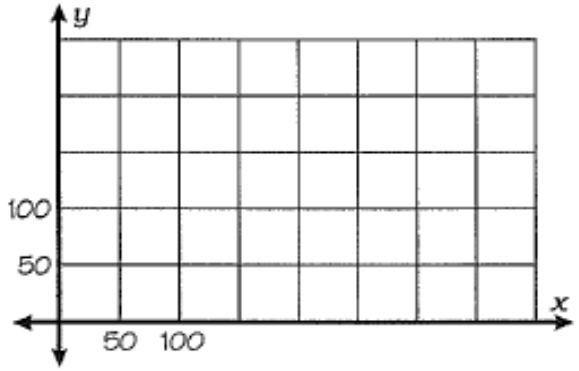
Situation 3. SPRING FLING.

Tickets for the Spring Dance cost \$3 per person or \$5 per couple. To cover expenses, at least \$750 worth of tickets must be sold. However, no more than 400 people can fit in the gym where the dance is being held.

Let x = number of \$3 tickets sold
Let y = number of \$5 tickets sold

inequality #1: _____

inequality #2: _____



Which of the following are solutions?

5·H (50, 110) **12·L** (150, 70) **9·Q** (280, 45) **6·U** (300, 60) **3·T** (0, 200)

1	2	3	4	5	6	7	8	9	10	11	12	13

Algebra 2 – Unit 2 Review
Inequalities/Linear Programming

1. Give a point that would be solution to the inequality $y > 3x - 4$.

2. Give a point that would not be a solution to the inequality $3x - 4y > 12$.

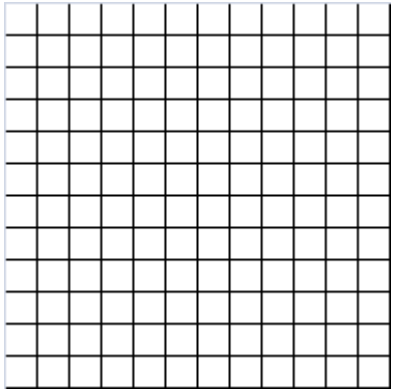
3. Determine if the point (2, -3) is a solution to the following system of inequalities. Why or why not?

$$y < 2x - 5$$
$$2x + y > -3$$

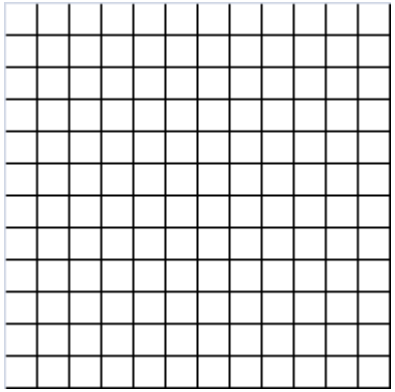
_____
_____
_____

In Questions 4-5 Graph the system of inequalities.

4. $y \geq x - 3$
   $y \geq -x + 1$



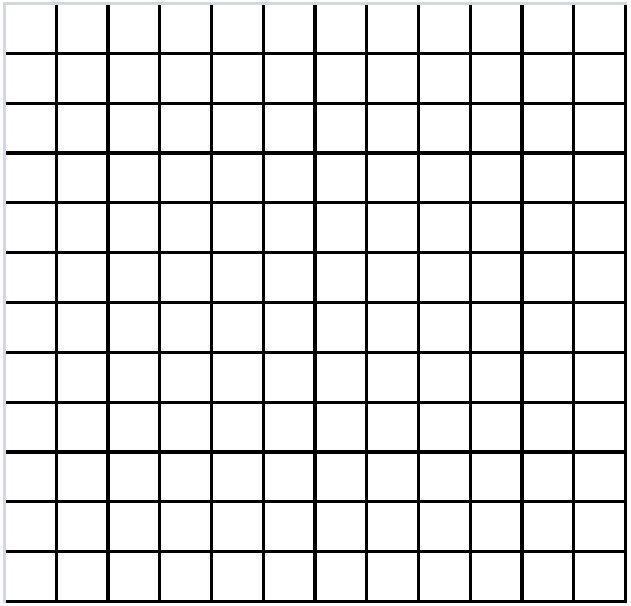
5. $x + 2y \geq 7$
   $3x - 4y < 12$



In Questions 6-7 Graph each system of inequalities, name the coordinates of the feasible region, and find the max and min values of the given function.

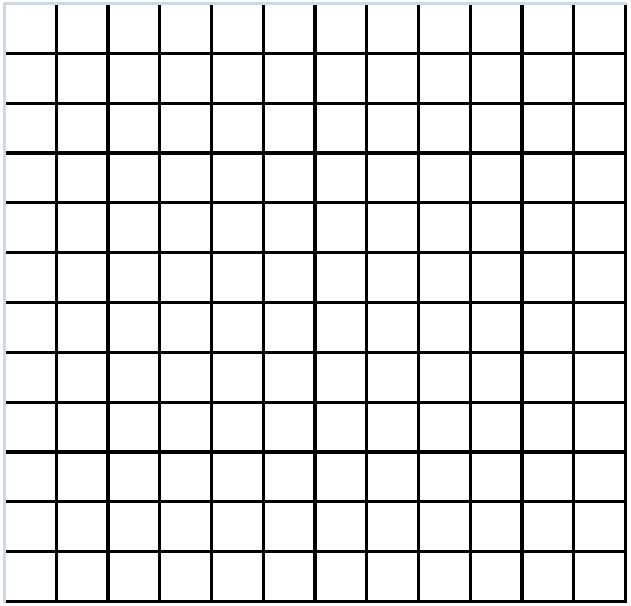
6. $y \leq 5$
   $y \geq -3$
   $4x + y \leq 5$
   $-2x + y \leq 5$

$f(x, y) = 4x - 3y$



7. $y \leq 4x + 6$
   $x + 4y \leq 7$
   $2x + y \leq 7$
   $x - 6y \leq 10$

$f(x, y) = 2x - y$



In questions 8-9

i. ***write a system of inequalities that represents the problem***
ii. ***graph the inequalities, showing the feasible region and labeling its vertices***
iii. ***create a function for optimization***
iv. ***determine the maximum and minimum***

8. A shoe manufacturer makes cleats and turfs for soccer. There is a two-step process required when manufacturing both types of shoes. Each pair of cleats requires 2 hours in step on and one hour in step two. Turfs require 1 hour in step 1 and 3 hours in step 2. Cleats produce a profit of $20, while turfs produce a profit of $15. If the company has only 40 hours of labor per day for step 1 and 60 hours of labor for step 2, how many of each should they produce?

9. MHS has contracted with a hypnotist to perform at school. The school has guaranteed an attendance of at least 1000 people and a total ticket receipt of at least $4800. The tickets are $4 for students and $6 for non-students. Of that money the hypnotist will receive $2.50 and $4.50 respectively. What would be the minimum amount that the hypnotist will get paid?

Rosalyn works no more than 20 hours a week during the school year. She is paid $10 an hour for tutoring geometry students and $7 an hour for delivering pizzas for Pizza Pal. She wants to spend at least 3 hours but no more than 8 hours a week tutoring. Find Rosalyn's maximum earnings.

Farmer Bill has 20 days in which to plant corn and soybeans. The corn can be planted at a rate of 60 acres per day and the soybeans at a rate of 70 acres per day. He has 1300 acres available to plant these two crops. If he makes a profit of $30 for every acre of corn and $26 for every acre of soybean then how much of each should he plant to maximize his profit?

1. A farmer has 10 acres to plant in wheat and rye. He has to plant at least 7 acres. However, he has only $1200 to spend and each acre of wheat costs $200 to plant and each acre of rye costs $100 to plant. Moreover, the farmer has to get the planting done in 12 hours and it takes an hour to plant an acre of wheat and 2 hours to plant an acre of rye. If the profit is $500 per acre of wheat and $300 per acre of rye how many acres of each should be planted to maximize profits?

2. A gold processor has two sources of gold ore, source A and source B. In order to keep his plant running, at least three tons of ore must be processed each day. Ore from source A costs $20 per ton to process, and ore from source B costs $10 per ton to process. Costs must be kept to less than $80 per day. Moreover, Federal Regulations require that the amount of ore from source B cannot exceed twice the amount of ore from source A. If ore from source A yields 2 oz. of gold per ton, and ore from source B yields 3 oz. of gold per ton, how many tons of ore from both sources must be processed each day to maximize the amount of gold extracted subject to the above constraints?

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Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Translate quantitative relationships into equations to model situations and use algebraic reasoning to solve.** **A-CED(***)** Create equations that describe numbers or relationships	**How…** **A.CED.4:** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*	**Students will:** **Multiple Variables Worksheet** Create equations that describe number **Equations and Variables Problem Solving** Jaclyn's garden.docx	**A.CED.4: Examples:** • The Pythagorean Theorem expresses the relation between the legs a and b of a right triangle and its hypotenuse c with the equation $a^2 + b^2 = c^2$. • Why might the theorem need to be solved for c? • Solve the equation for c and write a problem situation where this form of the equation might be useful. • Solve $V = \frac{4}{3}\pi r^3$ for radius *r*. • Motion can be described by the formula below, where *t* = time elapsed, *u*=initial velocity, *a* = acceleration, and *s* = distance traveled • $s = ut + \frac{1}{2}at^2$ • Why might the equation need to be rewritten in terms of *a*? • Rewrite the equation in terms of *a*.

Vocabulary: absolute value, coefficients, constant, equations, exponents, factor, inequalities, linear, parts of expressions, problems, properties of exponents, quadratic, radical expressions, rational exponents, solution method, terms, variable

Create equations that describe numbers or relationships.

1.) If given the formula $D = RT$, solve for the rate.

2.) When given the formula for Fahrenheit $F = \frac{9}{5}C + 32$, solve for Celsius.

3.) The Pythagorean Theorem expresses the relation between the legs a and b of a right triangle and its hypotenuse c with the equation $a^2 + b^2 = c^2$. Solve for one of the legs, given the hypotenuse.

4.) Given the volume of a sphere, $V = \frac{4}{3}\pi r^3$ solve for radius r.

5.) Motion can be described by the formula below, where t = time elapsed, u=initial velocity, a = acceleration, and s = distance traveled

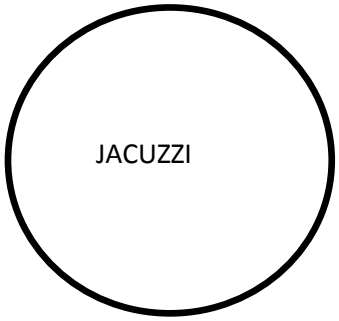
$$s = ut + \frac{1}{2}at^2$$

Why might the equation need to be rewritten in terms of a?
Rewrite the equation in terms of a.

Group Activity:

Jaclyn's Garden

(A.SSE.1, N.Q.1, N.Q.2, A.CED.4)



The above diagram shows Jaclyn's backyard. She is going to fill in her pool and jacuzzi and replace it with a garden and flower bed.

Jaclyn knows her pools volume is 544 ft^3. She also remembers her pool was only four feet deep.

She also wants to turn her jacuzzi into a flower bed. She knows the volume of her jacuzzi is 100 yd^3 and that it is only one yard deep.

What would the square footage of the garden and flower bed be if all she remembers is the volume of each and only some of the measurements?

Extension: How much mulch would she need to buy to fill in both the pool and jacuzzi if she needs her mulch to be two inches deep?

Research the price of mulch on the Home Depot website to find out how much mulch she needs to purchase to fill in both garden bed and pool. http://www.homedepot.com/

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8. Look for and express regularity in repeated reasoning.

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Translate quantitative relationships into equations to model situations and use algebraic reasoning to solve.** **A-REI(***)** Understand solving equations as a process of reasoning and explain the reasoning	**How…** **A.REI.1:** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	**Students will:** **Presentation Class work** Multistepequationsslidepresentation.ppt **Equation Problems** ***Algebra 1 Station Activities (Reasoning with Equations-set 1 pg.107)***	**A.REI.1:** Properties of operations can be used to change expressions on either side of the equation to equivalent expressions. In addition, adding the same term to both sides of an equation or multiplying both sides by a non-zero constant produces an equation with the same solutions. Other operations, such as squaring both sides, may produce equations that have extraneous solutions. Examples: • Explain why the equation $x/2 + 7/3 = 5$ has the same solutions as the equation $3x + 14 = 30$. Does this mean that $x/2 + 7/3$ is equal to $3x + 14$? • Show that $x = 2$ and $x = -3$ are solutions to the equation $x^2 + x = 6$. Write the equation in a form that shows these are the only solutions, explaining each step in your reasoning

Vocabulary: absolute value, coefficients, constant, equations, exponents, factor, inequalities, linear, parts of expressions, problems, properties of exponents, quadratic, radical expressions, rational exponents, solution method, terms, variable

Solving Multi-Step Equations

Work Backwards

Multi-step equations can be solved by undoing the operations in reverse of the order of operations.

Steps for Solving Equations

1. Use the Distributive Property to remove the grouping symbols.
2. Simplify the expressions on each side of the equals sign.
3. Use the Addition and /or Subtraction Properties of Equality to get the variables on one side of the equals sign and the numbers without variables on the other side of the equals sign.
4. Simplify the expressions on each side of the equals sign.
5. Use the Multiplication and /or Division Properties of Equalities to solve.

Example 1:

$$3(x-3) = x+11$$

$$3x-9 = x+11$$

Use Distributive Property to remove the grouping symbols

$$2x = 20$$

Use the addition/subtraction properties of equalities to get the variables on one side and the numbers without variables on the other.

$$x=10$$

Use the multiplication/division property of equalities to solve.

Example 2:

$$4t-2(t+3) = t$$

$$4t-2t-6 = t$$ Use the distributive property to remove the grouping symbols.

$$2t-6 = t$$ Simplify the expressions on each side of the equals sign.

$$t = 6$$ Use the multiplication/addition properties of equalities to solve.

Example 3:

$8.3h-2.2 = 6.1h-8.8$

$2.2h = -6.6$

Use the addition/subtraction properties of equality to get the variables on one side of the equal sign and the numbers without variables on the other side of the equal sign.

$h = -3$

Use the multiplication/division properties of equalities to solve.

Work these on your paper.

1. $4n + 8 = -62$
2. $1.5 – 2x = 3.6$
3. $3s/2 + 5 = 17$
4. $-2(2x + 3) = 2x +2$

Check your answers

1. $4n + 8 = -62$
2. $1.5 – 2x = 3.6$
3. $3s/2 + 5 = 17$
4. $-2(2x + 3) = 2x +2$

1. -17.5
2. -1.05
3. 8
4. 7/3

Unit 1 - Relationships Between Quantities and Reasoning with Equations

Domain: Algebra (A)

Essential Question: How do expressions and equations help us model and solve problems?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Translate quantitative relationships into equations to model situations and use algebraic reasoning to solve.** **A-REI(***)** Solve equations and inequalities in one variable	**How…** **A.REI.3:** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	**Students will:** **Multi-step Equation Worksheet** Solve equations and inequalities in one var **Inequality Problems** ***Algebra 1 Station Activities (Reasoning with Equations-set 3 pg. 132)***	**A.REI.3** Solve linear equations in one variable, including coefficients represented by letters. **A.REI.3** Solve linear inequalities in one variable, including coefficients represented by letters. Examples: • $-\frac{7}{3}y-8=111$ • $3x > 9$ • $ax + 7 = 12$ • $\frac{3+x}{7}=\frac{x-9}{4}$ Solve for x: $2/3x + 9 < 18$

Vocabulary: absolute value, coefficients, constant, equations, exponents, factor, inequalities, linear, parts of expressions, problems, properties of exponents, quadratic, radical expressions, rational exponents, solution method, terms, variable

Solve equations and inequalities in one variable

Name:_____ Date:_____

Multi-step equations Quiz. SHOW ALL WORK for full credit.

1. $-5x+13=-17$

2. $4x + 6 - 7x = -15$

3. $6+\frac{x}{2}=4$

4. $-3.8-13.4p=-460.606$

5. $2(x+3) = 12$

6. $9r + 15 \geq 24 + 10r$

7. $x \leq \frac{2x-15}{3}$

8. Sue went to the mall and bought a scarf for $12 and 2 pairs of jeans. If she spent $37 total, how much did each pair of jeans cost?

9. Solve for m. $12 = mx + 6$

MHS Algebra 1 Mathematics Curriculum

Unit 2 – Linear and Exponential Relationships

Domain: Number and Quantity (N)

Essential Question: How do linear relationships help model non-linear functions?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Compare and contrast linear and non-linear functions to develop appropriate models for given situations.** **N-RN**(***) Extend the properties of exponents to rational exponents	**How…** **N.RN.1:** Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.* **N.RN.2:** Rewrite expressions involving radicals and rational exponents using the properties of exponents.	**Students will:** **Rational Exponents** N.RN.1 (Unit 2).docx Fractional Exponents **Exponent Sample Questions** U6L1.ExponentProperties-MultDiv.pdf	**N.RN.2:** $\sqrt[3]{5^2} = 5^{\frac{2}{3}}$; $5^{\frac{2}{3}} = \sqrt[3]{5^2}$ Rewrite using fractional exponents: $\sqrt[5]{16} = \sqrt[5]{2^4} = 2^{\frac{4}{5}}$ Rewrite $\frac{\sqrt{x}}{x^2}$ in at least three alternate forms. Solution: $x^{-\frac{3}{2}} = \frac{1}{x^{\frac{3}{2}}} = \frac{1}{\sqrt{x^3}} = \frac{1}{x\sqrt{x}}$ Rewrite $\sqrt[4]{2^{-4}}$ .using only rational exponents. Rewrite $\sqrt[3]{x^3+3x^2+3x+1}$ in simplest form.

Vocabulary: Arithmetic and geometric sequences, arithmetic and geometric situations, base, effect on graph, end behavior, equal distance over equal intervals, equal factor over equal intervals, even and odd functions, functional inputs, function notation, graphs, graphical effects, intercepts, intervals, parameters of linear/exp, recursive and explicit formulae, relative maximum/minimum, slope, symmetry

Exploring Fractional Exponents and Matching Activity

Name:_____Date:_____Period:_____

We have dealt with exponents as whole numbers.

Now let's explore what happens if the exponent is a fraction?

Fractional Exponents

An exponent of $^1/_2$ is actually **square root** $4^{\frac{1}{2}} = \sqrt{4}$

And an exponent of $^1/_3$ is **cube root** $4^{\frac{1}{3}} = \sqrt[3]{4}$

An exponent of $^1/_4$ is **4th root** $4^{\frac{1}{4}} = \sqrt[4]{4}$

And so on!

etc...

Why?

Let's see why in an example.

First, the Laws of Exponents tell us how to handle exponents when we multiply:

Example: $x^2x^2 = (xx)(xx) = xxxx = x^4$

Which shows that $\mathbf{x^2x^2 = x^{(2+2)} = x^4}$

TRY:

What is $9^{\frac{1}{2}} \bullet 9^{\frac{1}{2}}$?

What is $\left(3^5\right)^{\frac{1}{5}}$?

How many time do you need to multiply $2^{\frac{1}{4}}$ by itself to get 2?

Can you find the general rule?

$$x^{\frac{1}{n}} = \sqrt[_]{_}$$

Example: What is $27^{1/3}$?

Answer: $27^{1/3} = \sqrt[3]{27} = 3$

More Complicated Fractions

1.) Using the general rule $x^{\frac{1}{n}} = \sqrt[n]{x}$, how can we rewrite $5^{\frac{2}{3}}$ with a radical? _____

2.) What do I need to multiply $5^{\frac{2}{3}}$ by to get 5?_____

3.) Rewrite $\sqrt[5]{16}$ using fractional exponents, can we rewrite 16 using an exponent?

4.) Rewrite $\sqrt[4]{32}$ using fractional exponents._____

Activity: Matching Game

Directions: Match your radical with its rewritten fractional exponent and value. The first group to find its equivalent wins!

Example: $\sqrt[3]{5^5} = 5^{\frac{5}{3}} = 5\sqrt[3]{25}$ OR $5^{-\frac{3}{2}} = \frac{1}{5^{\frac{3}{2}}} = \frac{1}{\sqrt[2]{5^3}} = \frac{1}{5\sqrt[2]{5}}$

$\sqrt[2]{5^3}$	$5^{\frac{3}{2}}$	$5\sqrt{5}$
$\sqrt[4]{81}$	$3^{\frac{4}{4}}$	3
$\sqrt[3]{16}$	$2^{\frac{4}{3}}$	$2\sqrt[3]{2}$
$\sqrt[4]{32}$	$2^{\frac{5}{4}}$	$2\sqrt[4]{2}$
$\sqrt[3]{81}$	$9^{\frac{2}{3}}$	$3\sqrt[3]{3}$
$\sqrt[3]{625}$	$5^{\frac{4}{3}}$	$5\sqrt[3]{5}$

$32^{-\frac{1}{5}}$	$\frac{1}{\sqrt[5]{32}}$	$\frac{1}{2}$
$3^{-\frac{3}{2}}$	$\frac{1}{\sqrt{27}}$	$\frac{1}{3\sqrt{3}}$
$5^{-\frac{4}{3}}$	$\frac{1}{\sqrt[3]{5^4}}$	$\frac{1}{5\sqrt[3]{5}}$
$2^{-\frac{5}{3}}$	$\frac{1}{\sqrt[3]{2^5}}$	$\frac{1}{8\sqrt[3]{4}}$
$6^{-\frac{1}{4}}$	$\frac{1}{6^{\frac{1}{4}}}$	$\frac{1}{\sqrt[4]{6}}$

Name: _____ Date: _____

Exponents and Their Properties - Multiplying and Dividing Monomials

Algebra 1

When we want to express a product of the same number like $3 \cdot 3 \cdot 3 \cdot 3$ we can use a shortcut notation

exponent

$$3^4$$

base

The exponent tells how many times the base is used as a **factor** in the product.

***Exercise* #1:** Write each of the following in the form of an expanded product.

(a) x^3 (b) 4^5 (c) $(2x)^3$ (d) $(x + 5)^2$

***Exercise* #2:** Express each of the following with an equivalent expression involving exponents.

(a) $z \cdot z \cdot z \cdot z \cdot z$ (b) $6 \cdot 6 \cdot 6$ (c) $2a \cdot 2a \cdot 2a \cdot 2a$

(d) $x \cdot x \cdot y \cdot y \cdot y$ (e) $x \cdot y \cdot x \cdot y \cdot x \cdot y$

***Exercise* #3:** Consider the product shown below:

$$x^4 \cdot x^3$$

(a) Write both parts of this product as extended products.

x^4 x^3

Name: _____ Date: _____

Exponents and Their Properties - Multiplying and Dividing Monomials

(b) Write the product $x^4 \cdot x^3$ as an expanded product and in terms of an equivalent expression involving an exponent.

***Exercise* #4:** Express each of the following products as a single variable raised to a power.

(a) $x^2 \cdot x^7$ (b) $x^6 \cdot x^2$ (c) $a^2 \cdot a^3 \cdot a^4$ (d) $y^2 \cdot y^6 \cdot y^{10}$

We also must be able to divide monomial expressions that have the same base. The key to understanding how this process works is the following:

Any quantity divided by itself, except for zero, is equal to 1.

***Exercise* #5:** Consider the quotient: $\frac{x^5}{x^3}$

(a) Fill in the following: $x^5 = x^2 \cdot$ _____

(b) Rewrite the numerator of the quotient using (a).

(c) Rewrite the quotient as the product of two fractions, one of them being equal to 1.

(d) Simplify the quotient using the Multiplicative Identity Property of Real Numbers.

Name: _____ Date: _____

Exponents and Their Properties - Multiplying and Dividing Monomials

***Exercise* #6:** Fill in the blanks in the box below:

EXPONENT PROPERTIES

For any real numbers a and b,

1. $x^a \cdot x^b = x$ _____

2. $\dfrac{x^a}{x^b} = x$ _____

***Exercise* #7:** Write each product or quotient in its simplest form:

(a) $a^4 \cdot a^4 =$

(b) $x^2 \cdot x^7 =$

(c) $3^4 \cdot 3^2 =$

(d) $y^2 \cdot y^6 =$

(e) $2^3 \cdot 2 \cdot 2^4 =$

(f) $y^5 \cdot y^2 \cdot y =$

(g) $(x^3)^2 =$

(h) $\dfrac{10y^8}{6y^2} =$

(i) $\dfrac{z^3}{z^2} =$

(j) $\dfrac{b^2}{b^7} =$

(k) $\dfrac{x^2y^5}{x^8y^2} =$

(l) $\dfrac{(x^2)^3}{x^4} =$

Name: _____ Date: _____

Exponents and Their Properties - Multiplying and Dividing Monomials
Algebra 1 Homework

Skill

Express the product with exponents.

1. $a \cdot a \cdot a \cdot b \cdot b =$

2. $(2x)(2x)(2x) =$

3. $(2x)(2x)\, y \cdot y =$

Express the product in simplest form.

4. $b^3 \cdot b =$

5. $y^4 \cdot y^9 =$

6. $x^2 \cdot x^3 \cdot x^4 =$

7. $n^4 \cdot n =$

8. $y \cdot y =$

9. $a^4 \cdot a^2 =$

10. $x^3 \cdot x^7 =$

11. $z^4 \cdot z^4 =$

Express the quotient in simplest form.

12. $\frac{x^5}{x^4} =$

13. $\frac{a^{10}}{a^4} =$

14. $\frac{x^5}{x^8} =$

15. $\frac{y^6}{y^{12}} =$

16. $\frac{x^{13}y^5}{x^2y^9} =$

17. $\frac{8x^5y^3}{4x^8y^{10}} =$

18. $\frac{y^4}{y^4} =$

Name: _____ Date: _____

Exponents and Their Properties - Multiplying and Dividing Monomials

Reasoning

Simplify.

19. $\frac{x^c}{x^d} =$ $c > d$

20. $z\,(2z)^3\,(2z) =$

21. $x^{4a} \cdot x^{2a} =$

22. $\frac{x^3 \cdot x^6 \cdot x^4}{x^5 \cdot x^2} =$

23. $\frac{y^{2a} \cdot y^{3a}}{y^a} =$

24. $\frac{x^3 \cdot x^4}{(x^2)^2} =$

25. $x^4 \cdot y^5 =$

Determine True or False for each. State the reason for your answer.

26. $\frac{x^4}{x^2} = 1^2$

27. $\frac{4^5}{2^3} = 2^2$

Name: _____ Date: _____

Exponents and Their Properties - Multiplying and Dividing Monomials

Additional Practice Problems

Skill

Find the product.

1. $x^5 \cdot x^3 =$
2. $y^2 \cdot y =$
3. $y^4 \cdot y^2 \cdot y^6 =$
4. $a \cdot a \cdot a \cdot a =$
5. $a^6 \cdot a^4 =$
6. $t^3 \cdot t^7 =$
7. $r^5 \cdot r^3 =$
8. $x^4 \cdot x^5 =$
9. $y^3 \cdot y^8 =$
10. $t \cdot t =$
11. $w \cdot w^2 \cdot w \cdot w^2 =$
12. $p^5 \cdot p^3 \cdot p =$
13. $x^6 \cdot x^3 \cdot x =$
14. $h^8 \cdot h^3 =$
15. $x^4 \cdot x^2 =$
16. $y^2 \cdot y =$
17. $x \cdot x \cdot x =$
18. $m^{23} \cdot m^{17} =$
19. $b^{15} \cdot b^5 =$

Find each quotient.

20. $\frac{4^3}{4} =$
21. $\frac{x^3}{x} =$
22. $\frac{x^5}{x^2} =$
23. $\frac{y^4}{y^2} =$
24. $\frac{t^2}{t^2} =$

Name: _____ Date: _____

Exponents and Their Properties - Multiplying and Dividing Monomials

25. $\frac{z^5}{z^3} =$

26. $\frac{z^9}{z^5} =$

27. $\frac{t^9}{t^3} =$

28. $\frac{5^3}{5} =$

29. $\frac{p^3}{p^2} =$

30. $\frac{6^3}{6^3} =$

31. $\frac{x^{12}}{x^{10}} =$

Reasoning

Simplify.

32. $\frac{4x^3}{4x} =$

33. $\frac{t^3 \cdot t^2}{t} =$

34. $\frac{a \cdot b^3}{b^2} =$

35. $\frac{(ab)^2}{ab^2} =$

36. $\frac{x^4y^2}{x^3y^5} =$

37. $\frac{4x^4y^2}{2xy^3} =$

38. $x^a \cdot x^a =$

39. $\frac{x^a}{x^b} =$

40. $\frac{x^4y^7z}{x^2yz^3} =$

41. $\frac{x^4y^2}{x^4y^2} =$

42. $\frac{xy^5}{x^2y^2} =$

43. $\frac{x^2y}{x^3y^2} =$

MHS Algebra 1 Mathematics Curriculum

Unit 2 – Linear and Exponential Relationships

Domain: <u>Algebra</u> (A)

Essential Question: How do linear relationships help model exponential functions?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Compare and contrast linear and non-linear functions to develop appropriate models for given situations** **A-REI** (**) Solve systems of Equations **A-REI** (***) Represent and solve equations and inequalities graphically	**How…** **A.REI.5:** Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. **A.REI 6:** Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. **A.REI.10:** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). **A.REI.11:** Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.	**Students will:** **Group Activity – Linear Systems** Practice with Linear Systems.docx **Graphing Functions – Using TI-83** A.REI.10 Graphing Functions from Tables	**A.REI.5:** Given that the sum of two numbers is 10 and their difference is 4, what are the numbers? Explain how your answer can be deduced from the fact that they two numbers, x and y, satisfy the equations $x + y = 10$ and $x - y = 4$. **A.REI 6:** See Unit Addendum **A.REI.10:** Example: Which of the following points is on the circle with equation $(x-1)^2 + (y+2)^2 = 5$? (a) (1, -2) (b) (2, 2) (c) (3, -1) (d) (3, 4) **A.REI 11:** See Unit Addendum

<u>Vocabulary:</u> Arithmetic and geometric sequences, arithmetic and geometric situations, base, effect on graph, end behavior, equal distance over equal intervals, equal factor over equal intervals, even and odd functions, functional inputs, function notation, graphs, graphical effects, intercepts, intervals, parameters of linear/exp, recursive and explicit formulae, relative maximum/minimum, slope, symmetry

Practice with Linear Systems – A Group Approach

Prior Learning: Students must have practice and experience solving systems of equations using graphing, substitution and elimination methods. They must also have background in modeling systems from applications and solving the systems in an appropriate manner.

Stations Directions: The purpose of stations is for students to experience each type of problem. The teacher circulates and assists as needed, but students rely on each other for support.

Divide students into five groups. Have each group work on the problems at their station for 10 minutes. When the time is up, have students move to the next station. It is not necessary for students to complete all problems at the station. Any unfinished problems can be completed for homework.

Relay Directions: The purpose of a relay is for students to work for accuracy in their practice. This may require students to have the teacher check their work and try again before moving on to the next sheet.

Students should be paired for this activity. Each pair receives a set of problems that they must do completely and accurately before they move on to the next sheet of problems. You may need to modify the worksheets to reduce the number of problems to ensure students have the opportunity to complete all variations of problems.

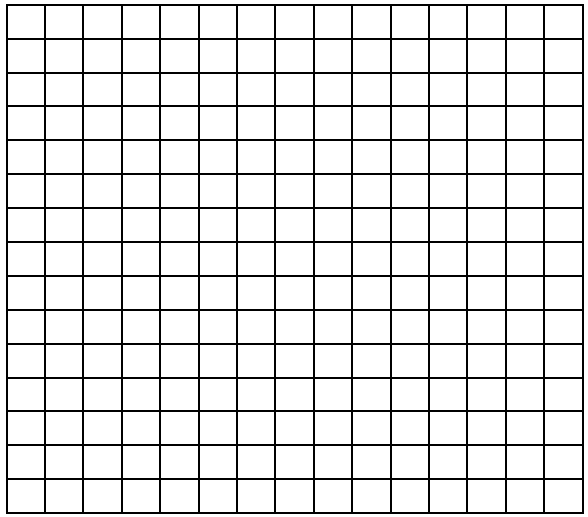
Materials: One copy of each station worksheet per student.

Name_____ Date_____

Station 1
Graphing Systems

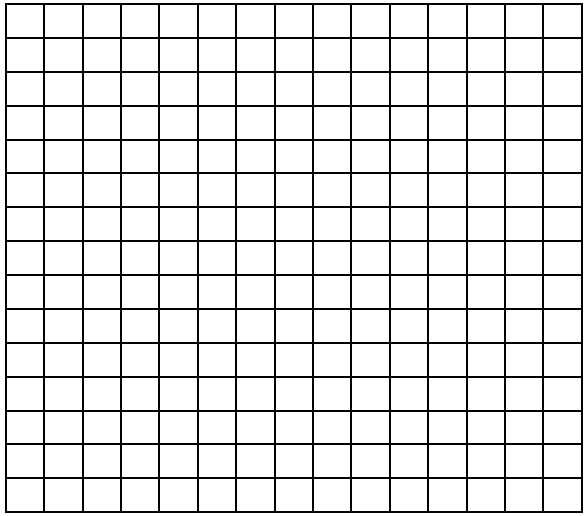
Instructions: *Solve each system using the graphing method.*

1.) $y = x + 5$
$y = -2x - 2$



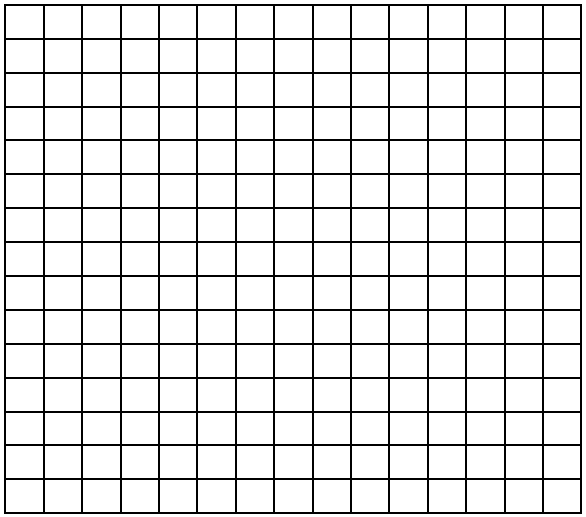
2.) $4y - 2x = 8$

$y = \frac{1}{2}x - 4$



3.) $x - 2y = -1$

$2x + 3y = -16$



4.) $8x - 10y = 14$
$4x - 5y = 7$

Name_____ Date_____

Station 2
Substituting Systems

Instructions: *Solve each system using the substitution method.*

1. $2x - 3y = 4$
   $x = -5y + 2$

2. $x - 4y = 7$
   $-y = x + 3$

3. $2x - y = 8$
   $2y + 3x = 12$

4. $4x - 2y = 14$
   $3x + 9y = 21$

Name_____ Date_____

Station 3
Eliminating Systems

Instructions: *Solve each of the following systems using the elimination method.*

1. $x + y = 5$
   $2x - y = 4$

2. $2x - y = 7$
   $x + 3y = 7$

3. $3x - 6y = -3$
   $6x + 3y = 9$

4. $-6y - 2x = 0$
   $11y + 3x = 4$

Name_____ Date_____

Station 4
Systems Applications

Instructions: *Write and solve a system of equations for each problem below. Be sure to define your variables, create appropriate equations and use whatever method you deem appropriate to solve your system. (Graphing is not required, although a graph has been provided should you choose.)*

1. Rent-A-Truck has rates of $50 a day plus $0.50 per mile. Sam's U-Drive has rates of $70 a day and $0.45 per mile. Determine under which conditions it is better to use each company.

Define the variables:

_____

_____

Write the system of equations:

_____

_____

Solve the system of equations:

Describe the conditions in which each company would be better.

_____

_____

_____

2. Our Algebra 1 class is going on field trip to an amusement park. Mr. Short and Ms. Miller have already compared the prices on admission tickets and graphed the results below. The Fun-Lover Pass (FL) is $15.50 and each ride cost .50 while the Walk-About Pass (WA) is $10 with each ride costing $1 each.

Define the variables:

_____

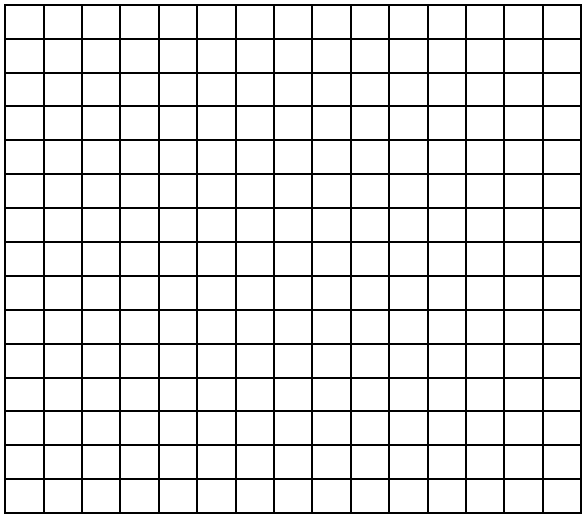
_____

Write the system of equations:

_____

_____

Solve the system of equations:



Describe the conditions in which each pass would be better.

_____

_____

_____

Name_____ Date_____

Station 5
Systems Applications

Instructions: *Write and solve a system of equations for each problem below. Be sure to define your variables, create appropriate equations and use whatever method you deem appropriate to solve your system. (Graphing is not required, although a graph has been provided should you choose.)*

1. At the Hebron Fair they use the same paper cups for both popcorn and soda. On Sunday they went through 700 cups and made $1062.50. If they sold sodas for $1.25 and popcorn for $2, how many of each item did they sell on Sunday?

Define the variables:

_____

_____

Write the system of equations:

_____

_____

Solve the system of equations:

Describe the conditions in which each company would be better.

_____

_____

_____

Mrs. Mazzotta has to work in the concession stand at her son's baseball game. During the game she sells cans of soda for $.75 and bottles of sports drink for $1.25. She sells a total of 30 drinks and collected $31.50. At the end of the game she must report the total number of each drink that was sold but she forgot to keep track. Please help her find the number of each drink sold at the baseball game.

Define the variables:

_____

_____

Write the system of equations:

_____

_____

Solve the system of equations:

Describe the conditions in which each company would be better.

_____

_____

_____

Name_____Date_____Period_____

Algebra 1 CCSS
Writing Equations from Tables
A.REI.10

Question 1: Given the function fill in the table of values.

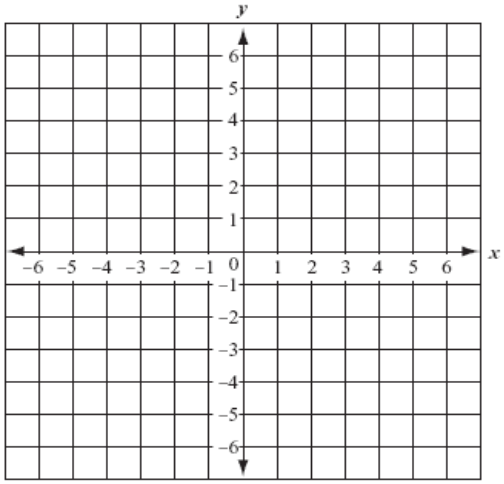
$\boldsymbol{y = 3x}$ $\boldsymbol{f(x) = 3x}$

Table

X (input)	$\boldsymbol{y = 3x}$ **or** $\boldsymbol{f(x) = 3x}$	Y or f(x) (output)	Ordered Pairs (x,y)
-2			
-1			
0			
1			

Now graph the function on the graphing calculator after that graph on your paper.

Graph



Question 2: Given the function fill in the table of values.

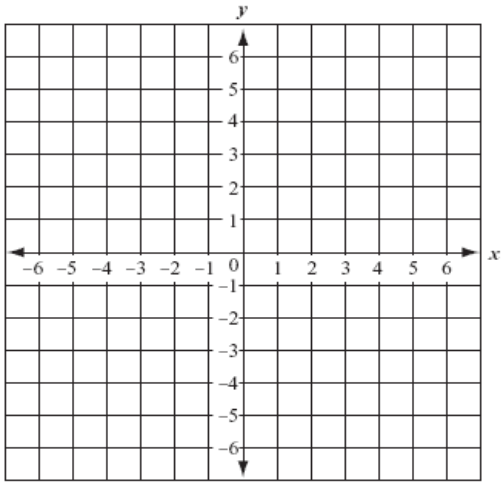
$\boldsymbol{y = x^2 + 2}$ $\boldsymbol{f(x) = x^2 + 2}$

Table

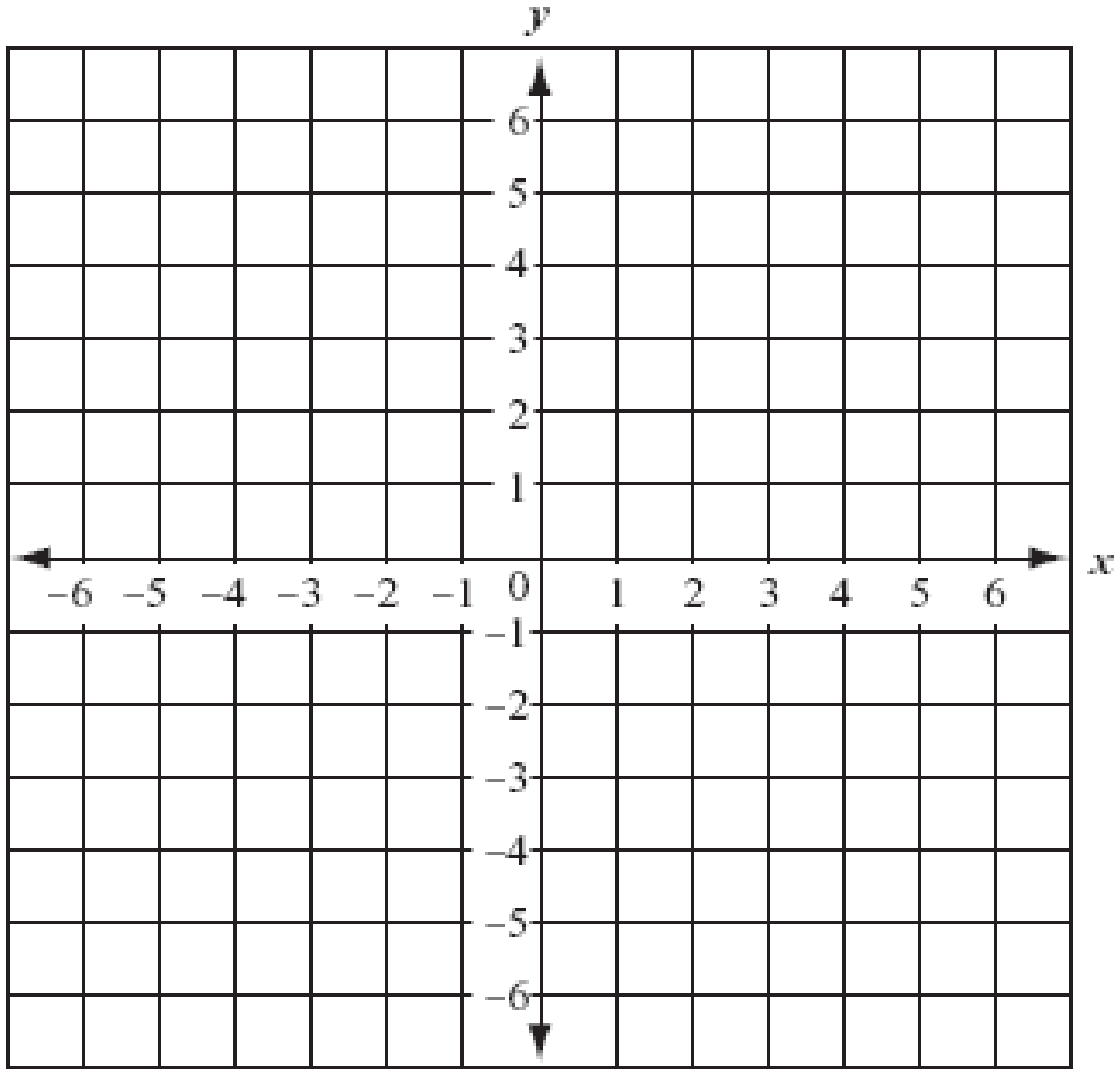
X (input)	$\boldsymbol{y = x^2 + 2}$ **or** $\boldsymbol{f(x) = x^2 + 2}$	Y or f(x) (output)	Ordered Pairs (x,y)
-2			
-1			
0			
1			
2			

Now graph the function on the graphing calculator after that graph on your paper.

Graph



Graph questions 1 and 2 on the same graph and try and find a point of intersection.



Do you see a clear point of intersection? If so what is it?_____

_____

Show how you could check this algebraically.

Question 3: Given the function fill in the table of values.

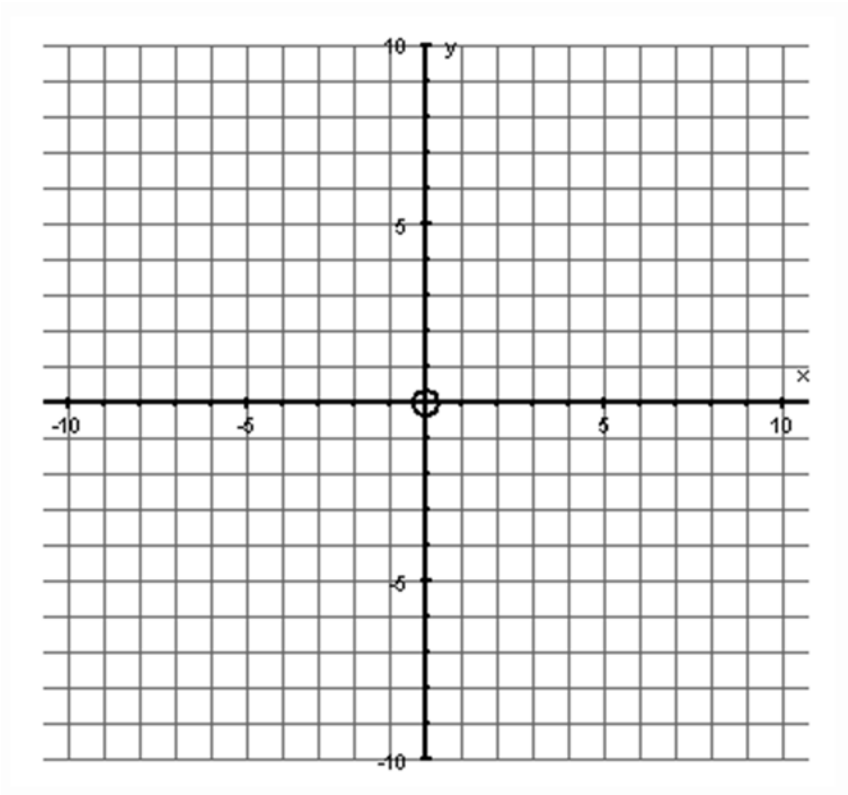
$\boldsymbol{y = x^3 - 1}$ $\boldsymbol{f(x) = x^3 - 1}$

Table

X (input)	$\boldsymbol{y = x^3 - 1}$ **or** $\boldsymbol{f(x) = x^3 - 1}$	Y or f(x) (output)	Ordered Pairs (x,y)
-2			
-1			
0			
1			
2			

Now graph the function on the graphing calculator after that graph on your paper.

Graph



Question 4: Given the function fill in the table of values.

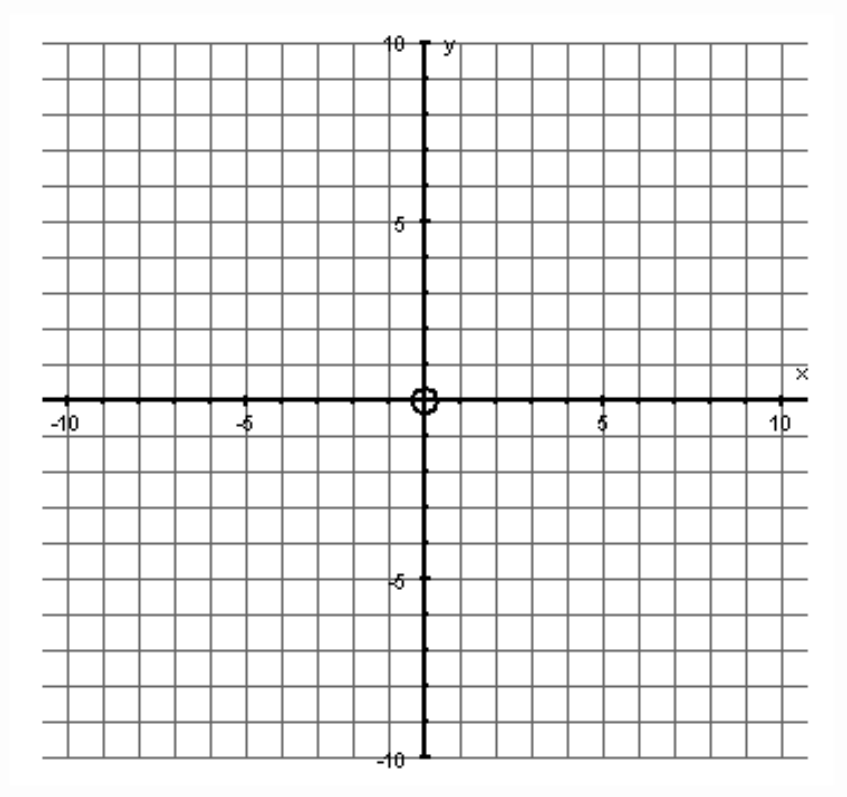
$\boldsymbol{y = \sqrt{x} - 1}$ $\boldsymbol{f(x) = \sqrt{x} - 1}$

Table

X (input)	$\boldsymbol{y = \sqrt{x} - 1}$ **or** $\boldsymbol{f(x) = \sqrt{x} - 1}$	Y or f(x) (output)	Ordered Pairs (x,y)
-1			
0			
1			
2			
4			

Now graph the function on the graphing calculator after that graph on your paper.

Graph



MHS Algebra 1 Mathematics Curriculum

Unit 2 – Linear and Exponential Relationships

Domain: <u>Algebra</u> (A)

Essential Question: How do linear relationships help model exponential functions?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Compare and contrast linear and non-linear functions to develop appropriate models for given situations** **A-REI** (***) Represent and solve equations and inequalities graphically	**How…** **A.REI.12:** Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	**Students will:** **Inequalities Exploration Lesson** A-REI.12 Lesson.pdf **Linear Program Group Activities** Optimization Problems- Boomerang Boomerangs (CCSS).docx Lettuce Farms.docx A-REI.12 Graphing inequalities.pdf	**A.REI.12:** See Unit Addendum

Vocabulary: Arithmetic and geometric sequences, arithmetic and geometric situations, base, effect on graph, end behavior, equal distance over equal intervals, equal factor over equal intervals, even and odd functions, functional inputs, function notation, graphs, graphical effects, intercepts, intervals, parameters of linear/exp, recursive and explicit formulae, relative maximum/minimum, slope, symmetry

CONCEPT DEVELOPMENT

Mathematics Assessment Project

CLASSROOM CHALLENGES

A Formative Assessment Lesson

Defining Regions Using Inequalities

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley

Beta Version

For more details, visit: http://map.mathshell.org
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Defining Regions Using Inequalities

MATHEMATICAL GOALS

This lesson unit is intended to help you assess how well students are able to use linear inequalities to create a set of solutions. In particular, the lesson will help you identify and assist students who have difficulties in:

- Representing a constraint by shading the correct side of the inequality line.
- Understanding how combining inequalities affects a solution space.

COMMON CORE STATE STANDARDS

This lesson relates to the following *Standards for Mathematical Content* in the *Common Core State Standards for Mathematics*:

A-REI: Represent and solve equations and inequalities graphically.

This lesson also relates to the following *Standards for Mathematical Practice* in the *Common Core State Standards for Mathematics*:

1. Make sense of problems and persevere in solving them.

INTRODUCTION

This lesson unit is structured in the following way:

- Before the lesson, students work individually on an assessment task that is designed to reveal their current levels of understanding and difficulties. You then review their work and create questions for students to answer in order to improve their solutions.
- After a whole-class introduction students work collaboratively, in pairs, on a game. One student decides on the position of a 'target point' on a coordinate grid and gives clues in the form of algebraic inequalities (e.g. $3y + 2x \leq 12$). The other student uses these clues to find the location of the target point. There is a strategic element to the game: what are the best clues to give, so that the target may be found quickly?
- In a whole-class discussion students review the main math concepts of the lesson.
- Students return to their original assessment tasks, and try to improve their own responses.

MATERIALS REQUIRED

Each student will need:

- Two copies of the assessment task *Combining Inequalities*.
- Either a mini-whiteboard (preferably with a square grid) or a sheet of squared paper, a pen, and an eraser.
- The game sheet *Give Us a Clue!*

There are some projector resources to support whole-class discussions.

TIME NEEDED

15 minutes before the lesson, one 70-minute lesson (or two 40-minute lessons), and 10 minutes in a follow-up lesson (or for homework). Timings given are approximate and will depend on the needs of the class.

BEFORE THE LESSON

Assessment task: *Combining Inequalities* (15 minutes)

Set this task, in class or for homework, a few days before the formative assessment lesson. This will give you an opportunity to assess the work, to find out the kinds of difficulties students have with it. You will then be able to target your help more effectively in the follow-up lesson.

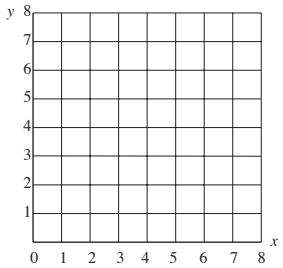
Give each student a copy of *Combining Inequalities*, a pencil and a ruler.

Briefly introduce the task:

> *Spend 15 minutes individually, answering these questions.*
>
> *Show all your work, so that I can understand your reasoning.*

It is important that students answer the questions without your assistance, as far as possible.

Combining Inequalities



Some treasure has been buried at a point (***x***,***y***) on the grid, where ***x*** and ***y*** are whole numbers.

Here are three clues to help you find the treasure:

Clue 1: $x > 2$ **Clue 2:** $x + y < 8$ **Clue 3:** $2y - x \geq 0$

1. Which of the following points could be a possible location for the treasure?
   The points must satisfy *all three* clues.
   Circle the answers you choose.

   (3,2) (2,3) (5,3) (3, 5) (4,3) (5, 2)

2. On the grid show *all* the possible places the treasure could be located.

3. Here are two more clues: **Clue 4:** $y > x - 4$ **Clue 5:** $y < x - 1$
   Which clue doesn't help at all? .....
   Explain why.
   .....
   .....
   At which point is the treasure located?

Students should not worry too much if they cannot understand or do everything, because in the next lesson they will engage in a similar task, which should help them. Explain to students that by the end of the next lesson, they should expect to answer questions such as these confidently. This is their goal.

Assessing students' responses

Collect students' written work for formative assessment. Read through their papers and make informal notes on what their work reveals about their current levels of understanding.

We strongly suggest that you do not write scores on students' work. Research shows that this is counterproductive, as it encourages students to compare scores, and distracts their attention from what they could do to improve their mathematics.

Instead, help students to make further progress by asking questions that focus their attention on aspects of their work. Some suggestions for these are given on the next page. These have been drawn from common difficulties observed in trials of this unit.

We suggest that you write your own lists of questions, based on your own students' work, using the ideas that follow. You may choose to write questions on each student's work. If you do not have time to do this, select a few questions that will help the majority of students. These can then be written on the board at the end of the lesson.

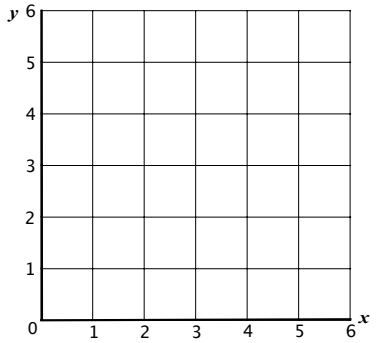
Common issues:	Suggested questions and prompts:
Student has difficulty distinguishing between > and ≥, or < and ≤ For example: The student includes (2,3) and (5,3) as possible locations for the target (Q1.) Or: The student states the treasure is located at (2,1), (4,3), (5,3), or (3,2) (Q3.) Or: The student does not use a dashed line for < or > inequalities (Q2 or Q3.)	• Write the inequalities into words. • What is the difference between > and ≥? • What is the difference between < and ≤? • The point (2,5) is outside the region where the treasure is located. Which clue tells you this? • Are points on the line $x = 2$ possible locations for the treasure? Are points on the line $2y - x = 0$ possible locations for the treasure? How can you distinguish graphically between the two? • Which points are not allowed?
Q2. Student uses guess and check to figure out the possible location for the treasure The student does not draw the inequality boundaries as lines on the grid, but instead guesses possible locations for the treasure and checks to see if they fit the clues.	• Can you think of a quicker way to figure out the possible locations? • How can you convince me there are no other possible points? • How can you use the graph to show the region where the treasure is located?
Q3. Student provides insufficient reasoning For example: The student does not explain the reason why Clue 4 is unhelpful.	• How does the clue affect the region where the treasure is located? • Does this clue help you find the position of the treasure?
Q3. Student assumes the treasure is located at one of the points chosen in Q1	• Check to see if your point fits your new clue.
Student correctly answers all the questions The student needs an extension task.	• Another treasure is at (6,5). Write just two clues that will locate the exact position of the treasure. Your clues should use the inequality symbols >, <, ≥, or ≤.

SUGGESTED LESSON OUTLINE

If you have a short lesson, or you find the lesson is progressing at a slower pace than anticipated, we suggest you end the lesson after the paired work, 'Preparing to play *Give Us a Clue!*', and continue in a second lesson.

Whole-class interactive introduction: *Hunting the Target* (15 minutes)

Give each student either a mini-whiteboard, pen, and eraser, or a sheet of squared paper.



Use slide P-1 of the projector resources to project the 6 × 6 coordinate grid on to the board.

Write the pair of coordinates, (2,2) on a piece of paper, fold it in half (hiding the coordinates) and stick this to the board.

I am thinking of a target point on this grid. I have written the coordinates on this paper. Both coordinates are integers.

Your task is to guess which point I am thinking of.

Here is the first clue: $3y + 2x \leq 12$

Does anyone know what this clue means?

Students may need careful leading through this idea, so take this stage slowly. Use questions such as the following, asking students to respond using their mini-whiteboards:

Show me the coordinates of a point that satisfies the clue.

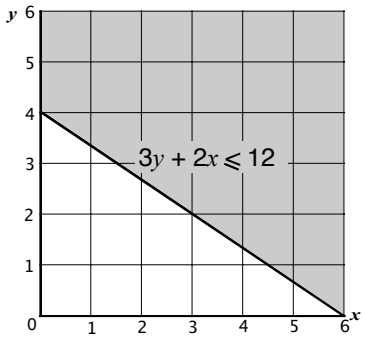
Can you show me another point? … and another? How do you know?

As students suggest possible points, mark these clearly on the grid.

Where are all the points that satisfy this clue?
[On or below the line $3y + 2x = 12$*.]*

Where are all the points that don't satisfy this clue?
[Above the line $3y + 2x = 12$*.]*

Give me a point that ***just*** *satisfies the clue.*
Give me a point that ***easily*** *satisfies the clue.*



Explain that for this lesson, the region that does not satisfy a clue is to be shaded out.

To help students keep track of each clue you may want to use a different color marker for each inequality.

Here's the second clue: $x > 1$

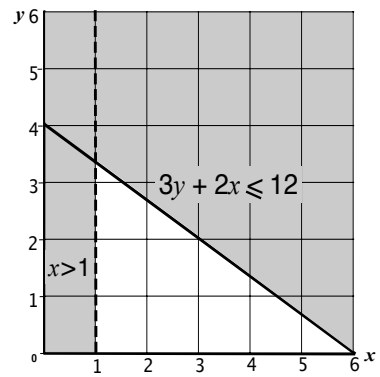
Shade out all the points that are eliminated.

Show me the new region.

Which points are possible now?

Is (1,2) a possible point?

Explain that we use a dashed line to show that the points on the line $x = 1$ are not included as possible points for the target.



Here's the third clue: $y > x - 1$

Shade out all the points that are eliminated.

Which points are possible now?

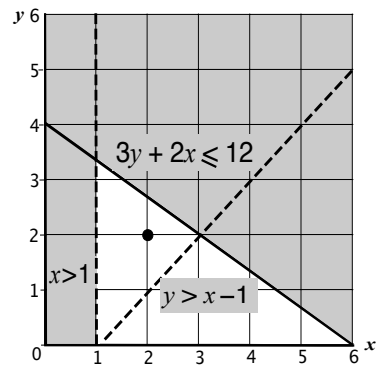
Show me the new region.

Do you know the point I am thinking of yet?

Is (2,2) the only possibility?

Why can't (3,2) be a possible point for the target?

Although there are many non-integer points that are possible, explain that for this lesson we will stick with integer coordinates.



Preparing to play *Give Us a Clue!* (10 minutes)

Give each student a copy of the sheet *Give Us a Clue!*

Use slide P-2 of the projector resource to project the 8 × 8 coordinate grid onto the board.

You are soon going to play a game called 'Give Us a Clue!'

You will use the lines on the small graphs on the handout.

Before beginning the game you need to figure out the inequalities for the regions to the left and right of each given line. You will use these inequalities as clues in the game.

For example, look at the line $2x - y = 8$.

Which side of the line are points that fit the inequality $2x - y \geq 8$?

Which side of the line are points that fit the inequality $2x - y \leq 8$?

In order to answer these two questions, it is helpful to test the inequality with specific pairs of coordinates. These are sometimes called test points.

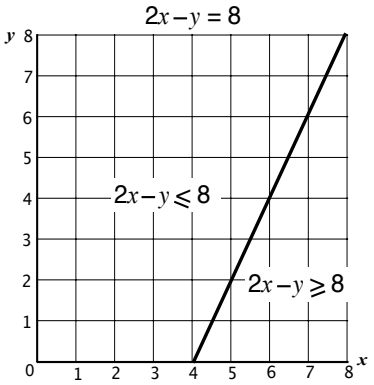
(0,0) is usually a good choice for a test point, since it makes the arithmetic easy, but if the line itself goes through the origin, then another point should be chosen:

Can you put the inequality into words?

Let's use the origin (0,0) as a test point. This point is to the left of the line.

Which of the two inequalities [$2x - y \geq 8$ or $2x - y \leq 8$] does it fit?

Now choose your own test point to the right of the line. Use its coordinates to check the inequality for this region.



Since $2(0) - 0 \leq 8$ is true, the origin is included in the region $2x - y \leq 8$. This region is to the left of the line.

Paired work: preparing to play *Give Us a Clue!* (10 minutes)

Organize the class into pairs of students.

Explain how students should work collaboratively.

Take it in turns to figure out the inequalities for each region of the twelve small graphs.

Once you have done this, explain to your partner how you came to your decision.

Your partner should either explain that reasoning again in his or her own words, or challenge the reasons you gave.

You need to agree on, and ***both*** *be able to explain, the inequalities for each region of each graph.*

Make sure you write all the inequalities on your own copy of Give Us a Clue!

There is no need to shade the graphs.

The purpose of this structured paired work is to make each student engage with their partner's explanations, and to take responsibility for their partner's understanding.

You have two tasks during the paired work: to note aspects of the task students find difficult, and support student reasoning.

Note aspects of the task students find difficult

For example, are students having difficulties using a test point? Do they understand the difference between inequality symbols? You can use information about particular difficulties to focus a whole-class discussion towards the end of the lesson.

Support student reasoning

Try not to make suggestions that move students towards a particular answer. Instead, ask questions to help students to reason together. For students struggling to understand the symbols, it may help if they put the inequalities into words.

How did you figure out the inequality for this region?

[Select a graph that goes through the origin.] Why is (0,0) not a good test point for this graph?

[Select one of the first four graphs.] Why is (4,4) not a good test point to use for this graph?

Sharing work: Preparing to play *Give Us a Clue!* (10 minutes)

Ask students to check their work with a neighboring pair of students.

Check to see which graphs are different.

When there is a disagreement, take turns to justify your decision. If you still don't agree, ask for further explanation.

Both of you need to agree and understand the math.

Students playing *Give Us a Clue*! (15 minutes)

When students are satisfied with their twelve graphs, use slide P-3 of the projector resource to introduce the game:

In your pairs, you are now going to play 'Give Us a Clue!'

One of you will be the target picker, and the other the target hunter.

The target picker decides on the position of the target, and gives the clues.

When giving clues, the target picker can use any inequality sign (≤, <, ≥, >), but not the '=' sign. Try to give helpful clues! As you give the clues, write them as a list on your mini-whiteboard.

The target hunter uses the clues to find the target.

The aim of the game for both partners is to find the target in the least number of tries.

Both partners should use a blank grid, to keep track of the clues that are given.

Each time a clue is given, shade out the region where the target cannot be located.

It is important that students cannot see each other's graphs. They could use a book or folder to hide the graph from their partner.

Encourage students to give clues using the correct inequality language, rather than using imprecise language such as "The point is above the line."

When the target picker has used all the useful inequalities on the handout, they could make up their own.

At the end of each game, students should check each other's graphs. If they are not the same, encourage them to work together to identify mistakes made. The mini-whiteboard listing the clues may help sort out disagreements. This should be seen as a collaborative rather than competitive activity.

Then students can reverse roles.

For students who have successfully completed this task, ask them to create their own inequalities, and use them to play the game with their partner.

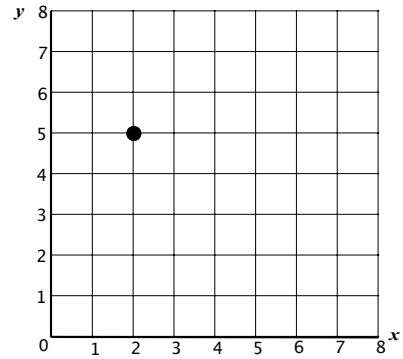
Whole-class discussion (10 minutes)

In the summary discussion you can explore the best strategy for giving a clue, while revising the main math concepts in the lesson. Students should use their mini-whiteboards to respond to your questions.

Use slide P-2 of the projector resource to project the 8 × 8 coordinate grid on to the board.

We will now investigate how to give the best clues for targets within an 8 × 8 grid. We are still using the inequalities on the work sheet.

Can anyone think of the best first clue for the point (2,5)? [$y > 2x$.]



Ask a few students to justify their answers. Use different color markers to draw their clues on the board.

In this case, is the clue $y \geq 2x$ better than the clue $y > 2x$? Why?

Once students are satisfied that they are using the best first clue ask:

What is the next best clue? [$y < x + 4$.]

Again ask students to explain their reasoning.

How do you know $y < x + 4$ is a better clue than $y < x$? Show me.

If students are struggling with the difference between a clue that uses < and one that uses ≤ ask:

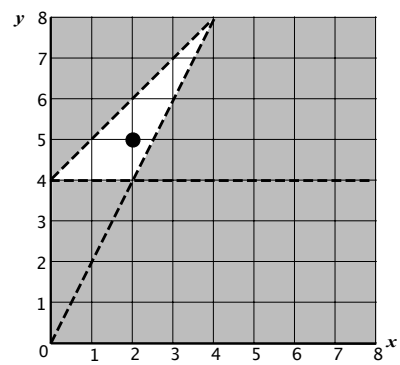
How many points could the target be if you use the clue $y < x + 4$?

How many points could the target be if you use the clue $y \leq x + 4$?

Now ask for a final clue:

And what is another good clue? [$y > 4$]

How many places could the target be now?



You could extend this further by asking:

Can you think of a target point within the 8 × 8 grid that only requires two clues?

It will be a point on a line. For example, when the target point is (8,4), the clues could be $y \leq 4$ and $x + 2y \geq 16$.

Improving individual solutions to the assessment task (10 minutes)

Return their original assessment *Combining Inequalities* to the students, as well as a second blank copy of the task.

Look at your original responses and think about what you have learned this lesson.

Using what you have learned, try to improve your work.

If you have not added questions to individual pieces of work, write your list of questions on the board. Students should select from this list only the questions they think are appropriate to their own work.

Explain to students that Questions 1 and 2 are concerned with just the first three clues. When answering these questions they should ignore Clues 4 and 5.

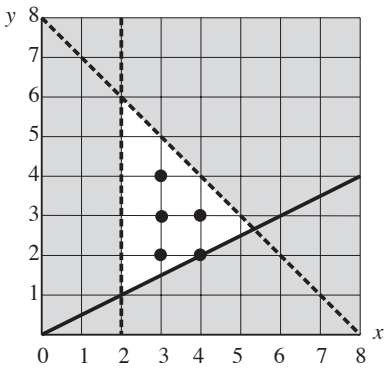
If you find you are running out of time, then you could set this task in the next lesson, or for homework.

SOLUTIONS

Assessment task: Combining Inequalities

1. The points (3,2) and (4,3) are the points that satisfy all clues.

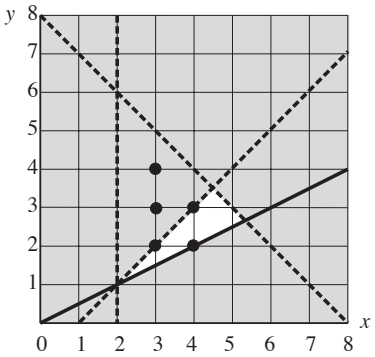
2. The possible places for the target are indicated by the bold dots on the grid below:



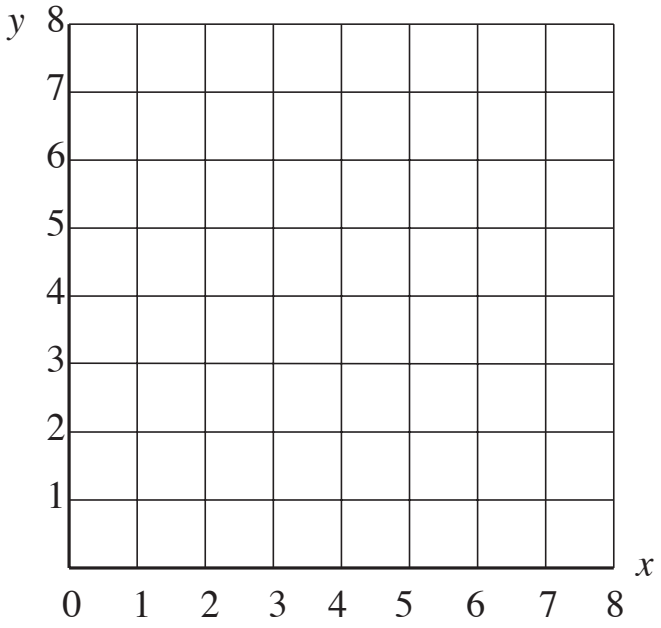
3. Clue 4 is unhelpful because it doesn't add any extra information.

   This is because the region $y > x - 4$ includes all of the above region.

   Clue 5 excludes all solutions but (4,2) so this is where the treasure is located. (See diagram)



Combining Inequalities



Some treasure has been buried at a point (***x,y***) on the grid, where ***x*** and ***y*** are whole numbers.

Here are three clues to help you find the treasure:

Clue 1: $x > 2$ **Clue 2:** $x + y < 8$ **Clue 3:** $2y - x \geq 0$

1. Which of the following points could be a possible location for the treasure?

   The points must satisfy *all three* clues.

   Circle the answers you choose.

   (3,2) (2,3) (5,3) (3, 5) (4,3) (5, 2)

2. On the grid show *all* the possible places the treasure could be located.

3. Here are two more clues: **Clue 4:** $y > x - 4$ **Clue 5:** $y < x - 1$

   Which clue doesn't help at all? .....

   Explain why.

   .....

   .....

   At which point is the treasure located?

   .....

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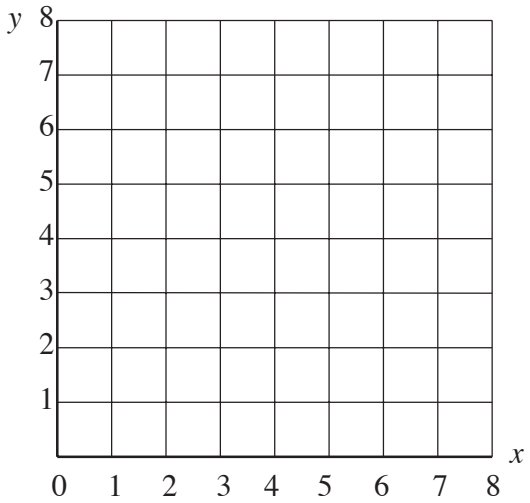
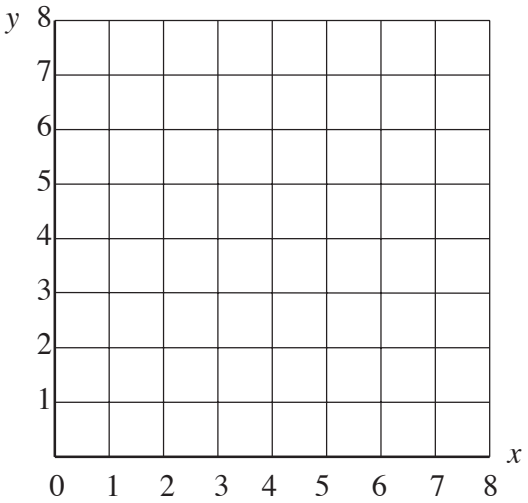
Give Us a Clue!

Use these grids to record the clues given by the teacher or your partner.

Game 1

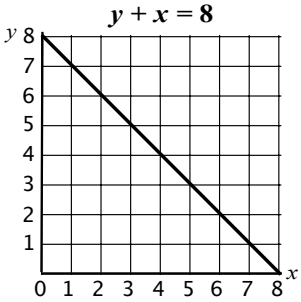


Game 2

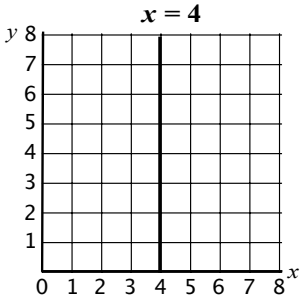


Use these graphs to invent your questions.

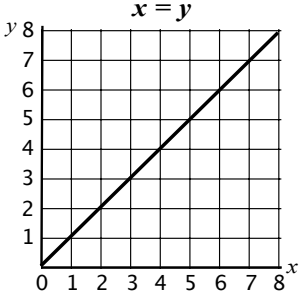
$y + x = 8$



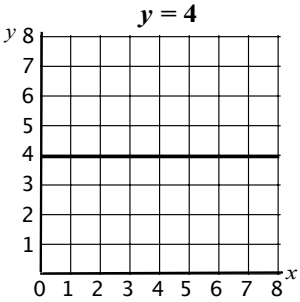
$x = 4$



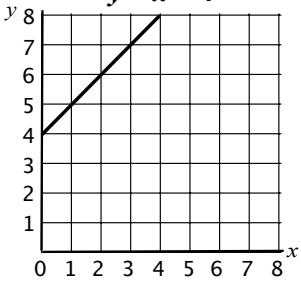
$x = y$



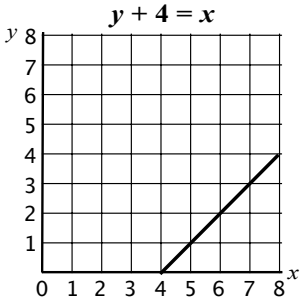
$y = 4$



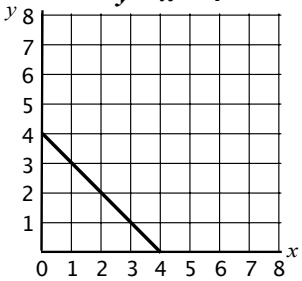
$y = x + 4$



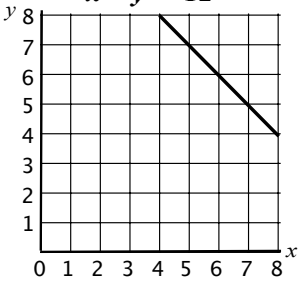
$y + 4 = x$



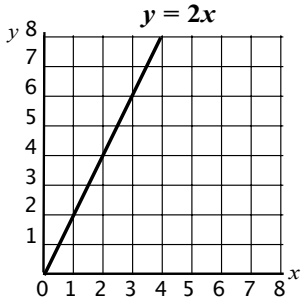
$y + x = 4$



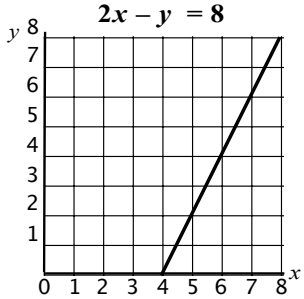
$x + y = 12$



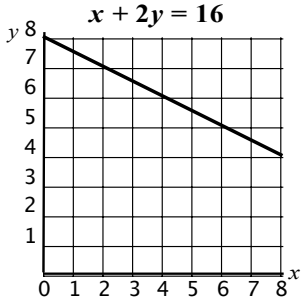
$y = 2x$



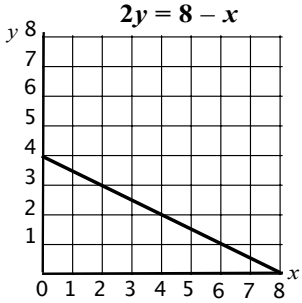
$2x - y = 8$



$x + 2y = 16$



$2y = 8 - x$



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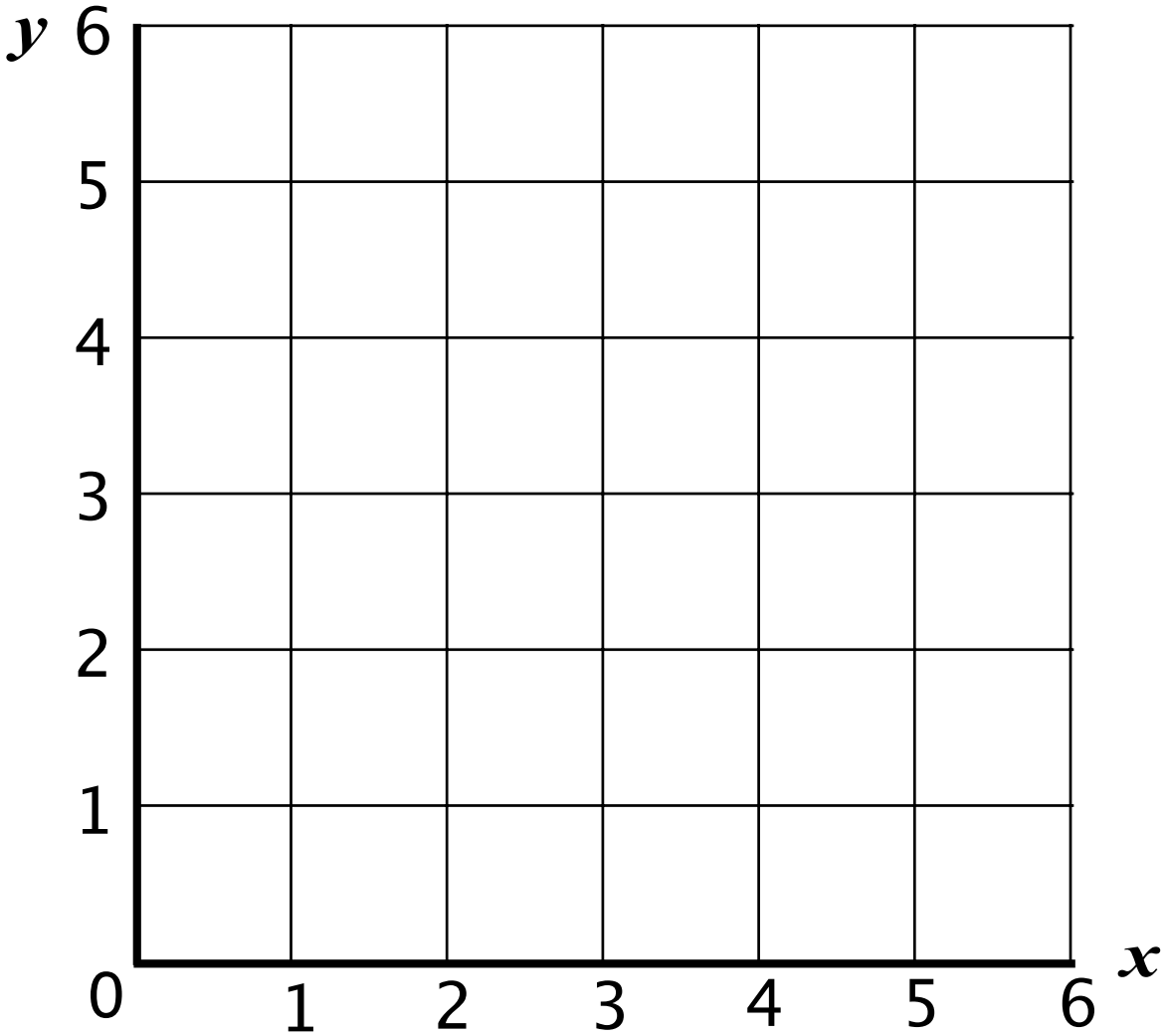
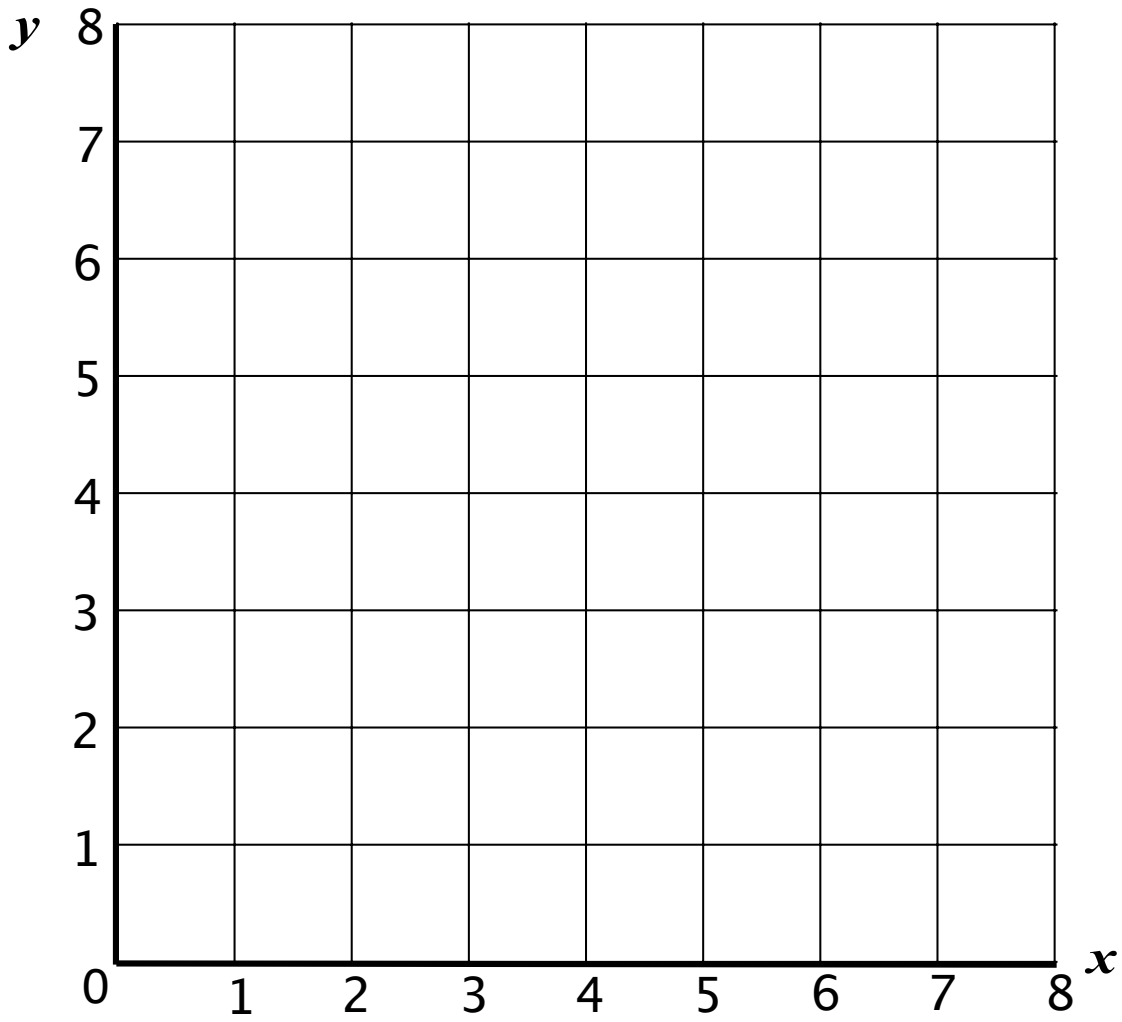
Hunting the Target



Give Us a Clue



Playing the Game

- One of you will be the target picker and the other the target hunter.
- The target picker decides on the position of the target and gives the clues.
- When giving clues, the target picker can use any inequality sign (≤, <, ≥, >), but not the '=' sign. Try to give helpful clues!
  As you give the clues, write them as a list on your mini-whiteboard.
- The target hunter uses the clues to find the target.
- The aim of the game for both partners, is to find the target in the least number of tries.
- Both partners must use a blank grid to keep track of the clues that are given.
- Each time a clue is given, shade out the region where the target cannot be located.

Mathematics Assessment Project

CLASSROOM CHALLENGES

This lesson was designed and developed by the
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at the
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It was refined on the basis of reports from teams of observers led by
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along with comments from teachers and other users.

This project was conceived and directed for
MARS: Mathematics Assessment Resource Service
by
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PROBLEM SOLVING

Mathematics Assessment Project
CLASSROOM CHALLENGES
A Formative Assessment Lesson

Optimization Problems:

Boomerangs

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley
Beta Version

For more details, visit: http://map.mathshell.org
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Optimization Problems: *Boomerangs*

MATHEMATICAL GOALS

This lesson unit is intended to help you assess how well students are able to:

- Interpret a situation and represent the constraints and variables mathematically.
- Select appropriate mathematical methods to use.
- Explore the effects of systematically varying the constraints.
- Interpret and evaluate the data generated and identify the optimum case, checking it for confirmation.
- Communicate their reasoning clearly.

COMMON CORE STATE STANDARDS

This lesson relates to the following *Mathematical Practices* in the *Common Core State Standards for Mathematics:*

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.

This lesson gives students the opportunity to apply their knowledge of the following *Standards for Mathematical Content* in the *Common Core State Standards for Mathematics:*

A-CED: Create equations that describe numbers or relationships.
A-REI: Solve equations and inequalities in one variable.
Solve systems of equations.

INTRODUCTION

This lesson is designed to help students develop strategies for solving optimization problems. Such problems typically involve scenarios where limited resources must be used to greatest effect, as in, for example, the allocation of time and materials to maximize profit.

- Before the lesson, students attempt the problem individually. You then review their work and formulate questions for students to answer in order to improve their solutions.
- At the state of the lesson, student work alone answering your questions about the same problem.
- Students are then grouped, and engage in a collaborative discussion of the same task.
- In the same small groups, students are given sample solutions to comment on and evaluate.
- In a whole-class discussion, students explain and compare the alternative solution strategies they have seen and used.
- Finally, students revise their individual solutions, and comment on what they have learned.

MATERIALS REQUIRED

- Each individual student will need a calculator and a copy of the assessment task *Boomerangs.*
- Each small group of students will need copies of the *Sample Responses to Discuss.*
- Graph paper should be kept in reserve, and used only when necessary or requested.
- There are also some projector resources to help you with whole-class discussions.

TIME NEEDED

15 minutes before the lesson, a 1-hour lesion, and 10 minutes in the next lesson (or for homework). Timings given are only approximate. Exact timings will depend on the needs of the class.

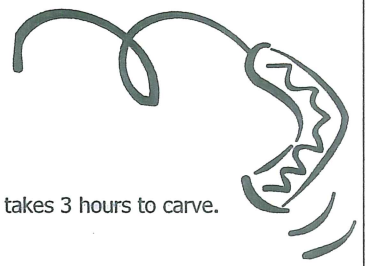
BEFORE THE LESSON

Assessment task: *Boomerangs* (15 minutes)

Have the students do this task in class or for homework, a day or more before the formative assessment lesson. This will give you an opportunity to assess the work and to find out the kinds of difficulties students have with it. Then you will be able to target your help more effectively in the follow-up lesson.

Give out the task *Boomerangs.* Introduce the task briefly to help the class to understand the problem and its context. You could show examples of boomerangs.

Boomerangs come from Australia where they are used as weapons or for sport. *When thrown, they travel in a roughly elliptical path and return to the thrower.* *Boomerangs are made in many different sizes.* *Read through the questions and try to answer them as carefully as you can. Show all your work so that I can understand your reasoning.* *As well as trying to solve the problem, I want you to see if you can present your work in an organized and clear manner.*	**Boomerangs** Phil and Cath make and sell Boomerangs for a school event. The money they raise will go to charity. They plan to make them in two sizes: small and large. Phil will carve them from wood. The small boomerang takes 2 hours to carve and the large one takes 3 hours to carve. Phil has a total of 24 hours available for carving. Cath will decorate them. She only has time to decorate 10 boomerangs of either size. The small boomerang will make $8 for charity. The large boomerang will make $10 for charity. They want to make as much money for charity as they can. How many small and large boomerangs should they make? How much money will they make?



It is important that, as far as possible, students are allowed to answer the questions without assistance.

Students who sit together often produce similar answers and then when they come to compare their work, they have little to discuss. For this reason, we suggest that when students do the task individually, you ask them to move to different seats. Then at the beginning of the formative assessment lesson, allow them to return to their usual seats. Experience has shown that this produced more profitable discussions.

Assessing students' responses

Collect students' responses to the task. Make some notes on what their work reveals about their current levels of understanding, and their different problem solving approaches. The purpose of doing this is to forewarn you of issues that will arise during the lesson itself so that you may prepare carefully.

We suggest that you do not score students' work. The research shows that this will be counterproductive as it will encourage students to compare their scores and distract their attention from what they can do to improve their mathematics.

Instead, help students to make further progress by summarizing their difficulties as a series of questions. Some suggestions for these are given on the next page. These have been drawn from common difficulties observed in trials of this lesson unit.

We suggest that you write a list of your own questions, based on your students' work, using the ideas below. You may choose to write questions on each students' work. If you do not have time to do this, just select a few questions that will be of help to the majority of students. These can be written on the board at the beginning of the lesson. If students have used graphs or simultaneous equations in their solutions, add the relevant questions to their work. You may also want to note students with a particular issue, so that you can ask them about their difficulties in the formative lesson.

Common issues:	Suggested questions and prompts:
Student has difficult getting started	• What do you know? • What do you need to find out?
Student makes an incorrect interpretation of the constraints and variables. For example: The student has applied just one constraint, such as "Phil has only 24 hours to make the boomerangs" or "Cath can only make 10 boomerangs". Or: The student has calculated the profit for making just one type of boomerang.	• What figures in the task are fixed? • What can you vary? • What is the greatest number of small/large boomerangs they can make? • Have you used any unnecessary restrictions on the number of small and large boomerangs to be made? • Why can't they make 50 boomerangs?
Student works unsystematically For example: The student shows three or four seemingly unconnected combinations, such as 5 small and 5 large boomerangs, then 10 large.	• Can you organize the numbers of large and small boomerangs made in a systematic way? • What would be sensible values to try? Why? • How can you check that you remember all the constraints? • Do you cover all possible combinations? If not, why not? • How do you know for sure your answer is the best option? • Can you organize your work in a table?
Student presents work poorly For example: The student presents the work as a series of unexplained numbers and/or calculations, or as a table without headings. Or: The student circles numbers and it is left to the reader to work out why this is the answer as opposed to any other combination.	• Would someone unfamiliar with your type of solution easily understand your work? • Have you explained how you arrived at your answer?
Student has technical difficulties when using graphs For example: Lines are plotted inaccurately, axes are not labeled or the purpose of the graph is not explained. **Student has technical difficulties when using simultaneous equations.** For example: A mistake is made when solving two correct simultaneous equations, or the correct solutions are obtained but the profit is not calculated.	• How would someone unfamiliar with your type of solution easily understand your work? • How can you check your answer? • How do your answers help you solve the problem?
Student produces a correction solution Student needs an extension task.	• Can you now use a different method? For example, a table or graph, or algebra? • Is this method better than your original one? Why? • In the problem investigated, how many boomerangs can be made in a month rather than 24 hours? Would any method(s) be preferable to others?

SUGGESTED LESSON OUTLINE

Improve individual solutions to the assessment task (10 minutes)

Return the assessment task papers to the students, and hand out calculators.

If you have not added questions to individual pieces of work, then write your list of questions on the board (excluding the ones for graphs and simultaneous equations). Students are to select questions appropriate to their own work, and spend a few minutes answering them.

Recall what we were looking at in a previous lesson. What was the task?
I have read your solutions and have some questions about your work. I would like you to work on your own to answer my questions for about ten minutes.

Collaborative small-group work (10 minutes)

Organize the class into small groups of two or three students, and give out a fresh piece of paper to each group. Ask students to try the task again, this time combing their ideas.

Put your own work aside until later in the lesson. I want you to work in groups now. Your task is to produce a solution that is better than your individual solutions.

While students work in small groups you have two tasks: to note different student approaches to the task and to support student problem solving.

Note different student approaches to the task

You can then use this information to focus a whole-class discussion towards the end of the lesson. In particular, note any common mistakes. For example, are students consistently using all the constraints, or are they imposing unnecessary constraints? Also note whether students are using algebra and, if so, how they are using it.

Support student problem solving

Try not to make suggestions that move students towards a particular approach to this task. Instead, ask questions that help students to clarify their thinking. You may discover that some students experience some difficulty in keeping more than one constraint at a time in mind. In that case, you may ask them to consider these three questions:

If they were to make only small boomerangs, how much money would they make?
If they were to make two small boomerangs, how many large ones could they also make? How much money would they make?

For the first question, Cath's time is the limiting constrain, whereas in the second question, Phil's time is more significant. Students who organize their work into a table may choose to use column headings for 'Time needed for Phil' and 'Time needed for Cath' which they can use to check that both constraints have been met.

To help students really struggling with the task, use the questions on the previous page to support your own questioning. In particular, if students find it difficult to get started, these questions may be useful:

Try some examples. What happens if they make three small and one large boomerang?
What would be sensible values to try? Why?
Can you organize the numbers of large and small boomerangs made in a systematic way?

If the whole class is struggling on the same issue, write relevant questions on the board. You could also ask students who performed well on the assessment to help struggling students. If students are having difficulty making any progress at all, you could hand out two pieces of sample work to model problem solving methods.

Collaborative analysis of *Sample Responses to Discuss* (20 minutes)

After students have had sufficient time to attempt the problem, give each group of students a copy of each of the four *Sample Responses to Discuss*, and ask for written comments. This task gives students the opportunity to evaluate a variety of possible approaches to the task, without providing a complete solution strategy.

Imagine you are the teacher and have to assess this work. Correct the work, and write comments on the accuracy and organization of each response.

Each of the sample responses poses specific questions for students to answer. In addition to these, you could ask students to evaluate and compare responses. To help them do more than check to see if the answer is correct, you may wish to use the projector resource *Evaluating Sample Responses to Discuss:*

What do you like about the work?
How has each student organized the work?
What mistakes have been made?
What isn't clear?
What questions would you like to ask this student?
In what ways might the work be improved?

You may decide there is not enough time for each group to work through all four pieces of work. In that case, be selective sabout what you hand out. For example, groups that have successfully completed the task using one method will benefit from looking at different approaches. Other groups that have struggled with a particular approach may benefit from seeing a student version of the same strategy.

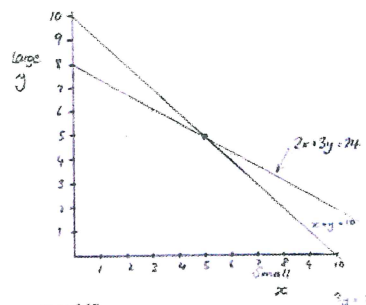
During the small-group work, support the students as before. Note similarities and differences between the sample approachs, and those approaches students took in the small-group work. Also check to see which methods students have difficulties in understanding. This information can help you focus the next activity, a whole-class discussion.

Whole-class discussion: comparing different approaches (10 minutes)

Organize a whole-class discussion to consider the different approaches used in the sample work. Focus the discussion on those parts of the small-group tasks that students found difficult. Ask the students to compare the different solution methods.

Which approach did you like best? Why?
Which approach did you find most difficult to understand?

To critique the different strategies use the questions on the slide *Evaluating Sample Responses to Discuss* and the worksheets *Sample Responses to Discuss.*

Alex has realized that you have to take account of both constraints: Phil's time for making the boomerangs and Cath's time for decorating them. Alex has not exampled different combinations of cases.	Phil can only make 12 small or 8 large boomerangs in 24 hours 12 small makes \$96 8 large makes \$80 Cath only has time to make 10, so \$96 is impossible. She could make 10 small boomerangs which will make \$80. So she either makes 8 large or 10 small boomerangs and makes \$80
Danny has found an effective way to organize his work, using a table. He has made some mistakes in this table, however. Part of the problem is that he loses track of the two constraints. It might have been helpful for him to include two additional columns headed: 'Time needed (≤ 24 hours)' and 'Total number made (≤ 10).' Then he could test each case and put a check mark if it satisfies both constraints.	No of small s / 5×8 / No of large / $l \times 10$ / Profit 0 / 0 / 8 / 80 / 80 1 / 8 / ~~8~~ 7 / 70 / 78 2 / 16 / 6 / 60 / 76 3 / 24 / 5 / 50 / 74 4 / 32 / 5 / 50 / 82 ← 5 / 40 / 4 / 40 / 80 6 / 48 / 3 / 30 / 78 The most Profit is \$82
Jeremiah has tried an algebraic approach and has hit upon the correct solution. However, he has used equalities rather than inequalities. He needs to calculate the total profit to complete the question.	Small boomerangs = x Large boomerangs = y Time to carve $2x + 3y = 24$ ① Only 10 can be decorated $x + y = 10$ ② $2x + 2y = 20$ ③ ①-③ $y = 4$ $x = 6$ So make 4 large boomerangs 6 small boomerangs
Tanya has used a graphical approach, but her graph of 2x + 3y = 24 is inaccurate and should be redrawn. This graph is powerful in that it shows the entire feasible solution space – the integer points on the grid. She has not explained why her method will give the greatest profit.	 $x+y \le 10$ $2x+3y \le 24$ 5 small + 5 large = 40+50 = \$90 $3y = 24-2x$ $y = 8-\frac{2}{3}x$ Slope = $-\frac{2}{3}$ int = 8

Review individual solutions to the assessment task (10 minutes)

Ask students to read through their original responses to the task.

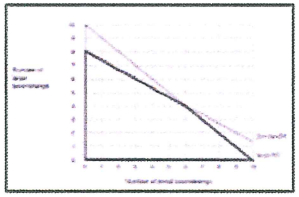
Read through your original solution and think about your work this lesson.
Write down what you have learned during the lesson.
Which method would you prefer to use if you were doing the task again? Why?

Encourage students to compare the new approaches they met during the lesson with their original method. Some teachers set this task as homework.

SOLUTIONS

If one assumes that ten boomerangs are made, then the following table of possibilities may be made. The constraint on carving hours is broken when more than four large boomerangs are made.

Number of small	Number of large	Total number (≤ 10)	Carving hours (≤ 24)	Profit made
10	0	10	20	$80
9	1	10	21	$82
8	2	10	22	$84
7	3	10	23	$86
6	4	10	24	$88
5	5	10	25	$90

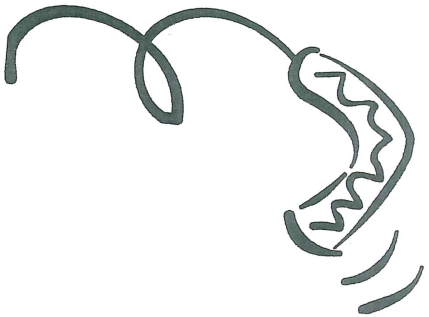


This approach, however, does not include the possibility of making fewer than ten boomerangs. A more complete approach would be to draw a graph showing all possibilities.

The possible combinations to be checked are the integer points within the bold region on the graph. The maximum profit occurs, however, when six small and four large boomerangs are made. This profit is $88.

(This can be seen graphically by drawing lines of constant profit on the graph, e.g. $8x + 10y = 80$. This idea may emerge in discussion.)

Boomerangs



Phil and Cath make and sell Boomerangs for a school event.

The money they raise will go to charity.
They plan to make them in two sizes: small and large.

Phil will carve them from wood.
The small boomerang takes 2 hours to carve and the large one takes 3 hours to carve.
Phil has a total of 24 hours available for carving.

Cath will decorate them.
She only has time to decorate 10 boomerangs of either size.

The small boomerang will make $8 for charity.
The large boomerang will make $10 for charity.

They want to make as much money for charity as they can. How many small and large boomerangs should they make?

How much money will they make?

Sample Responses to Discuss: Alex

Phil can only make 12 small or 8 large boomerangs in 24 hours
12 small makes $96
8 large makes $80
Cath only has time to make 10, so $96 is impossible.
She could make 10 small boomerangs which will make $80.
So she either makes 8 large or 10 small boomerangs
and makes $80

What assumptions has Alex made?

Are these assumptions correct? Explain your answer.

General comments:

Sample Responses to Discuss: Danny

No of Small s	Sx8	No of large	Lx10	Profit
0	0	8	80	80
1	8	~~8~~ 7	70	78
2	16	6	60	76
3	24	5	50	74
4	32	5	50	82 ←
5	40	4	40	80
6	48	3	30	78

The most Profit is $82

Why do you think Danny starts with 0 small and 8 large boomerangs and stops at 6 small and 3 large boomerangs?

What piece of information has Danny forgotten to use?

General comments:

Sample Responses to Discuss: Jeremiah

Small boomerangs = x

Large boomerangs = y

Time to carve $2x + 3y = 24$ ①

Only 10 can be decorated $x + y = 10$ ②

$2x + 2y = 20$ ③

① - ③ $y = 4$ $x = 6$

So make 4 large boomerangs

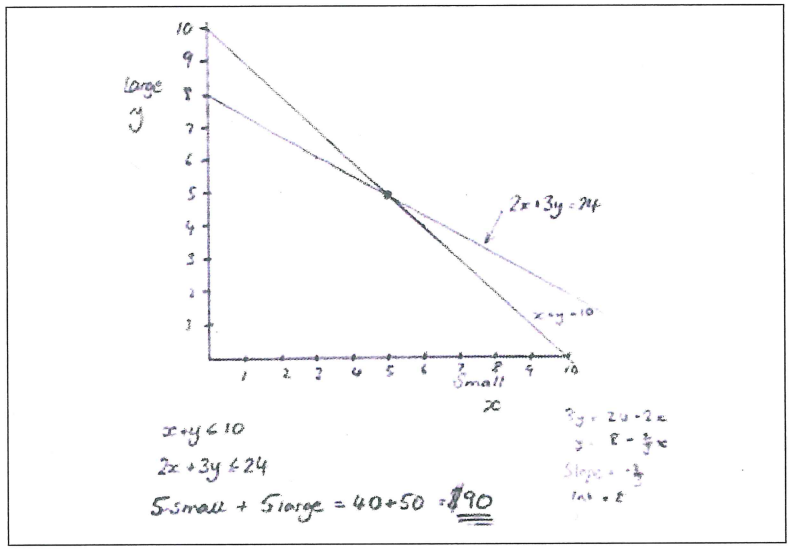
6 small boomerangs

Is it correct to use the equals sign in equations 1, 2, and 3? Explain your answer.

Why is Jeremiah's solution incomplete?

General comments:

Sample Responses to Discuss: Tanya



What is the purpose of the graph?

What is the point of figuring out the slope and intercept?

General comments:

Evaluating *Sample Responses to Discuss*

- What do you like about the work?
- How has each student organized the work?
- What mistakes have been made?
- What isn't clear?
- What questions do you want to ask this student?
- In what ways might the work be improved?

Mathematics Assessment Project
CLASSROOM CHALLENGES

This lesson was designed and developed by the
Shell Center Team
at the
University of Nottingham
Malcolm Swan, Nichola Clarke, Clare Dawson, Sheila Evans
with
Hugh Burkhardt, Rita Crust, Andy Noyes, and Daniel Pead

It was refined on the basis of reports from teams of observers led by
David Foster, Mary Bouck, and Diane Schaefer
based on their observations of trials in US classrooms along with comments from teachers and other users.

This project was conceived and directed for MARS: Mathematics Assessment Resource Service
by
Aland Schoenfeld, Hugh Burkhartd, Daniel Pead, and Malcolm Swan
and based at the University of California, Berkeley

We are grateful to the many teachers, in the UK and the US, who trialed earlier versions of these materials in their classrooms, to their students, and to Judith Mills, Carol Hill, and Alvaro Villanueva who contributed to the design.

This development would not have been possible without the support of
Bill & Melinda Gates Foundation

We are particularly grateful to
Carina Wong, Melissa Chabran, and Jamie McKee

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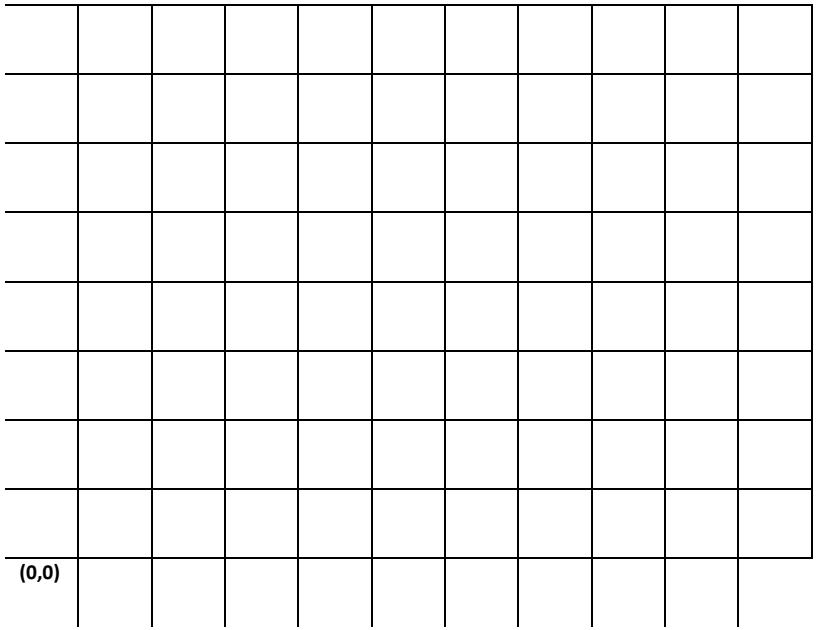
Name_____ Date_____ Period_____

Montanez Farms owns a 3600 acre field. The family of farmers wants to plant Iceberg lettuce, which yields $200 per acre and Romaine lettuce which yields $250 per acre. To prevent loss due to disease, the farmers should plant no more than 3000 acres of iceberg lettuce and 2500 acres of Romaine lettuce. How many acres of each crop should the farmers plant in order to maximize profits? What is the maximum profit?



MAT.HS.TE.1.0AREI.J.087

Sample Item ID:	MAT.HS.TE.1.0AREI.J.087
Grade:	HS
Claim(s):	**Claim 1: Concepts and Procedures** Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.
Assessment Target(s):	1 J: Represent and solve equations and inequalities graphically.
Content Domain:	Algebra
Standard(s):	A-REI.12
Mathematical Practice(s):	1, 5
DOK:	2
Item Type:	TE
Score Points:	2
Difficulty:	M
Key:	See Sample Top-Score Response.
Stimulus/Source:	
Target-specific attributes (e.g., accessibility issues):	
Notes:	TEI Template: Multi-lines; then select

Graph this system of inequalities below on the given coordinate grid.

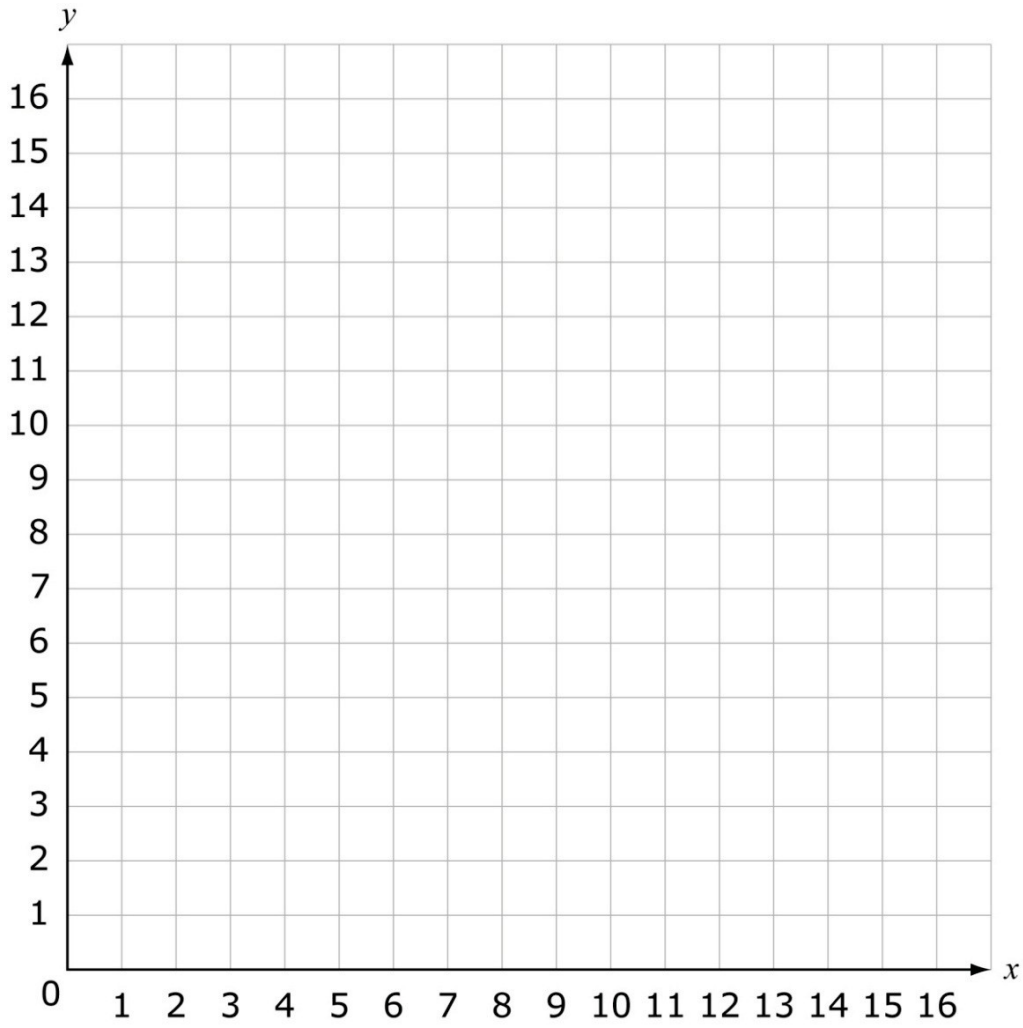
$$x + y \geq 12$$

$$20x + 30y \leq 300$$

To create a line, click in the grid to create the first point on the line. To create the second point on the line, move the pointer and click. The line will be automatically drawn between the two points. Use the same process to create additional lines.

When both inequalities are graphed, select the region in your graph that represents the solution to this system of inequalities. To select a region, click anywhere in the region.

To clear a selected region, click anywhere in the selected region.



Scoring Rubric:

Responses to this item will receive 0-2 points, based on the following:

2 points: The student has a solid understanding of how to solve a system of inequalities graphically. The student correctly graphs both inequalities and identifies the correct region that represents the solution to the system, region IV.

1 point: The student has some understanding of how to solve a system of inequalities graphically. The student correctly graphs both inequalities but does not identify the correct region that represents the solution to the system. **OR** The student incorrectly graphs one or both inequalities but identifies the correct region that represents the solution to the incorrectly graphed system.

0 points: The student demonstrates inconsistent understanding of how to solve a system of inequalities graphically. The student does not correctly graph both inequalities and/or does not identify the correct region that represents the solution to the system.

TE Information

Item Code: MAT.HS.TE.1.0AREI.J.087

Template: Multi-lines; then select

Interaction Space Parameters:
A: False (do not use default grid)
B: Bottom-left corner is (0,0); top-right corner is (15,15); grid line increment size is 1;
axes are labeled with X and Y C: Label each grid increment
D: True (support snap-to behavior)
E: Limit = true
F: Maximum number of lines is 2
G: Solid lines
H: Limit = true
I: Maximum number of sections that can be selected is 1

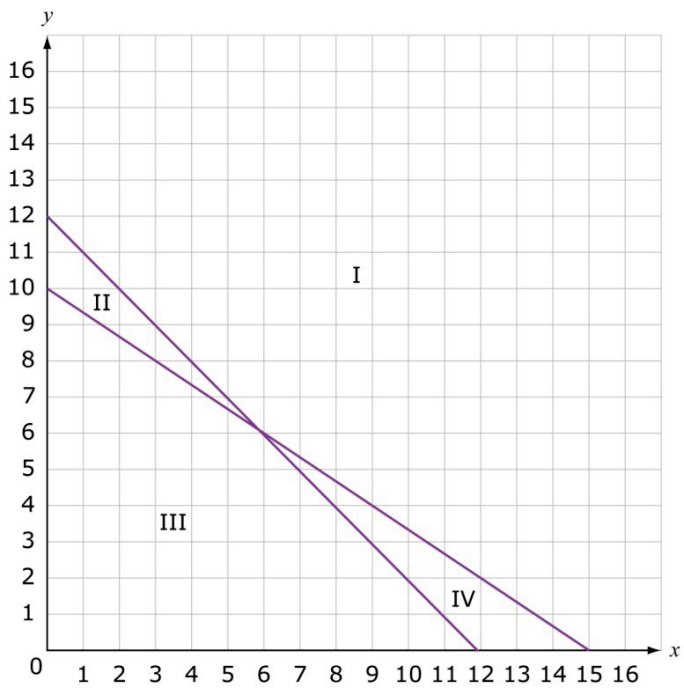
Scoring Data:
Line 1:
x-Intercept Consider = true (12,0)
0
y-Intercept Consider = true (0,12)
0

Slope
Consider = false
Line 2:
x-Intercept Consider = true (15,0)
0
y-Intercept
Consider = true
(0,10)
0
Slope
Consider = false

Grid section: II

The figure below represents how the four sections would be labeled: section I, section II, section III, and section IV.

One line contains the points (12, 0) and (0, 12). The other line contains the points (15, 0) and (0, 10).



Scoring Rule Explanation:
Based on the scoring rule and the scoring data for this particular item, students that create two lines representing $y = -x + 12$ and $y = -2/3x + 10$ and select the section of the plane represented by the intersection of $y \geq x + 12$ and $y \leq 2/3x + 10$ (IV, above) will receive 1 point. All other responses will receive 0 points.

MHS Algebra 1 Mathematics Curriculum

Unit 2 – Linear and Exponential Relationships

Domain: Functions (F)

Essential Question: How do linear relationships help model exponential functions?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Compare and contrast linear and non-linear functions to develop appropriate models for given situations** **F-IF** (**) Understand the concept of a function and use function notation	**How…** **F.IF.1:** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*). **F.IF.2:** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. **F.IF.3:** Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1.*	**Students will:** **Function Notation Exit Quiz** F.IF. 1 Lesson plans with links.docx **Function Sample Questions** F-IF.1 Functions.pdf **Evaluating Functions for inputs into their domain** Function Stations F.IF.2.docx	**F.IF.2:** The domain of a function given by an algebraic expression, unless otherwise specified, is the largest possible domain. Examples: • If $f(x) = x^2 + 4x - 12$, find $f(2)$. • Let $f(x) = 2(x+3)^2$. Find $f(3)$, $f(-\frac{1}{2})$, $f(a)$, and $f(a-h)$ If P(*t*) is the population of Tucson *t* years after 2000, interpret the statements P(0) = 487,000 and P(10)-P(9) = 5,900.

Vocabulary: Arithmetic and geometric sequences, arithmetic and geometric situations, base, effect on graph, end behavior, equal distance over equal intervals, equal factor over equal intervals, even and odd functions, functional inputs, function notation, graphs, graphical effects, intercepts, intervals, parameters of linear/exp, recursive and explicit formulae, relative maximum/minimum, slope, symmetry

Note Sheet

Vocabulary	Definition	Example
Function		
Function Rule		
Function Notation		
Domain		
Range		
Input		
Output		
Independent Variable		
Dependent Variable		
Vertical Line Test		
Ordered Pair		

Example

Number of Hours a Plumber Works	Cost to Home Owner
1	$65
2	$90
3	$115
4	$140

Words used for x-values are Domain, Independent, and Input Values.

Words used for y-values are Range, Dependent, and Output Values.

Question 1: For the above situation do you see a relationship between the input and output values?

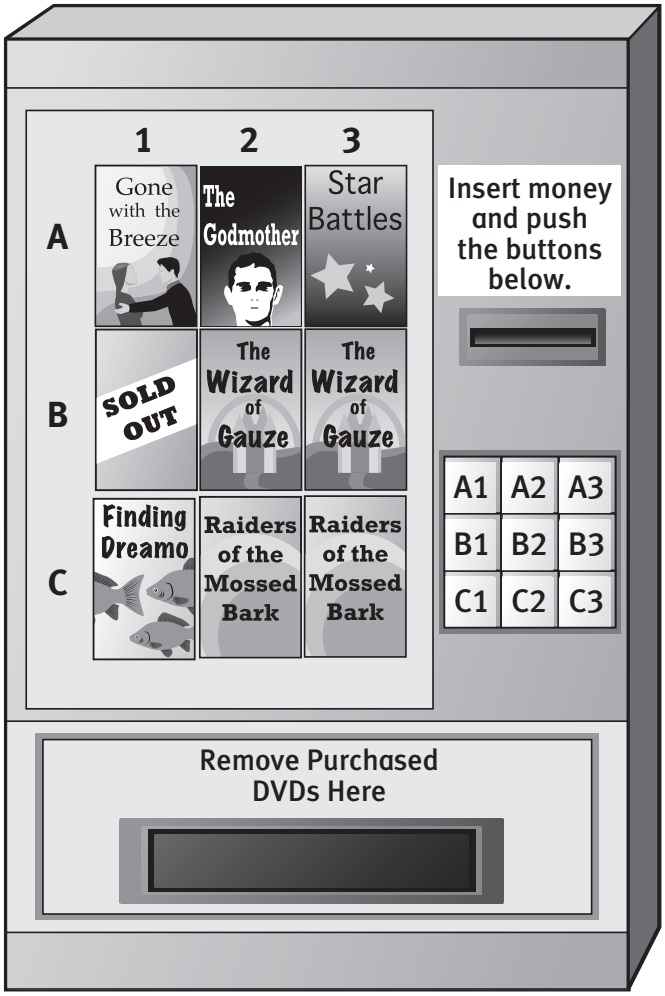
Introduction to Functions
Vending Machines

SUGGESTED LEARNING STRATEGIES: Visualization

My Notes

Use this machine to answer the questions on the next page.

DVD Vending Machine



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My Notes

SUGGESTED LEARNING STRATEGIES: Activating Prior Knowledge

1. Suppose you inserted your money and pressed A1. What item would you receive?

2. Suppose you inserted your money and pressed C2. What item would you receive?

3. Suppose you inserted your money and pressed B3. What item would you receive?

4. If the machine were filled properly, what would happen if you pressed any of those same buttons again?

Each time you press a button, an **input**, you may receive a DVD, an **output**.

5. In the DVD vending machine situation, does every input have an output? Explain your response.

6. Each combination of input and output can be expressed as a **mapping** written *input* → *output*. For example, B2 → Wizard of Gauze]

a. Write as mappings each of the possible combinations of buttons pushed and DVDs received in the vending machine.

MATH TERMS
A **mapping** is a visual representation of a relation in which an arrow associates each input with its output.

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SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Note Taking, Create Representations

My Notes

b. Mappings relating values from one set of numbers to another set of numbers can be written as **ordered pairs**. Write the following numerical mappings as ordered pairs.

Input		Output	Ordered Pairs
1	→	−2	(1, −2)
2	→	1	
3	→	4	
4	→	7	

MATH TERMS
An **ordered pair** shows the relationship between two values, written in a specific order using parentheses notation and a comma separating the two values.

A **relation** is a set of ordered pairs. The list of ordered pairs that you wrote in Item 6(b) is a relation.

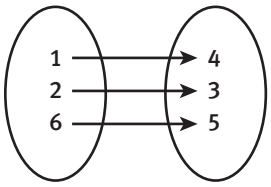
Relations can have a variety of representations. Consider the relation {(1, 4), (2, 3), (6, 5)}, shown here as a set of ordered pairs. This relation can also be represented in these ways.

MATH TERMS
relation

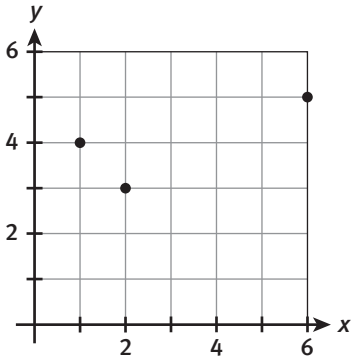
Table

x	y
1	4
2	3
6	5

Mapping



Graph



7. You represented the vending machine situation using mappings in Item 6. Other representations can also be used to illustrate how the inputs and outputs of the vending machine are related.

a. Create a table to illustrate how the inputs and outputs of the vending machine are related.

b. In representing the vending machine inputs and outputs, what decisions would need to be made to create the graph?

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SUGGESTED LEARNING STRATEGIES: Group Presentation, Create Representations

My Notes

ACADEMIC VOCABULARY

function

A **function** is a relation in which each input is paired with, at most, one output.

8. Compare and contrast the DVD Vending Machine with a function.

9. Suppose when pressing button C1 button on the vending machine both "Finding Dreamo" and "Raiders of the Mossed Bark" come out. How does this vending machine resemble or not resemble a function?

10. Imagine a machine where you input an age and the machine gives you the name of anyone who is that age. Compare and contrast this machine with a function. Explain by using examples and create a representation of the situation.

11. Create an example of a situation (math or real-life) that behaves like a function and another that does not behave like a function. Explain why you chose each example to fit the category.

a. Behaves like a function:

b. Does not behave like a function:

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SUGGESTED LEARNING STRATEGIES: Activating Prior Knowledge, Think/Pair/Share

My Notes

12. Identify whether each list of ordered pairs represents a function. Explain your answers.

a. {(5, 4), (6, 3), (7, 2)}

b. {(4, 5), (4, 3), (5, 2)}

c. {(5, 4), (6, 4), (7, 4)}

13. Using positive integers, write two relations as a list of ordered pairs below, one that is a function and one that is not a function.

Function:

Not a function:

The set of all inputs for a function is known as the **domain** of the function. The set of all outputs for a function is known as the **range** of the function.

ACADEMIC VOCABULARY

domain
range

14. Consider a vending machine where inserting 25 cents dispenses one pencil, inserting 50 cents dispenses 2 pencils, and so forth up to and including all 10 pencils in the vending machine.

a. What is the domain in this situation?

b. What is the range in this situation?

WRITING MATH

The **domain** and **range** of a function can be written using set notation.

For example for the function {(1,2), (3,4),(5,6)} the domain is {1,3,5} and the range is {2,4,6}.

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SUGGESTED LEARNING STRATEGIES: Quickwrite

My Notes

15. For each function below, identify the domain and range.

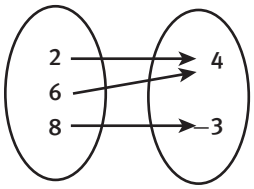
a.

input	output
7	6
3	−2
5	1

Domain:

Range:

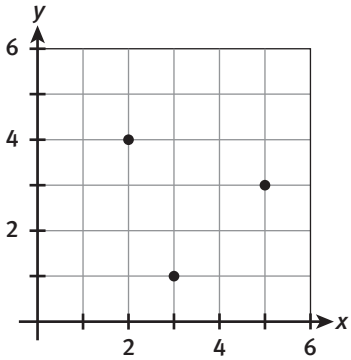
b.



Domain:

Range:

c.



Domain:

Range:

d. $\{(-7, 0)(9, -3)(-6, 2.5)\}$

Domain:

Range:

MATH TERMS
A **finite** set has a fixed countable number of elements. An **infinite** set has an unlimited number of elements.

16. Each of the functions that you have seen has a **finite** number of ordered pairs. There are functions that have an **infinite** number of ordered pairs. Describe any difficulties that may exist trying to represent a function with an infinite number of ordered pairs using the four representations of functions that have been described thus far.

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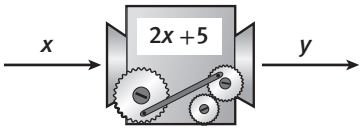
SUGGESTED LEARNING STRATEGIES: Activating Prior Knowledge

My Notes

17. Sometimes, machine diagrams are used to represent functions. In the function machine below, the inputs are labeled x and the outputs are labeled y. The function is represented by the expression $2x + 5$.



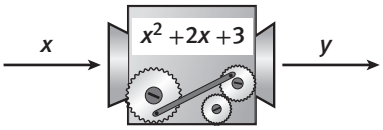
a. If $x = 7$ is used as an input, what is the output?

b. If $x = -2$ is used as an input, what is the output?

c. If $x = \frac{1}{2}$ is used as an input, what is the output?

d. Is there any limit to the number of input values that can be used with this expression? Explain.

Consider the function machine below.



18. Use the diagram to find the (input, output) ordered pairs for the following values.

a. $x = -5$

b. $x = \frac{3}{5}$

c. $x = -10$

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SUGGESTED LEARNING STRATEGIES: Create Representations, Activating Prior Knowledge, Group Discussion

My Notes

19. Make a function machine for the expression $10 - 5x$. Use it to find ordered pairs for $x = 3$, $x = -6$, $x = 0.25$, and $x = \frac{3}{4}$.

Creating a function machine can be time consuming and awkward. The function represented by the diagram in Item 17 can also be written algebraically as the equation $y = 2x + 5$.

20. Evaluate each function for $x = -2$, $x = 5$, $x = \frac{2}{3}$, and $x = 0.75$. For each x-value, find the corresponding y-value. Place the results in a table.

a. $y = 9 - 4x$

b. $y = \frac{1}{x}$

When referring to the functions in Item 20, it can be confusing to distinguish among them since each begins with "$y =$." Function notation can be used to help distinguish among different functions.

For instance, the function $y = 9 - 4x$ in Item 20(a) can be written:

This is read as "f of x" and $f(x)$ is equivalent to y.

$$f(x) = 9 - 4x$$

"f" is the name of the function.

x is the input variable.

MATH TIP

It is important to recognize that $f(x)$ *does not* mean f multiplied by x.

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SUGGESTED LEARNING STRATEGIES: Create Representations

My Notes

21. To distinguish among different functions, it is possible to use different names. Use the name h to write the function from Item 20b using function notation.

Function notation is useful for evaluating functions for multiple input values. To evaluate $f(x) = 9 - 4x$ for $x = 2$, you substitute 2 for the variable x and write $f(2) = 9 - 4(2)$. Simplifying the expression yields $f(2) = 1$.

MATH TIP

Notice that $f(x) = y$. For a **domain** value x, the associated **range** value is $f(x)$.

22. Use function notation to evaluate $f(x)$ shown above at $x = 5$, $x = -3$, and $x = 0.5$.

23. Use the values for x and $f(x)$ from Item 22. Display the values using each representation.

a. list of ordered pairs

b. table of values

c. mapping

d. graph

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My Notes

24. Evaluate each function for $x = -5$, $x = \frac{4}{3}$.

a. $f(x) = 2x - 7$

b. $g(x) = 6x - x^2$

c. $h(x) = \frac{2}{x^2}$

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

1. The set $\{(3,5), (-1,2), (2,2), (0,-1)\}$ represents a function. Identify the domain and range of the function. Then display the function using each representation.

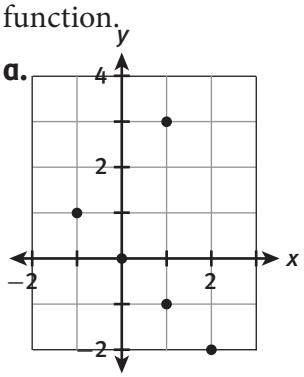
a. a table

b. a mapping

c. a graph

2. Explain why each of the following is *not* a function.

a.



b.

x	y
12	−8
17	3
−4	9
17	−5

3. Evaluate the functions for each domain value indicated.

a. $p(x) = 3x + 14$

$x = -5, 0, 4$

b. $h(t) = t^2 - 5t$

$t = -2, 0, 5, 7$

4. MATHEMATICAL REFLECTION Which representation of a function do you feel is most useful? Why? Which one do you feel is least useful? Why?

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Name_____Date_____Period_____

Algebra 1 CP
Relations and Functions
Exit Check: Quiz

1.) Which function rule best models the data in the table? **(1 point)**

a.) $y = x^2 + 8$

b.) $y = -x + 6$

c.) $y = -3x + 6$

d.) $y = 3x + 9$

x	y
-1	9
0	6
1	3

2.) Find the range of the function $f(x) = 3x^2 + 2x - 5$ for the domain {-2,0,2}. **(1 point)**

a.) {-21, 0, 16}

b.) {-21, -5, 11}

c.) {3, 0, 16}

d.) {3, -5, 11}

3.) Use the given rule to complete the table of values for the function: $f(x) = 4x - 9$ **(2 points)**

x	**4x - 9**	**f(x)**
-2		
0		
1		
2		

4.) State the domain and the range for each of the following relations, then determine whether or not each one is a function and explain why or why not. **(4 points each / 12 points total)**

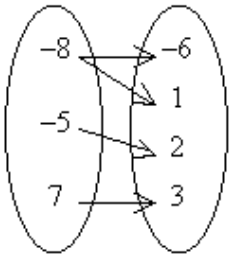
A. { (-3,4) , (2,9) , (1,4) , (5,-1) }

Domain:_____

Range:_____

Function?_____

B.

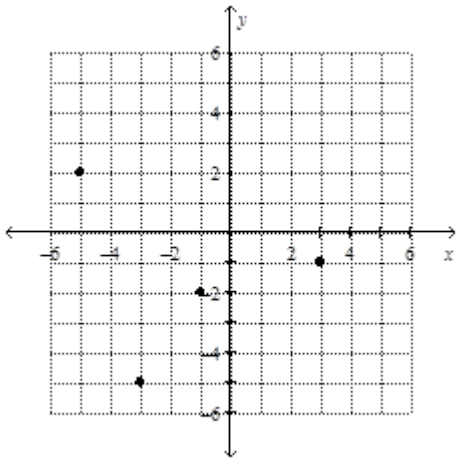


Domain:_____

Range:_____

Function?_____

C.



Domain:_____

Range:_____

Function?_____

5.) The table below shows the recorded temperatures for Mount Washington on five different days. Is this relation a function? Why or why not? Use may use a mapping or graph to help explain if needed: **(4 points)**

Mount Washington Temperatures (F)

At Base of Mountain	At Top of Mountain
80	62
65	48
93	74
98	91
74	59

Mapping or Graph?

6.) The children in the Ramirez family decided to record the number of hours and distances they drove on their cross country trip during summer vacation. They recorded data for each day of the trip and would now like to organize it for a school project. **(4 points)**

Day	Miles travelled
1	500
2	520
3	610
4	500
5	470

Give an explanation of how they should organize the data including:

Mapping or Graph?

Independent variable: _____

Dependent variable: _____

Is it a Relation or a Function? Why?

_____

_____

_____

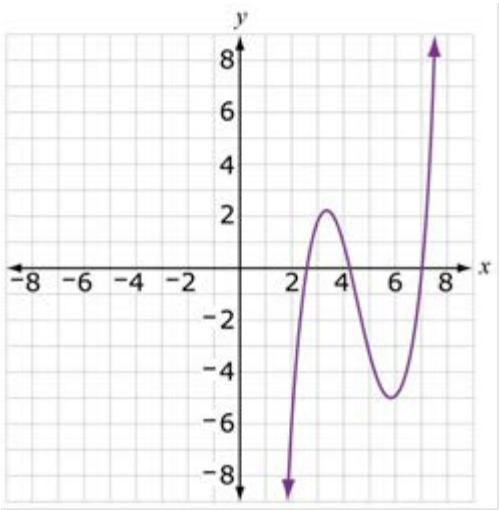
MAT.HS.SR.1.00FIF.K.082

Sample Item ID:	MAT.HS.SR.1.00FIF.K.082
Grade:	HS
Claim(s):	**Claim 1: Concepts and Procedures** Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.
Assessment Target(s):	1 K: Understand the concept of a function and use function notation.
Content Domain:	Functions
Standard(s):	F-IF.1
Mathematical Practice(s):	4, 6
DOK:	2
Item Type:	SR
Score Points:	1
Difficulty:	M
Key:	YNYN
Stimulus/Source:	
Target-specific attributes (e.g., accessibility issues):	
Notes:	Multi-Answer Item

For numbers 1a – 1d, determine whether each relation is a function.

1a. $\{(0,1),(1,2),(3,1),(4,2)\}$ ○ Yes ○ No

1b. $y = \pm\sqrt{4-x^2}$ ○ Yes ○ No

1c. 

○ Yes ○ No

1d. $\{(5,3),(2,4),(5,2)\}$ ○ Yes ○ No

Key and Distractor Analysis

1a. Y $\{(0,1),(1,2),(3,1),(4,2)\}$

All x-coordinates are unique, so it meets the definition of a function.

1b. N $y = \pm\sqrt{4 - x^2}$

An input of $x = 1$ has two corresponding outputs, $y = \sqrt{3}$ and $y = -\sqrt{3}$ so it fails to meet the definition of a function.

1c. Y This is a function since for each value chosen along the x-axis, there is exactly one y-value on the graph that corresponds to it.

1d. N This is not a function since the input of 5 has two corresponding output values, 3 and 2.

Name_____ Date_____ Period_____

Algebra 1 – Function Stations

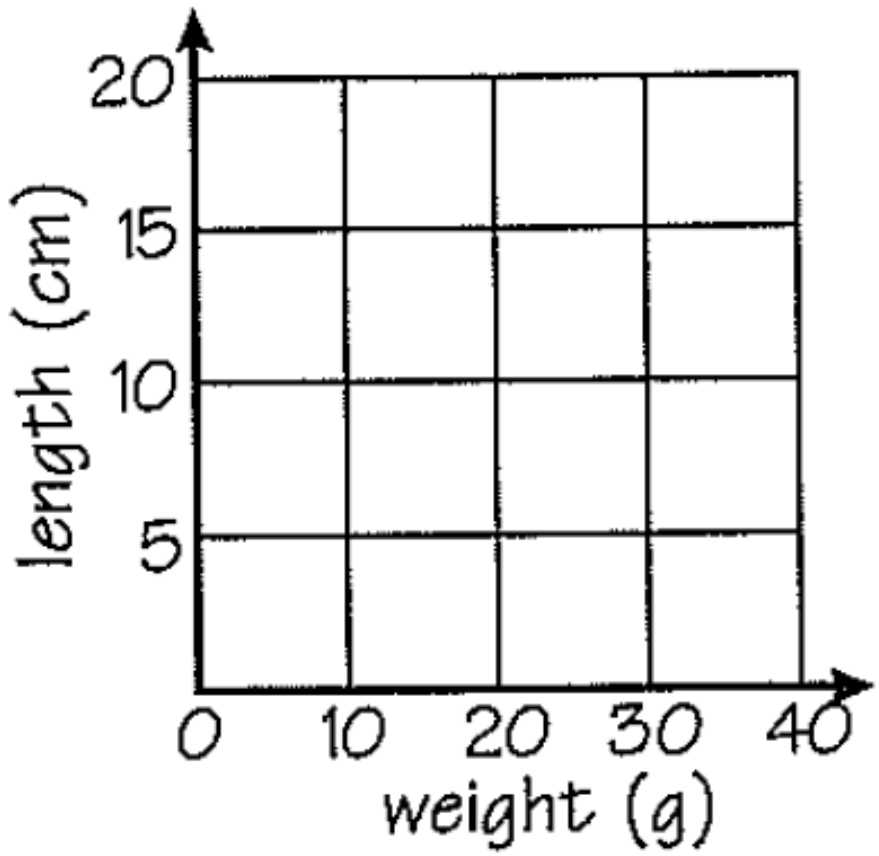
STATION 1

Suppose you suspend weights from a spring. The relationship between the length of the spring and the amount of weight suspended from it is given by the function:

$$L = 0.3w + 4$$

where L is length (cm), and w is weight (g)

w	L
0	
10	
20	
30	
40	



Name_____ Date_____ Period_____

Algebra 1 – Function Stations

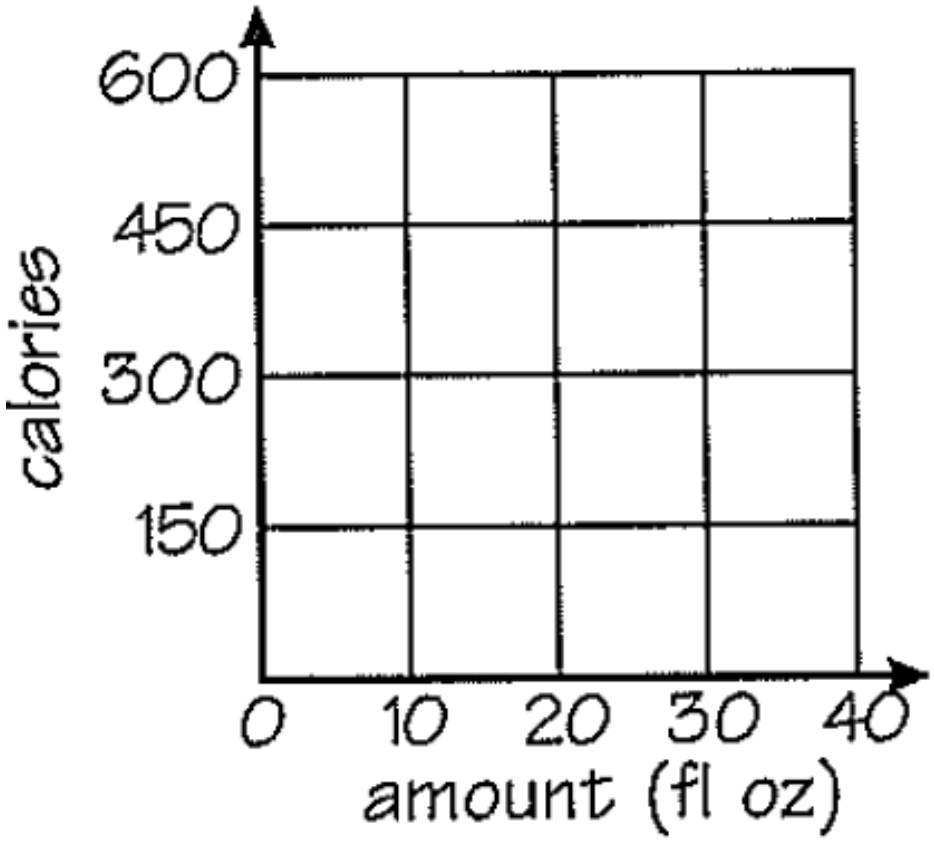
STATION 1

Suppose you order a pitcher of root beer.
The relationship between the number of calories in the root beer and the amount of root beer is given by the function:

$$C = 14b$$

where C is calories, and b is amount (fl oz)

b	C
0	
10	
20	
30	
40	



Name_____ Date_____ Period_____

Algebra 1 – Function Stations

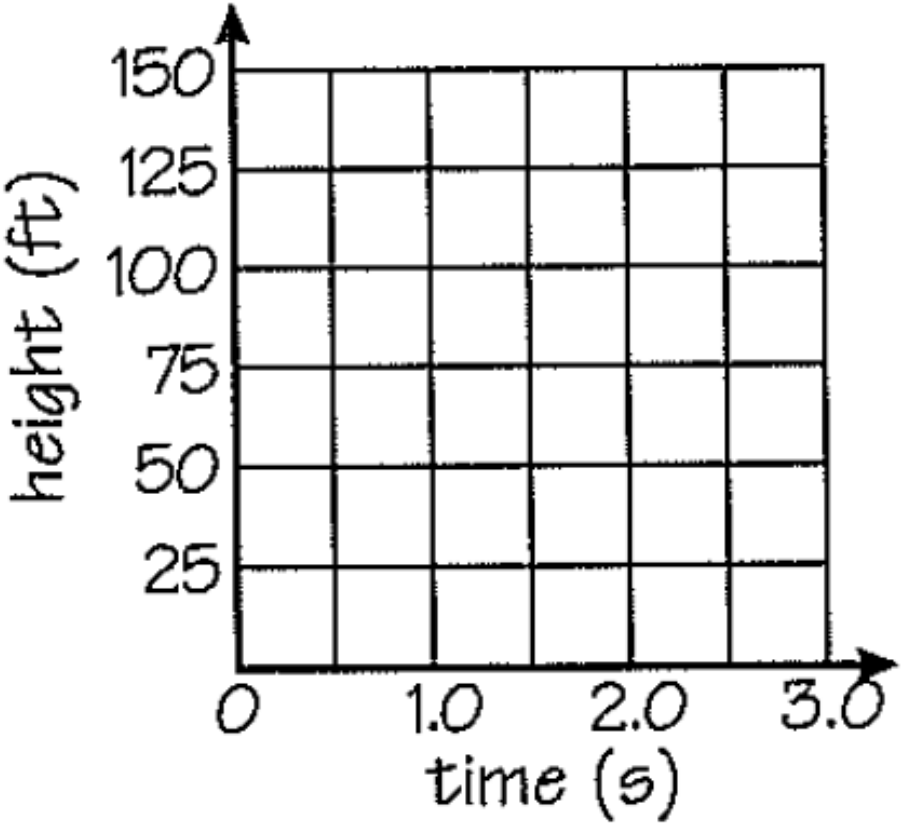
STATION 1

Suppose you are standing on a cliff 144 feet above the ocean surface. You drop a rock. The relationship between the height of the rock above the water and time since you dropped it is given by the function:

$$h = 144 - 16t^2$$

where h is height (ft), and t is time (s).

t	h
0	
0.5	
1.0	
1.5	
2.0	
2.5	
3.0	



Name_____ Date_____ Period_____

Algebra 1 – Function Stations

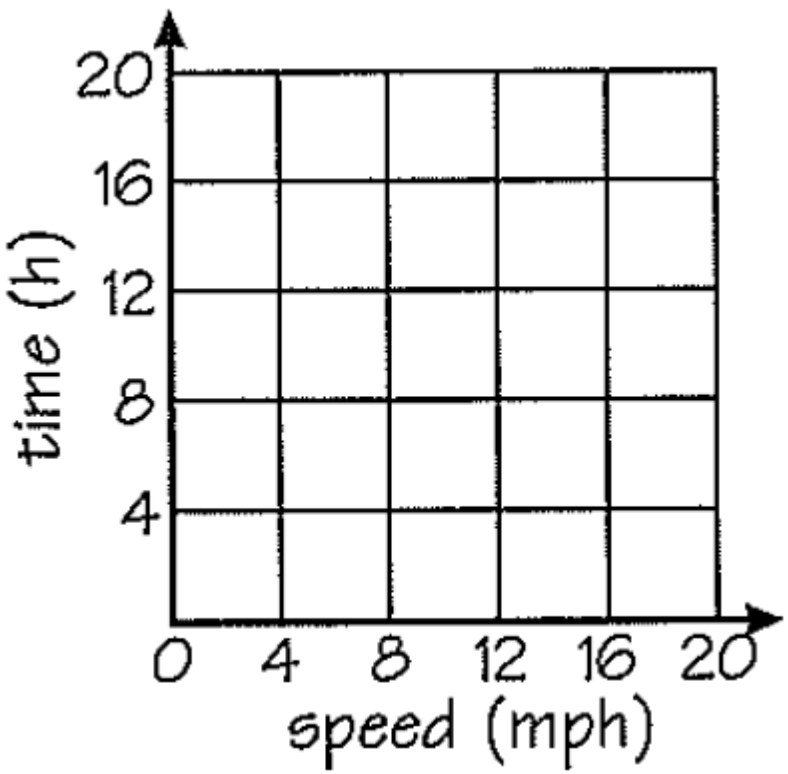
STATION 1

Suppose you plan to ride a bike 36 miles. The relationship between the time needed to complete the trip and your average speed is given by the function:

$$t = \frac{36}{r}$$

where t is time (h), and r is average speed (mph)

r	t
2	
3	
4	
6	
9	
12	
18	



Unit 2 – Linear and Exponential Relationships

Domain: Functions (F)

Essential Question: How do linear relationships help model exponential functions?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Compare and contrast linear and non-linear functions to develop appropriate models for given situations** **F-IF** (***) Analyze functions using different representations	**How…** **F.IF.4:** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* **F.IF. 5:** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* **F.IF.6:** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	**Students will:** **Functional Analysis Activity** Linear Investigations Lesson F.IF.3 and F.I **Domain Representations (F.IF.5)** DomainReps F.IF.5 (Unit 2).pdf **Rate of Change Growth Rate (F.IF.6)** GrowthRate-AS-HeightAgeChartF.IF.6(Un	**F.IF.4:** See Unit Addendum **F.IF.6:** See Unit Addendum

Vocabulary: Arithmetic and geometric sequences, arithmetic and geometric situations, base, effect on graph, end behavior, equal distance over equal intervals, equal factor over equal intervals, even and odd functions, functional inputs, function notation, graphs, graphical effects, intercepts, intervals, parameters of linear/exp, recursive and explicit formulae, relative maximum/minimum, slope, symmetry

Teacher Directions:

CLE:

1.1.1 Identify, describe and analyze patterns and functions (including arithmetic and geometric sequences) from real-world context using tables, graphs, words and symbolic rules.
1.1.3 Translate one representation of a pattern into another representation.
1.1.8 Compare and contrast linear and exponential growth.

Objectives: Students will be able to:

Create a table of values for a pattern and graph the relationship.
Describe the relationship between variables including if it is linear.

Materials:

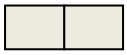
Copy of the worksheets Task One and Task Two – back to back, one set per student
Copy of Task Three – back-to-back
Copy of Task Four
Rulers

Procedure:

1. Open the class by demonstrating a pattern using toothpicks or drawing on the White Board.

2. Ask: What is the perimeter of this square? (4)
Yes, it takes 4 toothpicks to make this square.

3. Add a square like the picture below. Ask: What is the total toothpicks needed to make this shape made of 2 squares?

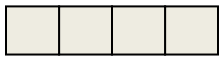


Yes, it takes 7 toothpicks to make this set of squares. The total toothpicks is 7.

4. Add a square like the picture below. Ask: What is the total toothpicks needed to make this figure?

Yes, it takes 10 toothpicks to make the shape.

5. Add a square like the picture below. Ask: What is the total toothpicks now?



Yes, it takes 13 toothpicks to make this figure.

4. Explain that today they will be creating tables of data based on patterns. Tell the students they will copy the table we just created graph the points and connect them on the paper that will be handed out.

5. Hand out Task One and Two. Have students use the tables just created to copy on the paper.

6. Demonstrate the words for the relationship must include both variables: As the squares increase by one, the total increases by 3.

6. Circulate as students work in pairs/groups.
7. Differentiation: Students who need more linear practice assign Task Three.
8. Assign Task Four to explore nonlinear relationships.

Closure: Exit ticket

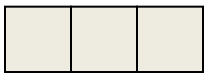
Linear Investigation Algebra One

Name _____ Date _____

Graphing Patterns

Task One: Use toothpicks to build the connected squares below.

1. Copy and complete the table below for the total toothpicks needed to build the connected squares.



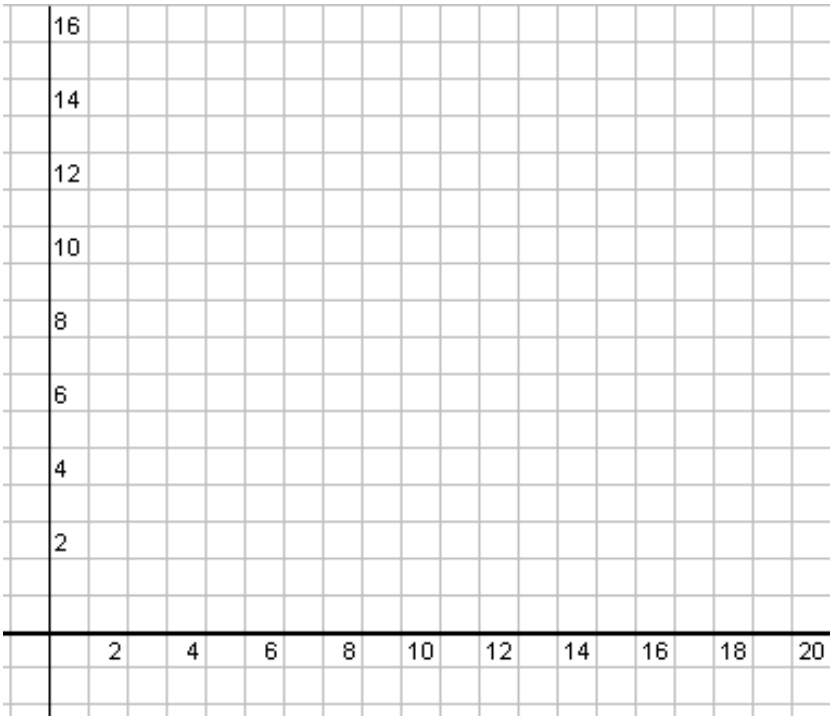
# of	**1**	**2**	**3**	**4**	**5**
Total Toothpicks	**4**				

2. Describe in words the patterns you see in the table.

_____

_____

3. Use the coordinate grid below to graph the data in the table. Label the x-axis as Number of Squares. Label the y- axis Total Toothpicks.



4. What is the total toothpicks needed for 7 squares? _____ 10 squares? _____

Linear Investigation Algebra One

Task Two

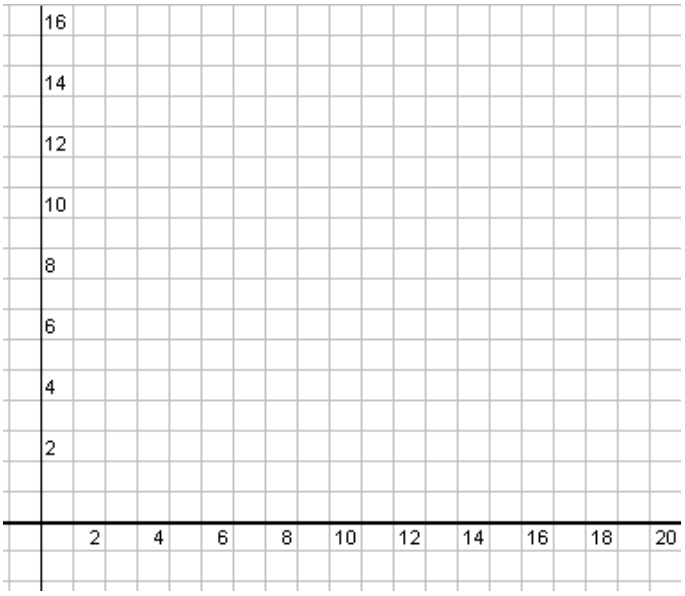
Kyle had $16 at the start of Computer Camp. He spent the same amount of money on snacks each day at camp. The table shows the amount of money, in dollars, he had left at the end of each day.

Snack Money	
Days	**Money Left**
0	$16
1	$14
2	$12
3	$10
4	$8
6	$4

5. How much money did Kyle have at the start of Computer Camp? _____
6. How much money is spent each day? _____
7. Describe in words the pattern you see in the table. _____

_____

8. Graph the relationship on the grid below. Label the x-axis Number of Days and the y-axis Money left.



9. How much money did Kyle have left on day 5? _____

10. When will he be out of money? _____

11. How is this graph the same as the graph in Task One? _____

12. How is this graph different than the graph in Task One? _____

_____

Linear Investigation Algebra One

Name _____ Date _____

Task Four

Bethany gets paid for doing chores around the house according to the following pattern:

$1 week one, $2 week two, $4 week three, $8 week four, $16 week five, $32 week six, and so on.

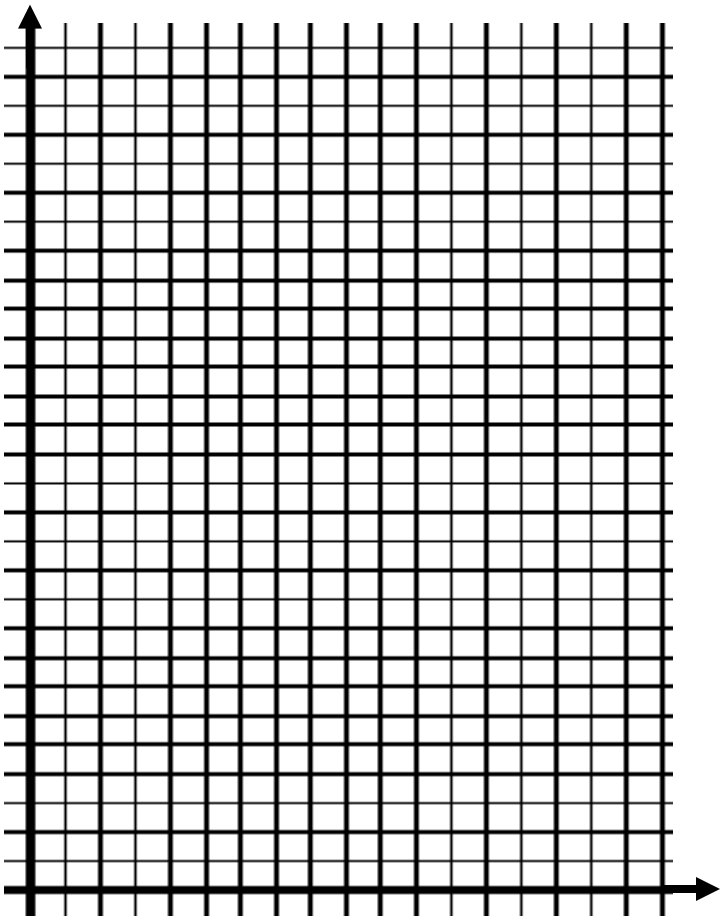
1. Complete the table below to show how much Bethany will get for the first 8 weeks of this plan.

Amount in Dollars								
Week Number	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**

2. Describe what dollar amounts are ADDED from week to week. _____

_____

3. Graph the relationship on the grid below. Connect the data points from week one to two to three etc.



4. How is this graph different than the graphs in Task One and Task Two?

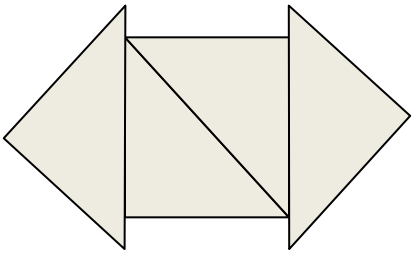
5. Look at the pattern you described in #2. Why do you think the graph is different than the graphs in Task One or Two?

Linear Investigation Algebra One

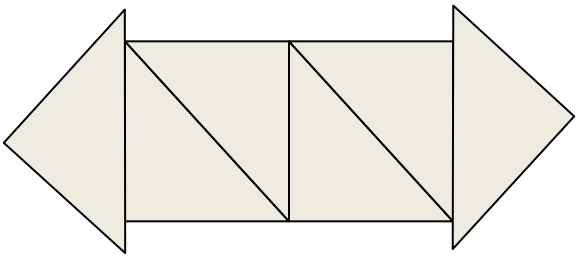
Name _____ Period _____ Date _____

Task Three

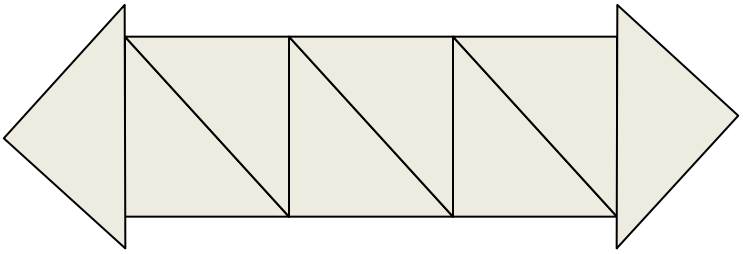
This is a one-day old worm.



This is a two-day old worm.



This is a three-day old worm.



1. In the space below, draw a four-day old worm.

2. In the space below, draw a five-day old worm.

Linear Investigation Algebra One

3. Complete the table for the worm patterns.

Number of Triangles					
Days Old					

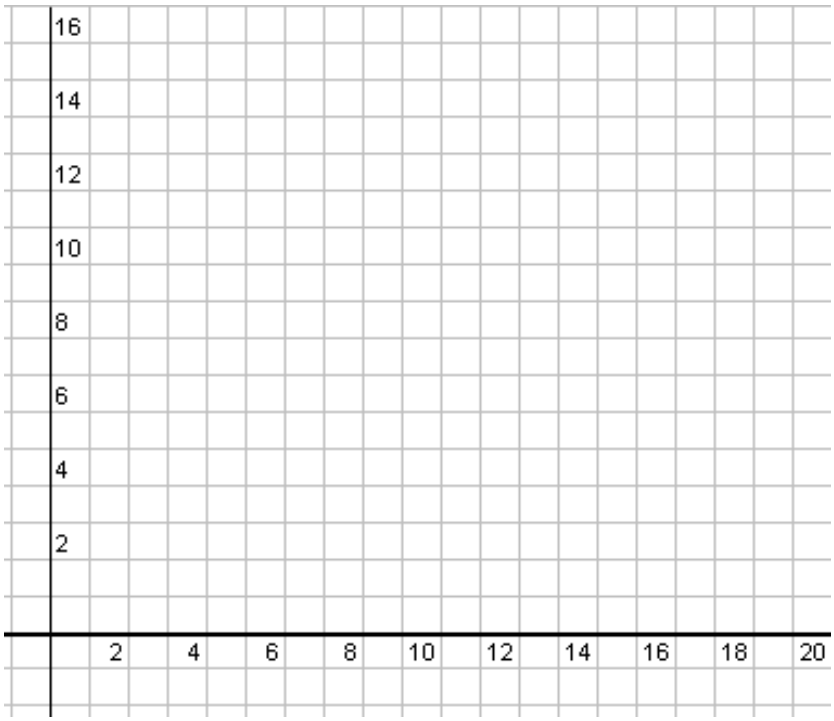
4. Describe in words the patterns between the days and number of triangles. _____

_____

5. How do you know from the table if this is a linear relationship? _____

_____

6. Use the coordinate grid below to graph the data in the table. Label the x-axis as Number of Days. Label the y- axis Total Triangles.



7. Is the graph linear? Explain how you know. _____

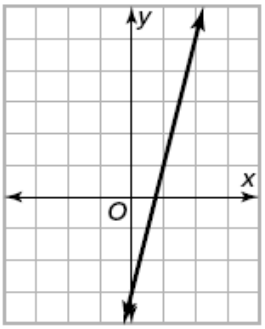
Linear Investigation Algebra One

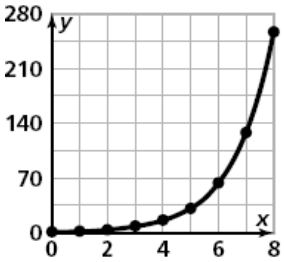
Exit Slip

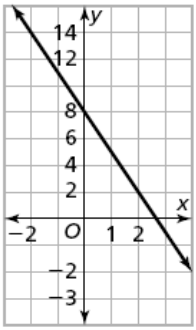
Name _____ Date _____

Classify each graph by writing linear or nonlinear.

1. _____ 2. _____ 3. _____







The table below shows the cost for books during a sale.

Cost	$3	$7	$11	$15	$19
Number of Books	1	2	3	4	5

4. Explain how you can tell if the pattern in the table is linear or nonlinear. _____

_____

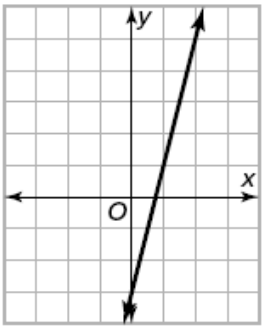
_____

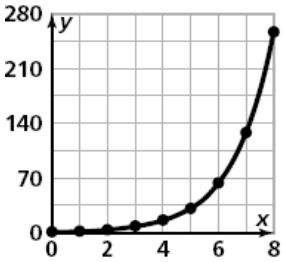
Exit Slip

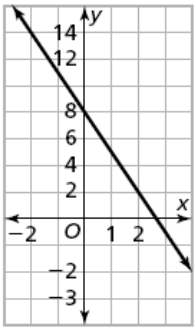
Name _____ Date _____

Classify each graph by writing linear or nonlinear.

1. _____ 2. _____ 3. _____







The table below shows the cost for books during a sale.

Cost	$3	$7	$11	$15	$19
Number of Books	1	2	3	4	5

4. Explain how you can tell if the pattern in the table is linear or nonlinear. _____

_____

_____

Domain Representations

NAME _____

For some functions, any x-value will have a corresponding $f(x)$-value. However, for other functions, certain x-values have no corresponding $f(x)$-values. Any x-value that corresponds to a specific $f(x)$-value is said to be in the *domain* of the function.

In this activity, you will analyze eight functions. You will use five different representations to determine the domain of each of those eight functions. The five different representations are:

- Graphical
- Tabular
- Number Line
- Verbal
- Symbolic

1. Graphical Representation

For each of the functions on the following page, sketch a graph on the grid provided.

- If you think that the function extends beyond the bounds of the grid, be sure to include arrows to indicate that it continues.

- Make a note if the function includes asymptotes or endpoints. If a function has asymptotes, be sure to indicate where it occurs. If a function has an endpoint, be sure to mark that point on the graph.

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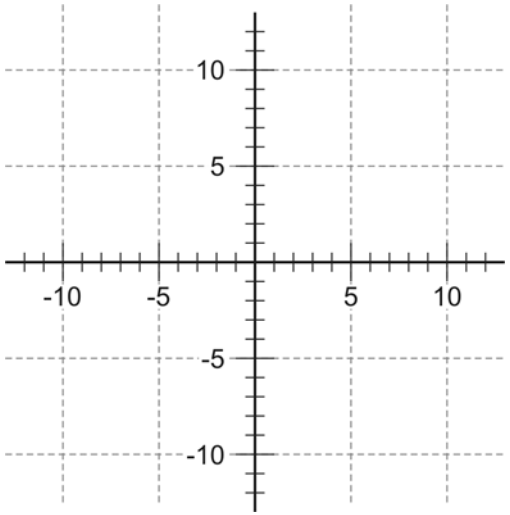
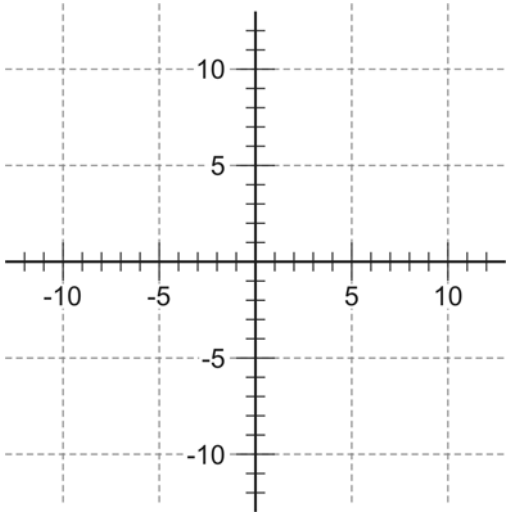
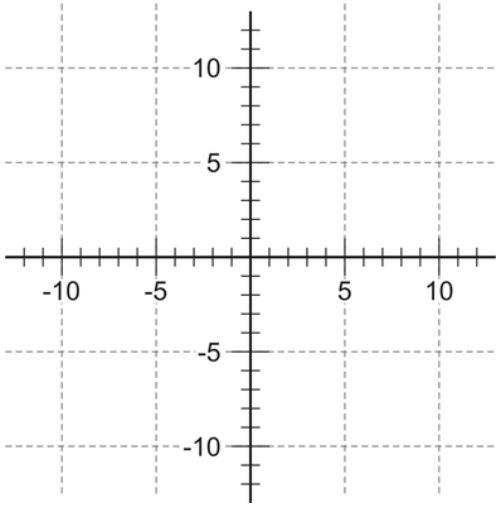
$f(x) = 3x - 2$



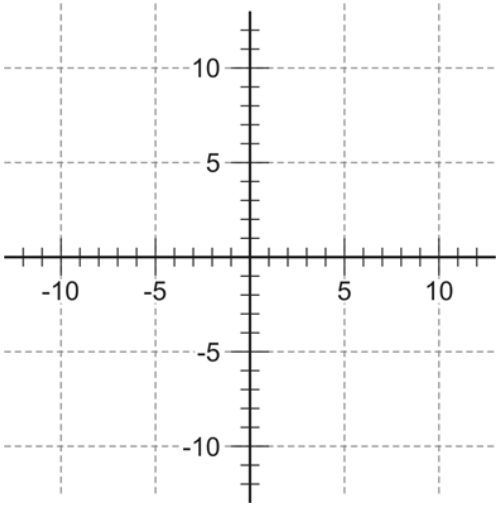
$f(x) = 4 - x^2$



$f(x) = \sqrt{9 - x^2}$



$f(x) = \frac{1}{x^2 - x - 6}$



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2. **Tabular Representation**
   - Complete the table of values for each function.
   - If a value of x produces an error for $f(x)$, indicate the error with an **E** in the table.

$f(x) = 3x - 2$

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
$f(x)$											

$f(x) = 4 - x^2$

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
$f(x)$											

$f(x) = x^3 - 2x - 1$

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
$f(x)$											

$f(x) = 1 + \frac{1}{x}$

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
$f(x)$											

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$f(x) = \sqrt{x+2}$

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
$f(x)$											

$f(x) = \dfrac{1}{x-3}$

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
$f(x)$											

$f(x) = \sqrt{9-x^2}$

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
$f(x)$											

$f(x) = \dfrac{1}{x^2-x-6}$

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
$f(x)$											

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3. Number Line Representation

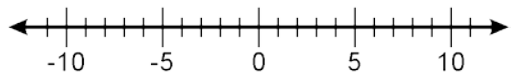
The set of all x-values for which there is a corresponding $f(x)$-value is called the **domain** of the function.

Whenever an x-value produces an error for an $f(x)$-value on the table, that x-value is not in the domain of the function.

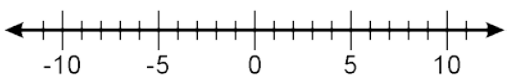
- Use the number lines below to plot x-values that are in the domain of each function.
- If an x-value has a corresponding $f(x)$-value, place a solid dot (●) on the number line.
- If an x-value produces an error for an $f(x)$-value, place an open dot (○) on the number line

$$f(x) = 3x - 2$$

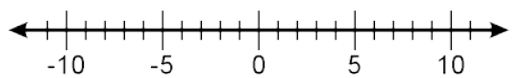
$$f(x) = \sqrt{x+2}$$



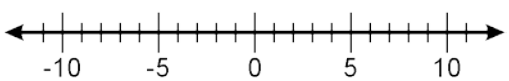
$$f(x) = 4 - x^2$$



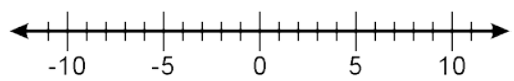
$$f(x) = \frac{1}{x-3}$$



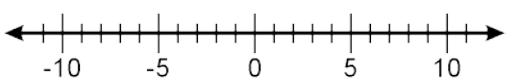
$$f(x) = x^3 - 2x - 1$$



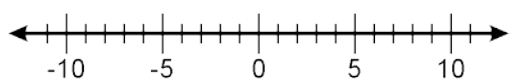
$$f(x) = \sqrt{9 - x^2}$$



$$f(x) = 1 + \frac{1}{x}$$



$$f(x) = \frac{1}{x^2 - x - 6}$$



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The solid and open dots on the number lines represent only a small portion of *x*-values included in the domain of each function.

- If you think that other *x*-values between the table values would also be included in the domain, shade the number lines to include those values.

- If you think *x*-values beyond the table values would also be included in the domain, shade the arrows on the number lines to indicate that the domain continues.

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4. Verbal and Symbolic Representations

To describe the domain of a function, you can use words or symbols.

- If the domain of a function includes all x-values, state that the domain extends "from $-\infty$ to ∞," or use symbols, $(-\infty, \infty)$.
- To express that the domain of a function includes all values except a particular x-value (2, for example), state that the domain includes "all x except 2," or use symbols, $x \neq 2$.
- To express that the domain of a function includes all values between a particular x-value (2, for example) and infinity, state that the domain extends "from 2 to ∞," or use symbols, $[2, \infty)$.

Express the domain of each function using words and symbols:

FUNCTION	DOMAIN (USING WORDS)	DOMAIN (USING SYMBOLS)
$f(x) = 3x - 2$		
$f(x) = 4 - x^2$		
$f(x) = x^3 - 2x - 1$		
$f(x) = 1 + \frac{1}{x}$		
$f(x) = \sqrt{x+2}$		
$f(x) = \frac{1}{x-3}$		
$f(x) = \sqrt{9 - x^2}$		
$f(x) = \frac{1}{x^2 - x - 6}$		

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Rates of Change

NAME _____

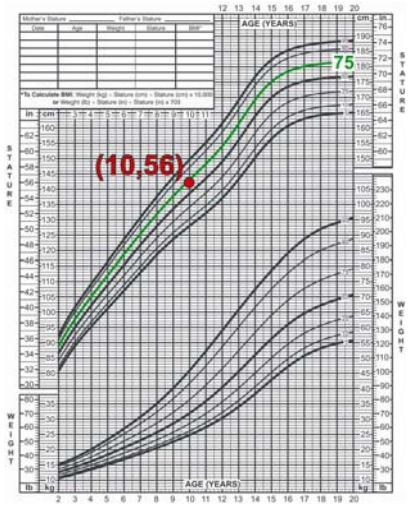
Height vs. Age charts are an important way to keep track of a child's progress.

Read the information on this page about Height vs. Age charts. Then, use the Height vs. Age charts for girls and boys to examine rates of change and answer the questions on the following page.

Height vs. Age Charts

Height vs. Age charts are used to keep track of a child's progress. They were developed by the Centers for Disease Control and Prevention (CDC), using data from thousands of children.

Based on height, these charts indicate the percentile at which a child falls compared to other children of the same age. Each curved line on the chart indicates a specific percentile. To use them, locate the point on the chart corresponding to a child's age along the horizontal axis and the child's height along the vertical axis. Then, identify the curve closest to that point, which indicates the percentile.



For instance, Monique is 10 years old and 56 inches tall. On the girls' Height vs. Age chart, the point (10 years, 56 inches) lies just below the curve for the 75th percentile. This means that Monique is taller than approximately 75% of all 10-year-old girls, but she is not as tall as the other 25% of 10-year-old girls.

Four separate charts are published by the National Center for Health Statistics (NCHS), a division of the CDC:

- Boys, birth to 36 months
- Boys, 3 to 18 years of age
- Girls, birth to 36 months
- Girls, 3 to 18 years of age

Parents often become fixated with their child's percentile, especially if their children fall below the 50th percentile. However, parents have nothing to worry about, as long as the child is consistently within the normal range. (The "normal range" is usually defined as a height between the 10th and 90th percentiles.) Physicians will become concerned, however, if a child repeatedly falls below the lower limit. A physician will also have concern if the percentile continues to decrease. For instance, a physician may worry about a child who

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measured at the 90th percentile at age two, but then fell to the 50th percentile by age three, and then fell to the 10th percentile by age four. From a health standpoint, it is important for children to maintain roughly the same percentile as they get older.

1. Consider a girl at the 50th percentile. Estimate the rate of change in her height (inches per year) at the following ages.

AGE	3	5	7	9	11	13	15	17
RATE OF CHANGE IN HEIGHT (INCHES PER YEAR)								

2. For a girl at the 50th percentile, at what age is her rate of change in height the greatest? That is, when is she growing the fastest? Estimate her rate of change in height (inches per year) at this age.

3. As a girl at the 50th percentile gets close to 20 years of age, explain what happens to her rate of change in height. Use the graph of Height vs. Age to justify your answer.

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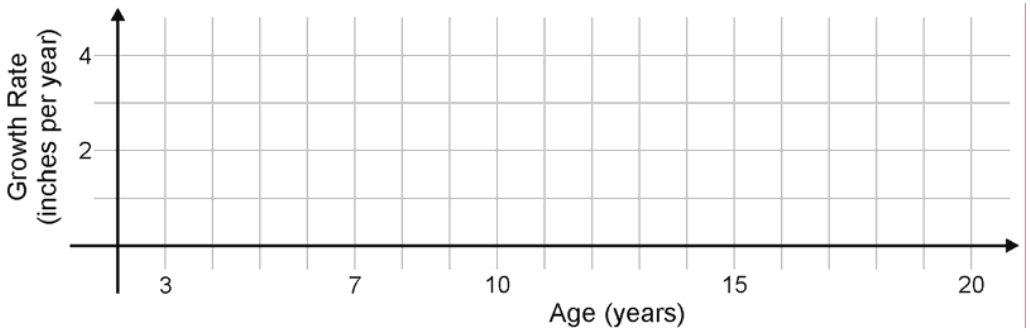
4. Use your results from Question 1 to sketch a graph representing the approximate **rate of change in height** vs. the age for a girl at the 50th percentile from age 3 to 20. What happens to a girl's rate of change in height over time? Explain.



5. Consider a boy at the 50th percentile. Estimate the rate of change in his height (inches per year) at the following ages.

AGE	3	5	7	9	11	13	15	17
RATE OF CHANGE IN HEIGHT (INCHES PER YEAR)								

6. For a boy at the 50th percentile, at what age is his rate of change in height the greatest? That is, when is he growing the fastest? Estimate his rate of change in height (inches per year) at this age.

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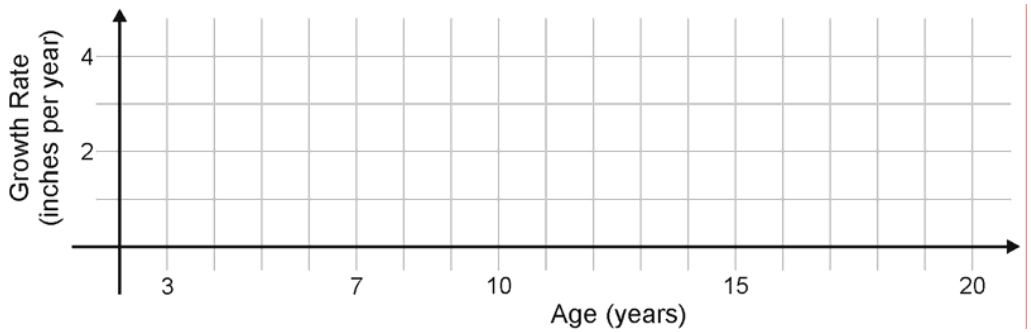
7. As a boy at the 50th percentile gets close to 20 years of age, explain what happens to his rate of change in height. Use the graph of Height vs. Age to justify your answer.

8. Use your results from Question 5 to sketch a graph representing the approximate **rate of change in height** vs. the age for a boy at the 50th percentile from age 3 to 20. What happens to a boy's rate of change in height rate over time? Explain.



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2 to 20 years: Girls
Stature-for-age and Weight-for-age percentiles

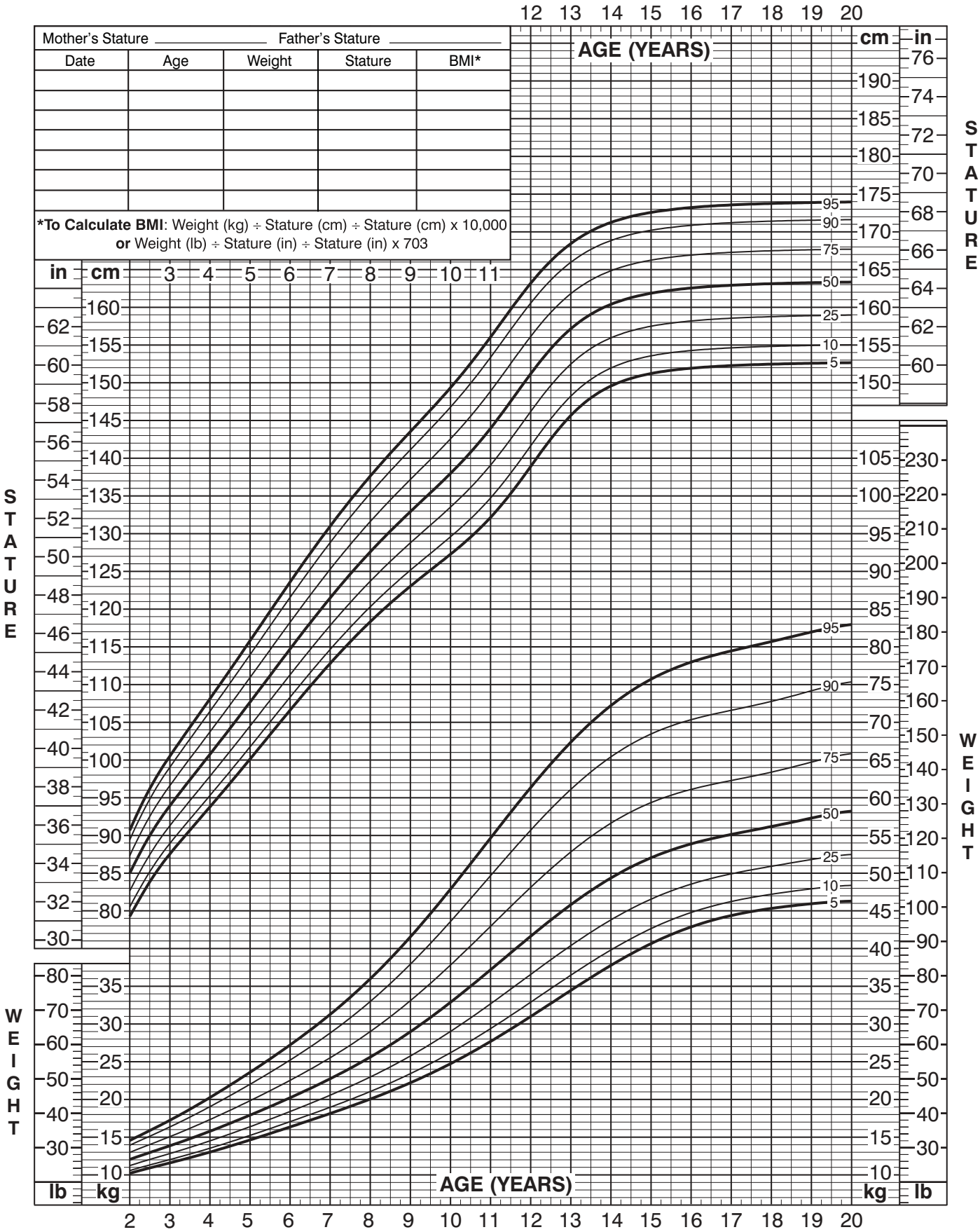
NAME _____

RECORD # _____

Mother's Stature _____ Father's Stature _____

Date	Age	Weight	Stature	BMI*

***To Calculate BMI**: Weight (kg) ÷ Stature (cm) ÷ Stature (cm) x 10,000
or Weight (lb) ÷ Stature (in) ÷ Stature (in) x 703



Published May 30, 2000 (modified 11/21/00).
SOURCE: Developed by the National Center for Health Statistics in collaboration with the National Center for Chronic Disease Prevention and Health Promotion (2000).
http://www.cdc.gov/growthcharts

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2 to 20 years: Boys
Stature-for-age and Weight-for-age percentiles

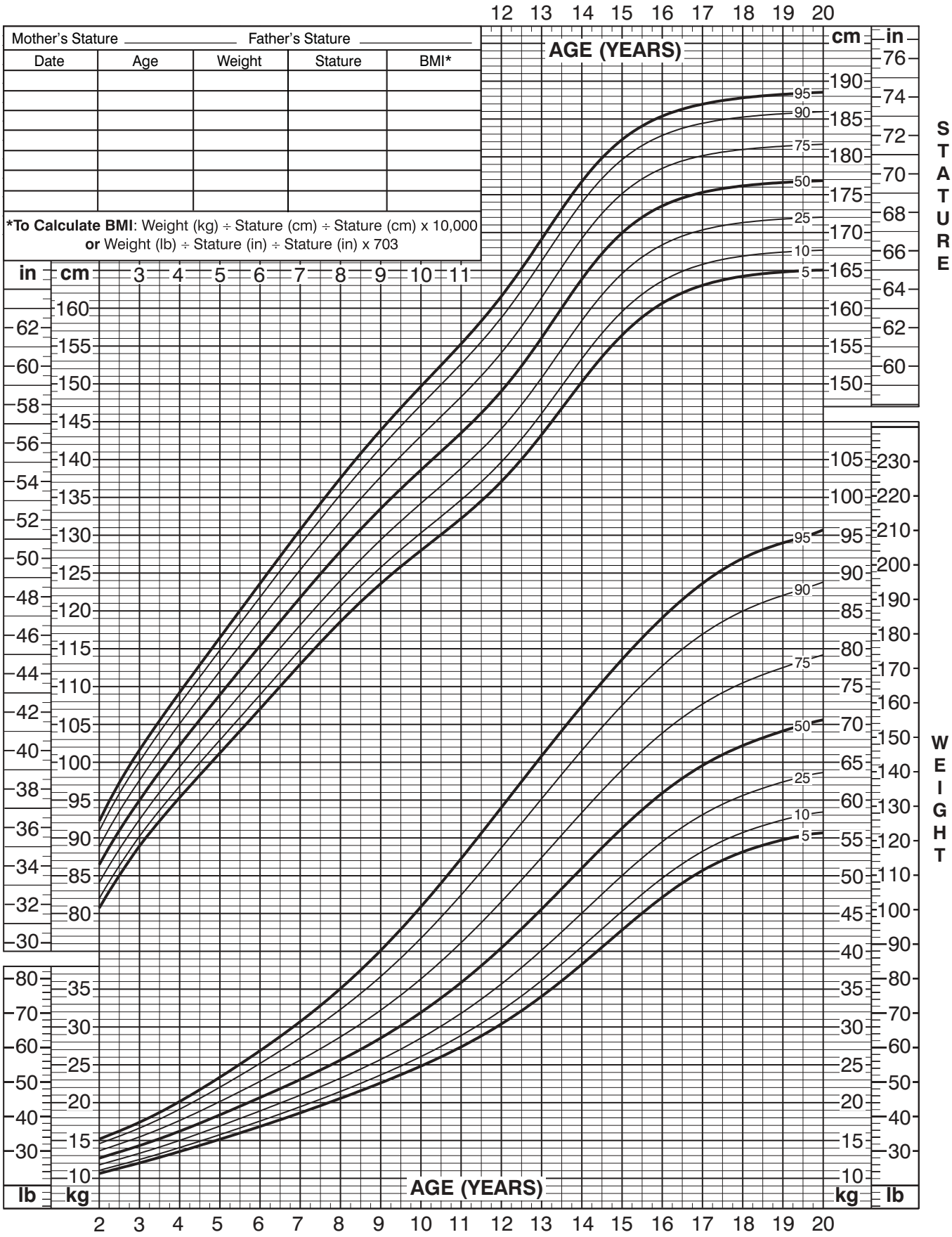
NAME _____

RECORD # _____

Mother's Stature _____ Father's Stature _____

Date	Age	Weight	Stature	BMI*

***To Calculate BMI**: Weight (kg) ÷ Stature (cm) ÷ Stature (cm) x 10,000
or Weight (lb) ÷ Stature (in) ÷ Stature (in) x 703



Published May 30, 2000 (modified 11/21/00).
SOURCE: Developed by the National Center for Health Statistics in collaboration with the National Center for Chronic Disease Prevention and Health Promotion (2000).
http://www.cdc.gov/growthcharts

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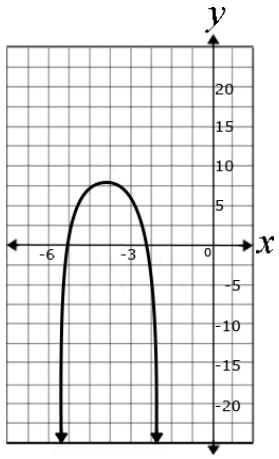
Unit 2 – Linear and Exponential Relationships

Domain: Functions (F)

Essential Question: How do linear relationships help model exponential functions?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Compare and contrast linear and non-linear functions to develop appropriate models for given situations** **F-IF** (***) Analyze functions using different representations	**How…** **F.IF.7:** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. **F.IF.9:** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*	**Students will:** **Graphing Comparisons – Group Activity – Graphing Calculator** F-LE.2,F-IF.7,A-REI.11.pdf	**F.IF.7:** See Unit Addendum **F.IF.9:** Example: • Examine the functions below. Which function has the larger maximum? How do you know? $f(x) = -2x^2 - 8x + 20$ 

Vocabulary: Arithmetic and geometric sequences, arithmetic and geometric situations, base, effect on graph, end behavior, equal distance over equal intervals, equal factor over equal intervals, even and odd functions, functional inputs, function notation, graphs, graphical effects, intercepts, intervals, parameters of linear/exp, recursive and explicit formulae, relative maximum/minimum, slope, symmetry

Sample 3:
HS Mathematics Sample ER Item

Sample Item ID:	MAT.HS.ER.2.00FLE.C.224
Grade:	HS
Primary Claim:	**Claim 2: Problem Solving** Students can solve a range of well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies.
Secondary Claim(s):	Claim 1: Conceptual Understanding and Procedural Fluency Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.
Primary Content Domain:	**Functions**
Secondary Content Domain(s):	Algebra
Assessment Target(s):	2 C: Interpret results in the context of a situation. 2 A: Apply mathematics to solve well-posed problems asking in everyday life, society and the workplace. 2 B: Select and use appropriate tools strategically. 1 N: Build a function that models a relationship between two quantities.
Standard(s):	F-LE.2, F-IF.7, A-REI.11
Mathematical Practice(s):	1, 2, 4, 5
DOK:	2
Item Type:	ER

Score Points:	3
Difficulty:	M
Key:	See Top-Score Response
Stimulus/Source:	Based (roughly) on accounts offered by TD Bank and Chase Bank
Target-specific attributes(e.g., accessibility issues)	
Notes:	Requires TE template, AI scoring and handscoring

Albert plans to put $1000 in a bank account. He considered two different accounts at a bank:

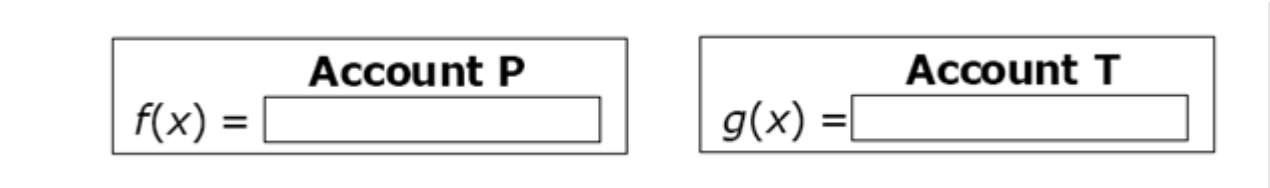
- Account P Offers a 1% interest rate, compounded annually.
- Account T offers a $100 bonus for opening the account, but no interest is earned.

Albert wants to compare how much money will be in that accounts over time. He decides to calculate what would happen to $1000 placed into each account, if no more money is added or removed.

Use the graphing applet below to explore which account will have more money in it over time.

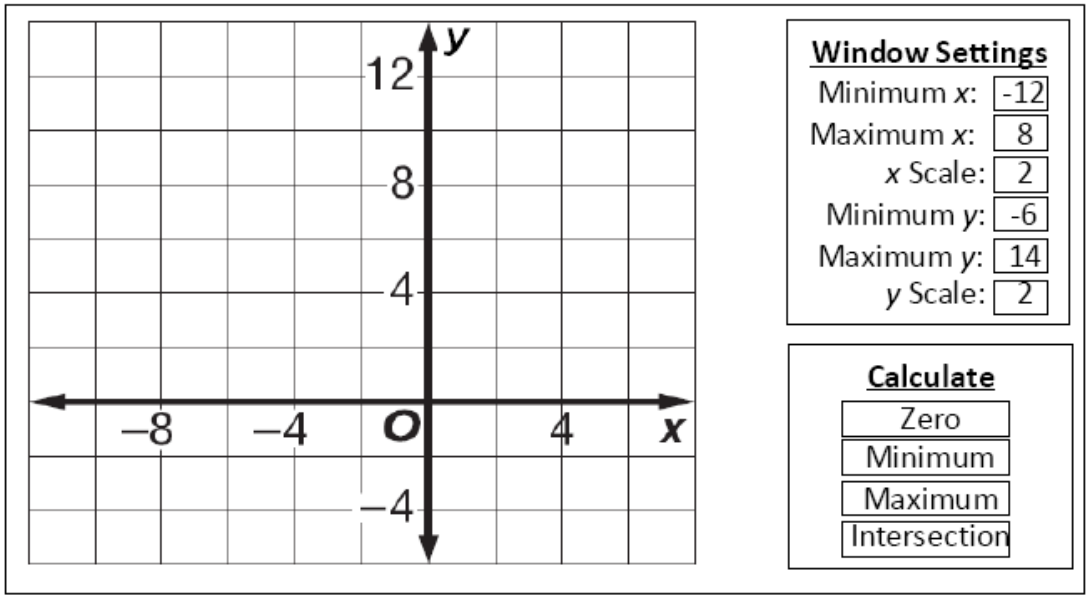
Part A

Write functions for the amount of money in each of the two accounts. Let x be the number of years since Albert opens the account. Your two functions will be automatically graphed in the applet below.



Part B

Enter appropriate values into the “Window Settings” menu below so that graphs of the two functions can be viewed in the same window.



Part C

Based on the graph, under what circumstances should Albert choose account P? Under what circumstances should he choose Account T? Explain how he arrived at your answer. You can use the buttons under the “calculate” menu to help you.

Sample Item ID:	MAT.HS.ER.3.0AAPR.F.045
Grade:	HS
Primary Claim:	**Claim 3: Communicating Reasoning** Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reason.
Secondary Claim(s):	Claim 1: Conceptual Understanding and Procedural Fluency Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.
Content Domain:	**Algebra**
Assessment Target(s):	3 F: Base arguments on concrete referents such as objects, drawing, diagrams, and actions. 3 B: Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures.

	1 F: Perform arithmetic operations on polynomials.
Standard(s):	A-APR.1
Mathematical Practice(s):	1, 2, 3, 6
DOK:	3
Item Type:	ER
Score Points:	3
Difficulty:	H
Key:	See Top-Score Response
Stimulus/Source:	
Target-specific attributes(e.g., accessibility issues)	
Notes:	Mutli-part item – Part of PT set

Unit 2 – Linear and Exponential Relationships

Domain: Functions (F)

Essential Question: How do linear relationships help model exponential functions?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

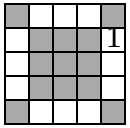
Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Compare and contrast linear and non-linear functions to develop appropriate models for given situations** **F-BF** (*) Build new functions from existing functions	**How…** **F.BF.1:** Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.* **F.BF.2:** Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. **F.BF.3:** Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.	**Students will:** **Building Functions Activities** F-BF.1 Sidewalks.pdf F-BF.2 Best Buy Tickets.pdf F.BF.2 (Unit 2).pdf F-BF.2 Patchwork.pdf **Sequence Analysis** F.BF.2 Patterns to Functions Activity.do **Function Parameters-Investigation** F-BF.3 Func Fam Shifting Graphs.docx F-BF.3 Function Family Summary.doc	**F.BF.1:** See Unit Addendum **F.BF.2:** See Unit Addendum **F.BF.3:** See Unit Addendum

Vocabulary: Arithmetic and geometric sequences, arithmetic and geometric situations, base, effect on graph, end behavior, equal distance over equal intervals, equal factor over equal intervals, even and odd functions, functional inputs, function notation, graphs, graphical effects, intercepts, intervals, parameters of linear/exp, recursive and explicit formulae, relative maximum/minimum, slope, symmetry

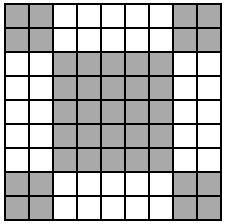
Sidewalk Patterns

In Prague some sidewalks are made of small square blocks of stone.

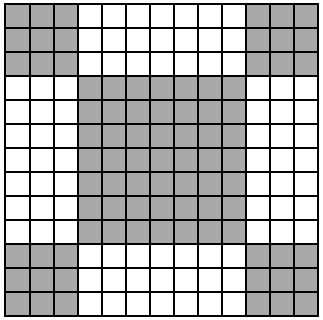
The blocks are in different shades to make patterns that are in various sizes.



Pattern # 1

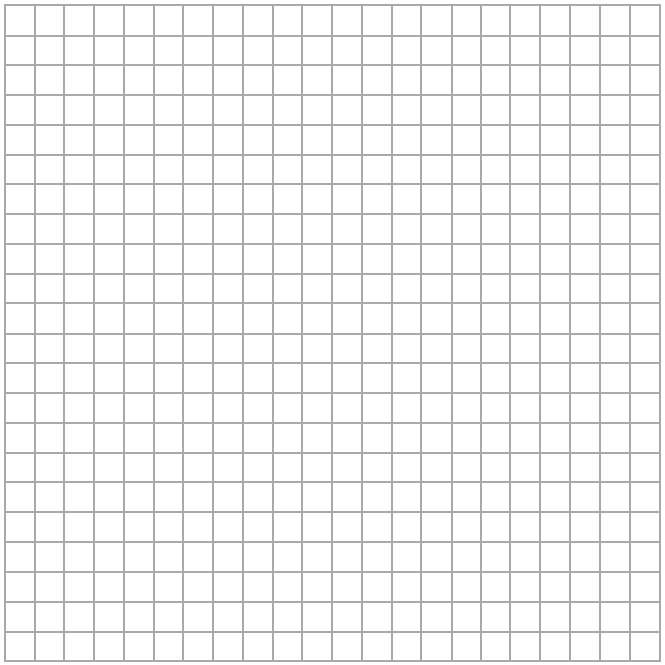


Pattern # 2



Pattern # 3

Draw the next pattern in this series.



Pattern # 4

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Sidewalk Patterns

1. Complete the table below

Pattern number, *n*	1	2	3	4
Number of white blocks	12	40		
Number of gray blocks	13			
Total number of blocks	25			

2. What do you notice about the number of white blocks and the number of gray blocks?

_____

3. The total number of blocks can be found by squaring the number of blocks along one side of the pattern.

a. Fill in the blank spaces in this list.

$25 = 5^2$ $\quad$ $81 =$ _____ $\quad$ $169 =$ _____ $\quad$ $289 = 17^2$

b. How many blocks will pattern #5 need? _____

b. How many blocks will pattern #*n* need? _____

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Sidewalk Patterns

4. a. If you know the total number of blocks in a pattern you can work out the number of white blocks in it. Explain how you can do this.

_____

_____

_____

b. Pattern # 6 has a total of 625 blocks.
How many white blocks are needed for Pattern # 6? _____
Show how you figured this out.

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Best Buy Tickets

Susie is organizing the printing of tickets for a show her friends are producing. She has collected prices from several printers and these two seem to be the best.

SURE PRINT
Ticket Printing
$25 Tickets for $2

BEST PRINT
Tickets Printed
$10 Setting Up
plus
$1 for 25 tickets

Susie wants to go for the best buy.

She doesn't yet know how many people are going to come.

Show Susie a couple of ways in which she could make the right decision, whatever the number.

Illustrate your advice with a couple of examples.

_____

_____

_____

_____

_____

Please continue your work on the page opposite

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Best Buy Tickets (continued)

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MAT.HS.CR.1.00FBF.N.276

Sample Item ID:	MAT.HS.CR.1.00FBF.N.276
Grade:	HS
Claim(s):	**Claim 1: Concepts and Procedures** Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.
Assessment Target(s):	1 N: Build a function that models a relationship between two quantities.
Content Domain:	Functions
Standard(s):	F-BF.2
Mathematical Practice(s):	1, 2, 4, 7
DOK:	2
Item Type:	CR
Score Points:	1
Difficulty:	M
Key:	$f(n) = 24{,}500 - 4900n$
Stimulus/Source:	
Target-specific attributes (e.g., accessibility issues):	
Notes:	Equation editor (or some equivalent functionality) will need to be available.

A company purchases $24,500 of new computer equipment. For tax purposes, the company estimates that the equipment decreases in value by the same amount each year. After 3 years, the estimated value is $9800.

Write an explicit function that gives the estimated value of the computer equipment *n* years after purchase.

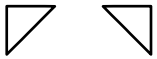
Sample Top-Score Response:

Correct responses to this item will receive 1 point

1 point: $f(n) = 24{,}500 - 4900n$

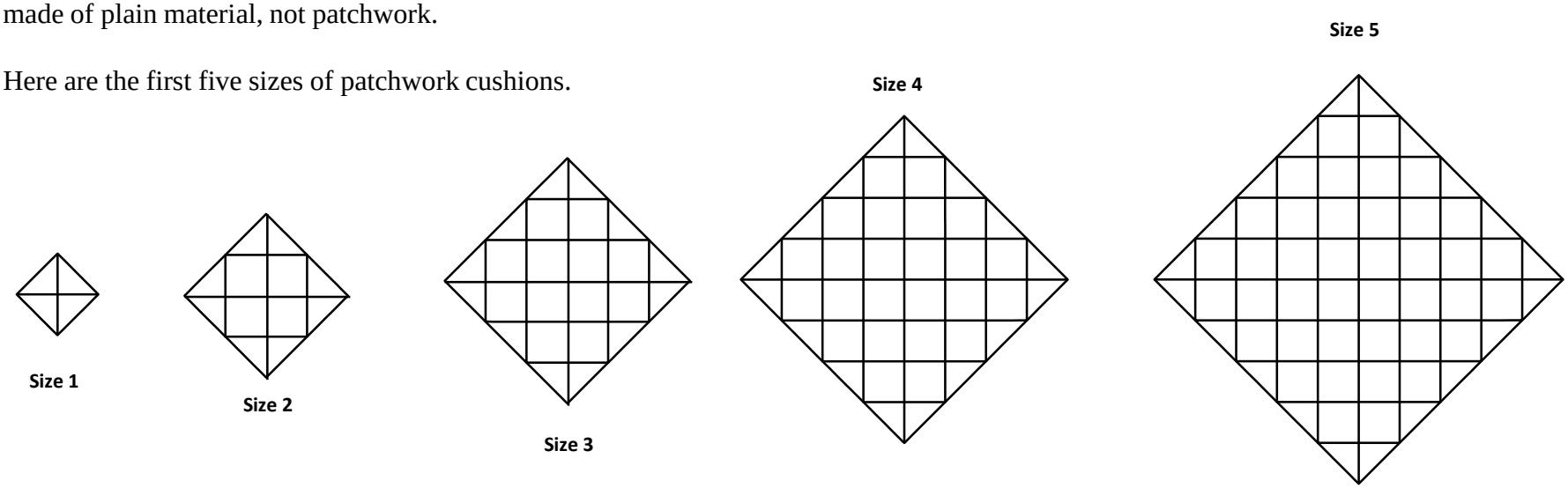
Patchwork

A sheet of square dot paper is provided for use with this item.

Kate makes patchwork cushions.
She uses right triangles  and squares.

She uses triangles along the edges of each cushion. The rest is made from squares. The backs of the cushions are made of plain material, not patchwork.

Here are the first five sizes of patchwork cushions.



Kate makes cushions in many other different sizes.

She begins to figure out how many triangles and squares she needs for each size. For size 1, she needs 4 triangles and 0 squares.

For size 2, she needs 8 triangles and 4 squares.

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Patchwork

1. Complete this table to show how many triangles and squares she needs for each of these five sizes?

Size *(n)*	**Number of triangles *(t)***	**Number of squares *(s)***
1		
2		
3		
4		
5		

2. Find a rule, or a formula, that will help Kate figure out the number of triangles that she needs for cushions of different sizes. Explain how you figured it out.

_____

_____

_____

_____

_____

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Patchwork

3. Use the number patterns in the table to find a rule, or a formula, that will help Kate figure out the number of squares she needs for cushions of different sizes. Explain why your rule works.

_____

_____

_____

_____

_____

4. Kate has a cushion made with 180 squares.
   How many triangles are in this cushion? _____
   Show how you found the number of triangles

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Name: _____ Date: _____

Patterns to Functions

1.) 2, 6, 10, 14, 18…

Do you notice a pattern? _____ What is the pattern you notice? _____

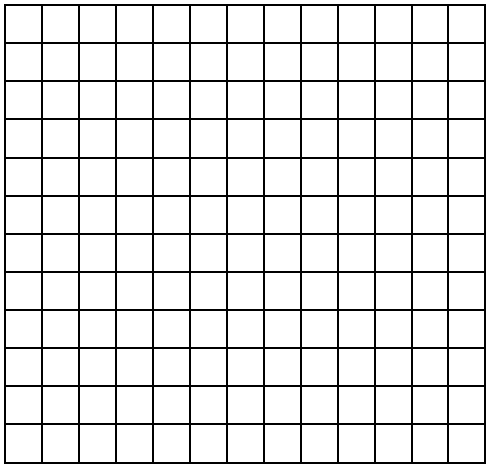
What is your starting point? _____

Write the recursive formula: f(n) = _____

Use the recursive formula to now write the function: f(x) = _____

Complete the table and plot the points on the given coordinate plane.

X	Y
0	
1	
2	
3	
4	
5	



2.) 53, 46.5, 40, 33.5, 27…

Do you notice a pattern? _____ What is the pattern you notice? _____

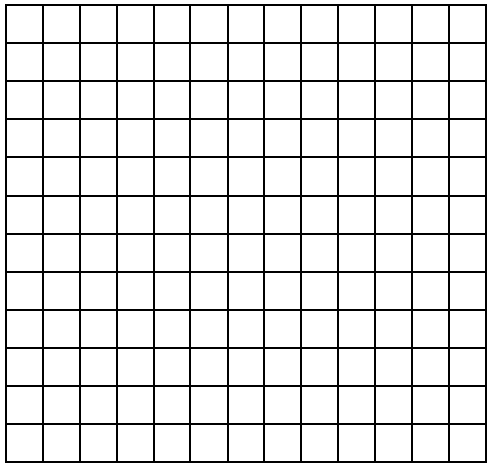
What is your starting point? _____

Write the recursive formula: f(n) = _____

Use the recursive formula to now write the function: f(x) = _____

Complete the table and plot the points on the given coordinate plane.

X	Y
0	
1	
2	
3	
4	
5	



3.) $-5, -2\frac{1}{2}, 0, 2\frac{1}{2}, 5 \ldots$

Do you notice a pattern? _____ What is the pattern you notice? _____

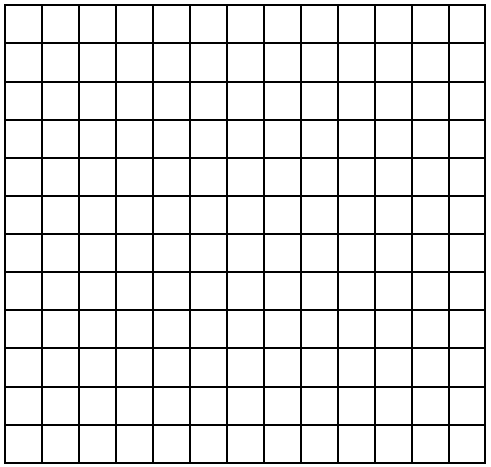
What is your starting point? _____

Write the recursive formula: f(n) = _____

Use the recursive formula to now write the function: f(x) = _____

Complete the table and plot the points on the given coordinate plane.

X	Y
0	
1	
2	
3	
4	
5	



4.) $11, 6\frac{3}{4}, 2\frac{1}{2}, -1\frac{3}{4}, -6$

Do you notice a pattern? _____ What is the pattern you notice? _____

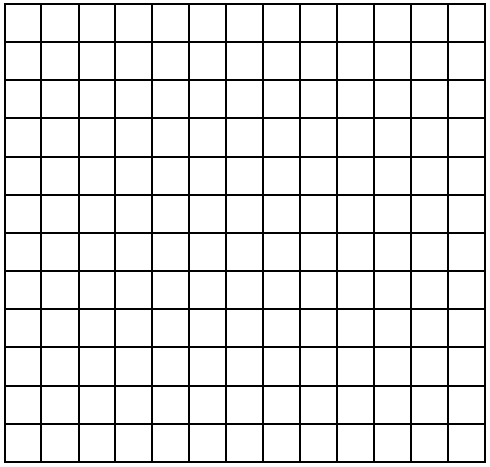
What is your starting point? _____

Write the recursive formula: f(n) = _____

Use the recursive formula to now write the function: f(x) = _____

Complete the table and plot the points on the given coordinate plane.

X	Y
0	
1	
2	
3	
4	
5	



5.) 5, 15, 45, 135, 405…

Do you notice a pattern? _____ What is the pattern you notice? _____

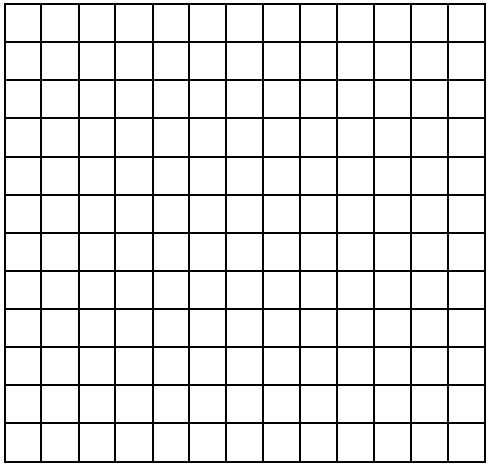
What is your starting point? _____

Write the recursive formula: f(n) = _____

Use the recursive formula to now write the function: f(x) = _____

Complete the table and plot the points on the given coordinate plane.

X	Y
0	
1	
2	
3	
4	
5	



6.) 80, 40, 20, 10, 5…

Do you notice a pattern? _____ What is the pattern you notice? _____

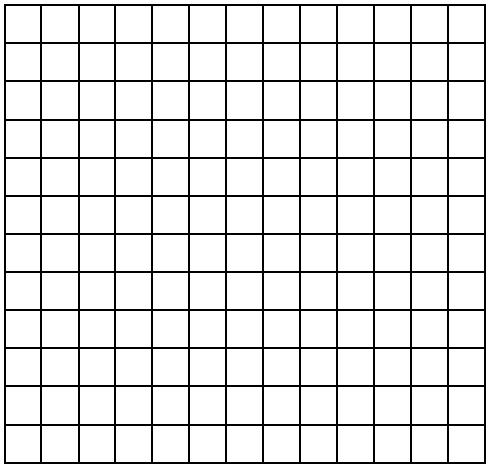
What is your starting point? _____

Write the recursive formula: f(n) = _____

Use the recursive formula to now write the function: f(x) = _____

Complete the table and plot the points on the given coordinate plane.

X	Y
0	
1	
2	
3	
4	
5	



7.) 3, 4.5, 6.75, 10.125, 15.1875…

Do you notice a pattern? _____ What is the pattern you notice? _____

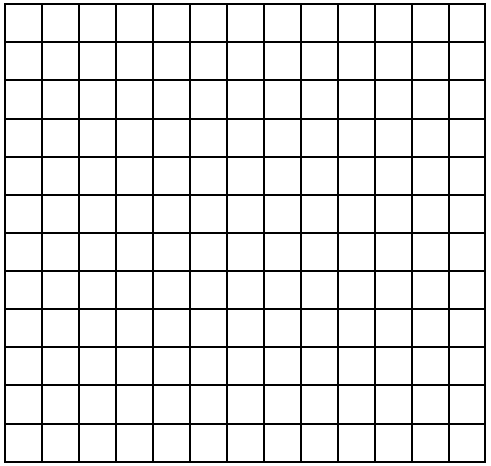
What is your starting point? _____

Write the recursive formula: f(n) = _____

Use the recursive formula to now write the function: f(x) = _____

Complete the table and plot the points on the given coordinate plane.

X	Y
0	
1	
2	
3	
4	
5	



8.) 10, 7.5, 5.625, 4.21875, 3.1640625…

Do you notice a pattern? _____ What is the pattern you notice? _____

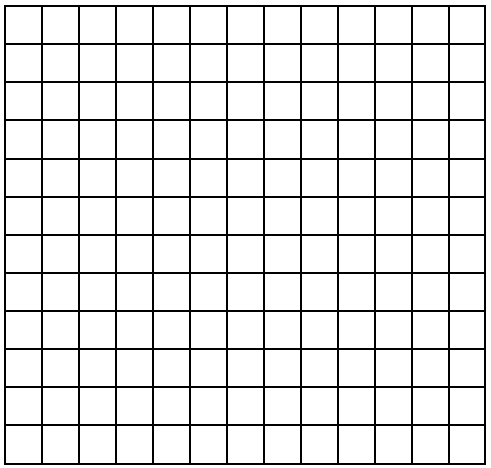
What is your starting point? _____

Write the recursive formula: f(n) = _____

Use the recursive formula to now write the function: f(x) = _____

Complete the table and plot the points on the given coordinate plane.

X	Y
0	
1	
2	
3	
4	
5	



1. Mrs. Garrahan's husband planted tomato plants in his garden before he had hip surgery. When he planted them they were 1 foot tall. She has been watching them grow all summer at a rate of 3 inches per week.

Week	Height
0	
1	
2	
3	
4	
5	

a. Use the data in the table to write a recursive formula for the plant height.

b. Now write an explicit function of the height of the plant after n weeks.

c. When will the tomato plant reach a height of 39 inches?

2. Mr. Tedesco needs to go on a diet after all of his holiday eating. He weighs 178lbs on the first of the year, and wants to lose 3 ½ pounds a month.

Month	Weight
0	
1	
2	
3	
4	
5	

a. Use the data in the table to write a recursive formula for Mr. Tedesco's weight.

b. Now write an explicit function of his weight after n months.

c. When will Mr. T reach his goal weight of 160lbs?

3. An ad for a special baseball card that was posted on the Internet claims that the value of the card "doubles every year." Jerome buys the card for $40 at the end of the year 2001.

Year	Value
0	
1	
2	
3	
4	
5	

a. Use the data in the table to write a recursive formula for the value of the baseball card.

b. Now write an explicit function of the value of the card after n years.

c. If the value of the card does indeed double every year, in what year will the value of the card first reach $5000?

4. The half-life of caffeine in a persons' body is approximately 4 hours (depending on weight, gender, etc). Mrs. Mazzotta ingests a large coffee at 8am that contains 30mg of caffeine.

Time	Caffeine
0	
1	
2	
3	
4	
5	

a. Use the data in the table to write a recursive formula for the amount of caffeine in Mrs. Mazzotta's system.

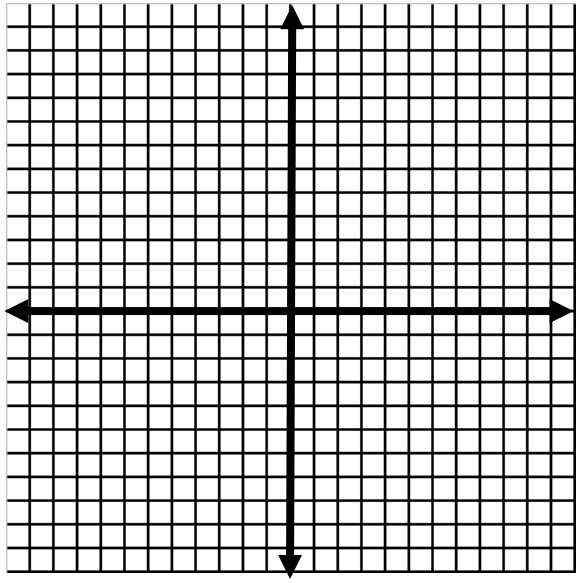
b. Now write an explicit function of the caffeine left in her system.

c. When will the level of caffeine in her system drop below 5mg?

On each of the following pages, use the given graph of the parent function to graph the 3 other functions given. Be sure to label at least 3 points, and state the domain and range of each function.

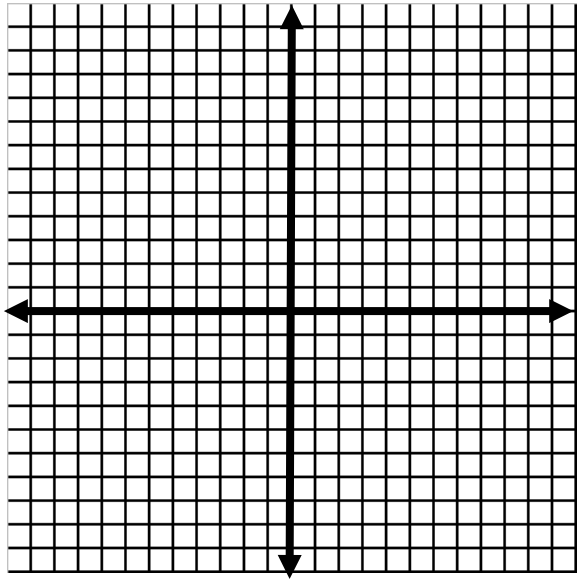
The Quadratic Family

1. $f(x) = x^2$



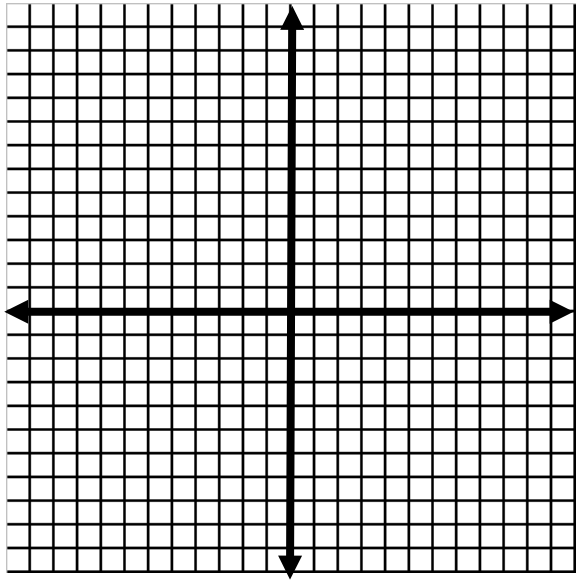
Description:

2. $f(x) = (x - 5)^2$



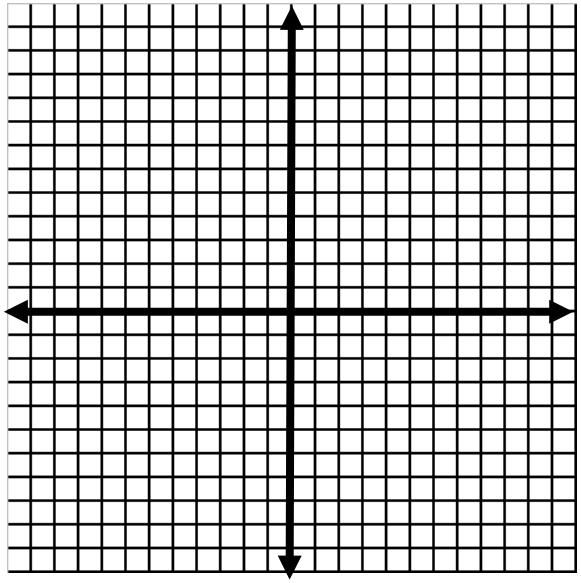
Description:

3. $f(x) = 3x^2$



Description:

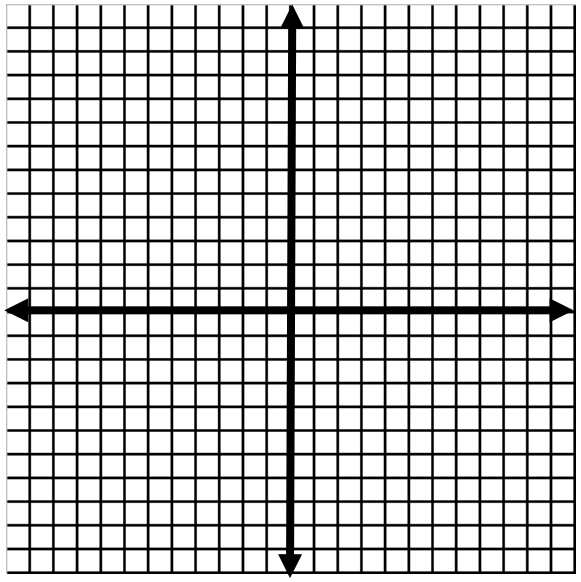
4. $f(x) = (x - 4)^2$



Description:

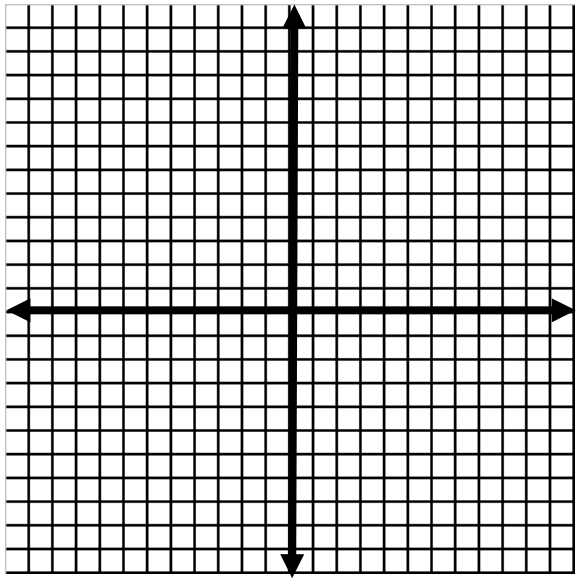
The Cubic Family

1. $f(x) = x^3$



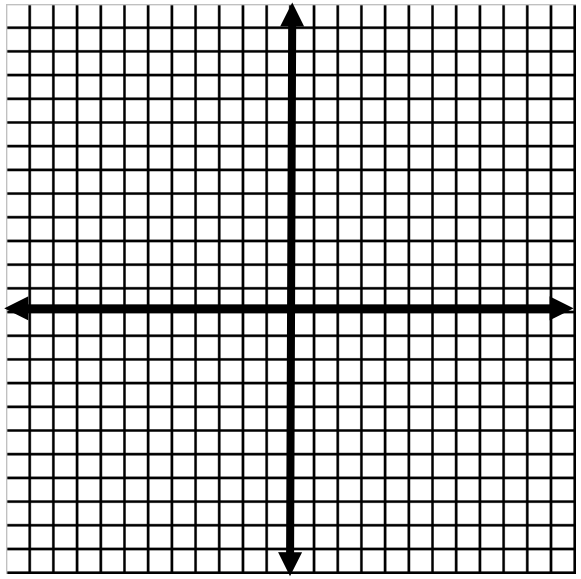
Description:

2. $f(x) = -x^3$



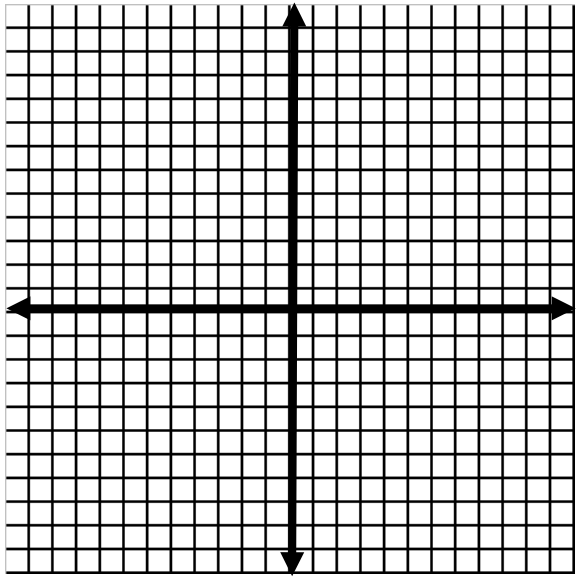
Description:

3. $f(x) = (x - 2)^3$



Description:

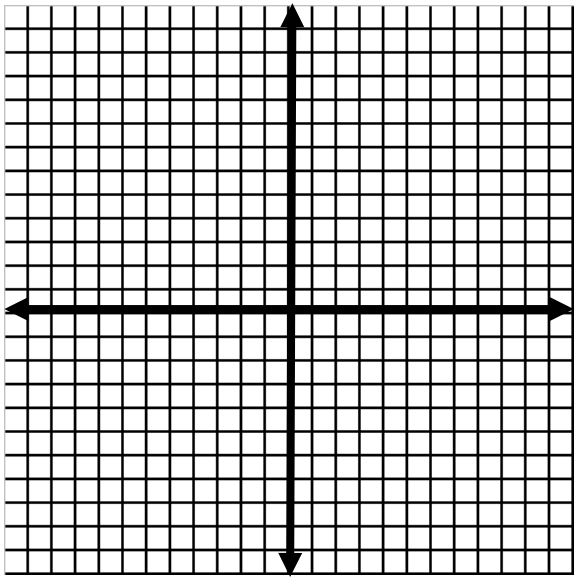
4. $f(x) = x^3 + 4$



Description:

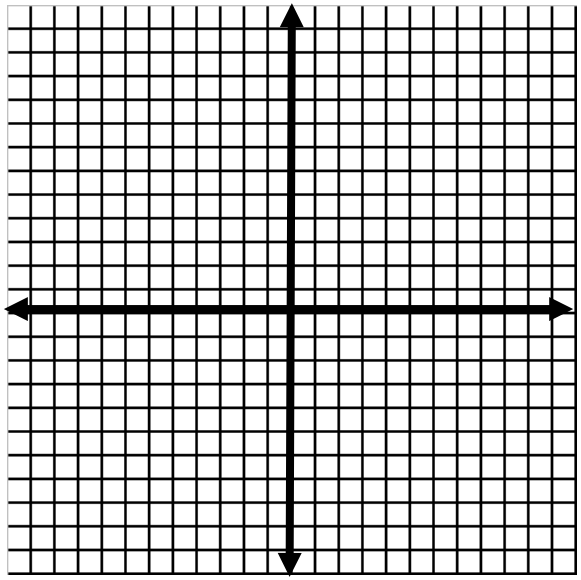
The Exponential Family

1. $f(x) = 2^x$



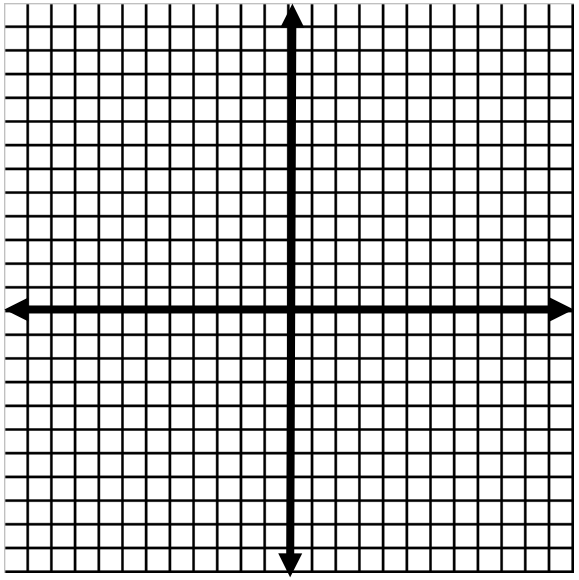
Description:

2. $f(x) = 1/2^x$



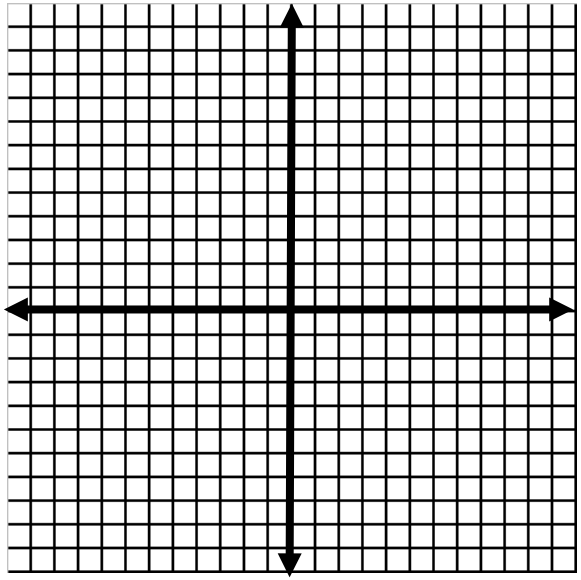
Description:

3. $f(x) = 3^x + 2$



Description:

4. $f(x) = .25^x$



Description:

Function Family Shifts Summary

Now that we've taken a look at a few "function families" and their graphs let's try to generalize how "parent function" are shifted/transformed based on the addition of a constant.

For each of the following let's assume f(x) is any "parent function" and c is some constant where C70.

Shift/Transformation Rules

To graph y= f(x) + C, the "parent graph", f(x), is shifted _____

To graph y= f(x) –C, the "parent graph", f(x), is shifted _____

To graph y= f(x+C), the "parent graph", f(x), is shifted _____

To graph y= f(x-C), the "parent graph", f(x), is shifted _____

To graph y= -f(x), the "parent graph", f(x), is shifted _____

To graph y= f(-x), the "parent graph", f(x), is shifted _____

-We can combine any series of shifts that we choose.

Practice

-Even without knowing the graph you can describe the shift of the “parent function”.

-For each of the following describe the shifts performed on the “parent function”.

Parent Function	Transformed Function	Describe Shifts
$F(x)=x^2$	$Y = (x-1)^2$	
$F(x)=x^2$	$Y = 4x^2$	
$F(x)=x^2$	$Y = x^2+2$	
$F(x)=x^2$	$Y = -2x^2$	
$F(x)=x^3$	$Y = 3x^3$	
$F(x)=x^3$	$Y = x^3+ 2$	
$F(x)=x^3$	$Y = (x – 4)^3$	
$F(x)= 3^x$	$Y = 3^{x - 4}$	
$F(x)= 3^x$	$Y = -3^x$	
$F(x)= 3^x$	$Y = 1/3^x$	
$F(x)= 3^x$	$Y =3^x + 1$	

Unit 2 – Linear and Exponential Relationships

Domain: Functions (F)

Essential Question: How do linear relationships help model exponential functions?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Compare and contrast linear and non-linear functions to develop appropriate models for given situations** **F-LE** (***) Construct and compare linear, quadratic, and exponential models and solve problems	**How…** **F.LE.1:** Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. **F.LE.2:** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	**Students will:** **Linear-Exponential Exit Task** F-LE.1 Exit Task.pdf	**F.LE.1:** See Unit Addendum **F.LE.2:** See Unit Addendum

Vocabulary: Arithmetic and geometric sequences, arithmetic and geometric situations, base, effect on graph, end behavior, equal distance over equal intervals, equal factor over equal intervals, even and odd functions, functional inputs, function notation, graphs, graphical effects, intercepts, intervals, parameters of linear/exp, recursive and explicit formulae, relative maximum/minimum, slope, symmetry

Mathematics Assessment Program College and Career Readiness Mathematics

Short Tasks: Linear and Exponential Models 1

1a. One of these tables represents a linear relationship, one represents an exponential growth and one represents an exponential decay.
Label each table correctly.

x	y
1	6
2	9
3	12
4	15

x	y
1	56
2	28
3	14
4	7

x	y
1	6
2	9
3	13.5
4	20.25

1b. Sketch graphs showing each of these relationships.

1c. Write an equation representing the linear relationship.

_____

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Unit 2 – Linear and Exponential Relationships

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7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Compare and contrast linear and non-linear functions to develop appropriate models for given situations** **F-LE:**(***) Interpret expressions for functions in terms of the situation they model	**How…** **F.LE.3:** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. **F.LE.5:** Interpret the parameters in a linear or exponential function in terms of a context.	**Students will:** **Linear-Exponential Comparison** F-LE.3+5 Linear and Exponential comparis **Mr. Miller Salary Prediction** F-LE 1, 3, 5 Mr. Miller.pdf	**F.LE.3:** Example: Contrast the growth of the $f(x)=x^3$ and $f(x)=3^x$. **F.LE.5:** Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model and interpret parameters in linear, quadratic or exponential functions. **Example:** • A function of the form $f(n) = P(1 + r)^n$ is used to model the amount of money in a savings account that earns 5% interest, compounded annually, where n is the number of years since the initial deposit. What is the value of r? What is the meaning of the constant P in terms of the savings account? Explain either orally or in written format.

Vocabulary: Arithmetic and geometric sequences, arithmetic and geometric situations, base, effect on graph, end behavior, equal distance over equal intervals, equal factor over equal intervals, even and odd functions, functional inputs, function notation, graphs, graphical effects, intercepts, intervals, parameters of linear/exp, recursive and explicit formulae, relative maximum/minimum, slope, symmetry

MAT.HS.ER.4.00FLE.E.566

Sample Item ID:	MAT.HS.ER.4.00FLE.E.566
Grade:	HS
Primary Claim:	**Claim 4: Modeling and Data Analysis** Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.
Secondary Claim(s):	Claim 3: Communicating Reasoning Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others. Claim 1: Concepts and Procedures Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.
Primary Content Domain:	Functions
Secondary Content Domain(s):	
Assessment Target(s):	4 E: Analyze the adequacy of and make improvements to an existing model or develop a mathematical model of a real phenomenon. 3 E: Distinguish correct logic or reasoning from that which is flawed and—if there is a flaw in the argument—explain what it is. 4 A: Apply mathematics to solve problems arising in everyday life, society, and the workplace. 3 B: Construct, autonomously, chains of reasoning to justify mathematical models used, interpretations made, and solutions proposed for a complex problem. 3 C: State logical assumptions being used. 4 D: Interpret results in the context of a situation. 3 F: Base arguments on concrete referents such as objects, drawings, diagrams, and actions.
Standard(s):	F-LE.1, F-LE.3, F-LE.5
Mathematical Practice(s):	1, 3, 4, 6, 7, 8
DOK:	4
Item Type:	ER
Score Points:	4
Difficulty:	M
Key:	See Sample Top-Score Response.
Stimulus/Source:	
Target-specific attributes (e.g., accessibility issues):	
Notes:	Part of PT set

Mr. Miller starts working for a technology company this year. His salary the first year is $40,000. According to the company's employee handbook, each following year Mr. Miller works at the company, he is eligible for a raise equal to 2–5% of his previous year's salary.

Mr. Miller calculates the range of his raise on his first year's salary. He adds that amount as his raise for each following year. Mr. Miller thinks that:

- in his second year working at the company, he would be earning a salary between $40,800 and $42,000, and
- in his third year, he would be earning a salary between $41,600 and $44,000.

Part A

1. Based on this reasoning, what salary range would Mr. Miller expect to earn in his **tenth** year at the company?

2. Mr. Miller's reasoning is incorrect. Show with diagrams, equations, expressions, or words why his reasoning is incorrect.

Part B

Create a table of values to compare the expected salary increases for an employee with a starting salary of $100,000 based on Mr. Miller's incorrect reasoning and the more reasonable expected salary increases. List these ranges in separate columns of the table up to the employee's sixth year at this company.

Sample Top-Score Response:

Part A

1. $47,200 – $58,000

2. Mr. Miller's reasoning is incorrect because he is treating the range of percent increases linearly instead of exponentially. He calculates each following year's increase range by

adding the amount calculated based on his first year's salary. What he should do is add the increase ranges from the first year to the first year's salary to find the range for his second year's salary. Then, he should multiply the higher second year's salary range by the range in percents and add those increase amounts to find the following year's amounts. Each following year's percent increases should be based off the prior year's increased salary ranges.

Part B

Expected Salary Increases for $100,000 Starting Salary

	Based on Incorrect Reasoning		More Reasonable Increases	
Year	**Low Range (2%)**	**High Range (5%)**	**Low Range (2%)**	**High Range (5%)**
1	$100,000.00	$100,000.00	$100,000.00	$100,000.00
2	$102,000.00	$105,000.00	$102,000.00	$105,000.00
3	$104,000.00	$110,000.00	$104,040.00	$110,250.00
4	$106,000.00	$115,000.00	$106,120.80	$115,762.50
5	$108,000.00	$120,000.00	$108,243.22	$121,550.63
6	$110,000.00	$125,000.00	$110,408.08	$127,628.16

Scoring Rubric:

Responses to ***Part A*** of this item will receive 0–2 points, based on the following:

2 points: The student demonstrates a solid understanding of how to analyze complex, real-world scenarios to interpret and critique the reasoning of others. The student identifies the correct salary range for year ten and provides an accurate and complete critique of why the given reasoning for calculating the salary range is flawed.

1 point: The student has a limited understanding of how to analyze complex, real-world scenarios to interpret and critique the reasoning of others. The student identifies the correct salary range for year ten but provides a partially accurate critique of why the given reasoning for calculating the salary range is flawed. **OR** The student miscalculates the salary range for year ten but provides an accurate and complete critique of why the given reasoning for calculating the salary range is flawed.

0 points: The student demonstrates inconsistent understanding of how to analyze complex, real-world scenarios to interpret and critique the reasoning of others. The student does not determine the correct salary range for year ten and does not provide an accurate and complete critique of why the given reasoning for calculating the salary range is flawed.

Responses to ***Part B*** of this item will receive 0–2 points, based on the following:

2 points: The student demonstrates a solid understanding of how to construct mathematical models to make improvements to an existing model. The student provides a fully accurate table for each calculation.

1 point: The student has some understanding of how to construct mathematical models to make improvements to an existing model. The student has a general understanding of which formulas are used to make the calculations in each column of the table but makes some minor calculation errors.

0 points: The student demonstrates inconsistent understanding of how to construct mathematical models to make improvements to an existing model. The student does not provide an accurate table for each calculation.

Name: _____ Date: _____ Period: _____

Algebra 1 CCSS
Unit 2 – Linear and Exponential Relationships

In the 2000 Census, the population of Manchester was recorded as 139,739. In the 2010 Census, the recorded population of Manchester had declined to 127,275. Government planners are trying to project the city's population in 2040. One planner Ashley makes the assumption that the population will continue to decrease by the SAME AMOUNT of people in each of the next three ten-year periods. Another planner Mariah makes the assumption that the population will continue to decrease by the SAME PERCENT in each of the next three ten-year periods.

a. Calculate the projected population of Manchester in 2040 using each of the two planners' approaches and the difference between the two projections. **Show your work or explain how you found your answer.**

YEAR	Ashley	Mariah
2000	139,739	139,739
2010	127,275	127,275
2020		
2030		
2040		

A) Using vocabulary, describe the pattern of Ashley's population predictions. _____

_____

Is this pattern linear or exponential? Why?_____

_____

Write the equation for Ashley's population predictions._____

_____

B) Using vocabulary, describe the pattern of Mariah's population predictions. _____

_____

Is this pattern linear or exponential? Why?_____

_____

Write the equation for Mariah's population predications._____

_____

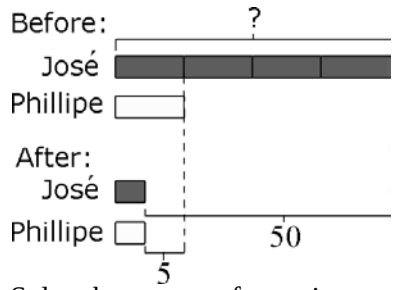
C) Which assumption – decreasing by the Ashley or by the Mariah – do you think most accurately projects the population? **Explain you reasoning.**

_____

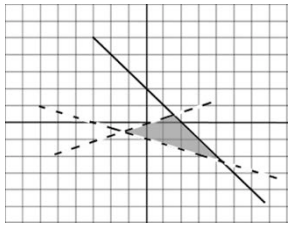
_____

_____

MHS Algebra 1 Mathematics Curriculum
Unit 2 Addendum

Performance Standard	Instructional Strategies
A.REI 6:	The system solution methods can include but are not limited to graphical, elimination/linear combination, substitution, and modeling. Systems can be written algebraically or can be represented in context. Students may use graphing calculators, programs, or applets to model and find approximate solutions for systems of equations. Examples: • José had 4 times as many trading cards as Phillipe. After José gave away 50 cards to his little brother and Phillipe gave 5 cards to his friend for this birthday, they each had an equal amount of cards. Write a system to describe the situation and solve the system.  • Solve the system of equations: $x + y = 11$ and $3x - y = 5$. Use a second method to check your answer. • Solve the system of equations: $x - 2y + 3z = 5$, $x + 3z = 11$, $5y - 6z = 9$. The opera theater contains 1,200 seats, with three different prices. The seats cost \$45 dollars per seat, \$50 per seat, and \$60 per seat. The opera needs to gross \$63,750 on seat sales. There are twice as many \$60 seats as \$45 seats. How many seats in each level need to be sold?
A.REI 11:	Students need to understand that numerical solution methods (data in a table used to approximate an algebraic function) and graphical solution methods may produce approximate solutions, and algebraic solution methods produce precise solutions that can be represented graphically or numerically. Students may use graphing calculators or programs to generate tables of values, graph, or solve a variety of functions. Example: • Given the following equations determine the x value that results in an equal output for both functions. $f(x) = 3x - 2$ $g(x) = (x+3)^2 - 1$

MHS Algebra 1 Mathematics Curriculum
Unit 2 Addendum

Performance Standard	Instructional Strategies
A.REI.12:	Students may use graphing calculators, programs, or applets to model and find solutions for inequalities or systems of inequalities. Examples: • Graph the solution: $y < 2x + 3$. • A publishing company publishes a total of no more than 100 magazines every year. At least 30 of these are women's magazines, but the company always publishes at least as many women's magazines as men's magazines. Find a system of inequalities that describes the possible number of men's and women's magazines that the company can produce each year consistent with these policies. Graph the solution set. • Graph the system of linear inequalities below and determine if (3, 2) is a solution to the system. $\begin{cases} x - 3y > 0 \\ x + y \le 2 \\ x + 3y > -3 \end{cases}$ Solution:  (3, 2) is not an element of the solution set (graphically or by substitution).

MHS Algebra 1 Mathematics Curriculum
Unit 2 Addendum

Performance Standard	Instructional Strategies

F.IF.4:	Students may be given graphs to interpret or produce graphs given an expression or table for the function, by hand or using technology.
	
	Examples:
	• A rocket is launched from 180 feet above the ground at time $t = 0$. The function that models this situation is given by $h = -16t^2 + 96t + 180$, where t is measured in seconds and h is height above the ground measured in feet.
	  o What is a reasonable domain restriction for t in this context?
	  o Determine the height of the rocket two seconds after it was launched.
	  o Determine the maximum height obtained by the rocket.
	  o Determine the time when the rocket is 100 feet above the ground.
	  o Determine the time at which the rocket hits the ground.
	  o How would you refine your answer to the first question based on your response to the second and fifth questions?
	• Compare the graphs of $y = 3x^2$ and $y = 3x^3$.
	• Let $R(x) = \frac{2}{\sqrt{x-2}}$. Find the domain of $R(x)$. Also find the range, zeros, and asymptotes of $R(x)$.
	• Let $f(x) = 5x^3 - x^2 - 5x + 1$. Graph the function and identify end behavior and any intervals of constancy, increase, and decrease.
	It started raining lightly at 5am, then the rainfall became heavier at 7am. By 10am the storm was over, with a total rainfall of 3 inches. It didn't rain for the rest of the day. Sketch a possible graph for the number of inches of rain as a function of time, from midnight to midday.

MHS Algebra 1 Mathematics Curriculum
Unit 2 Addendum

<table>
<tr><th>Performance Standard</th><th>Instructional Strategies</th></tr>
<tr><td>

F.IF.6:

</td><td>

The average rate of change of a function $y = f(x)$ over an interval [a,b] is $\frac{4y}{4x} = \frac{f(b)-f(a)}{b-a}$. In addition to finding average rates of change from functions given symbolically, graphically, or in a table, students may collect data from experiments or simulations (ex. falling ball, velocity of a car, etc.) and find average rates of change for the function modeling the situation.

Examples:

- Use the following table to find the average rate of change of *g* over the intervals [-2, -1] and [0,2]:

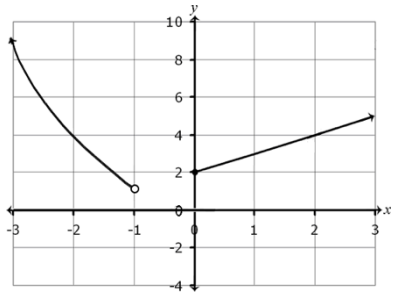
<table>
<tr><th>x</th><th>g(x)</th></tr>
<tr><td>-2</td><td>2</td></tr>
<tr><td>-1</td><td>-1</td></tr>
<tr><td>0</td><td>-4</td></tr>
<tr><td>2</td><td>-10</td></tr>
</table>

- The table below shows the elapsed time when two different cars pass a 10, 20, 30, 40 and 50 meter mark on a test track.
  - For car 1, what is the average velocity (change in distance divided by change in time) between the 0 and 10 meter mark? Between the 0 and 50 meter mark? Between the 20 and 30 meter mark? Analyze the data to describe the motion of car 1.
  - How does the velocity of car 1 compare to that of car 2?

<table>
<tr><th></th><th>Car 1</th><th>Car 2</th></tr>
<tr><th>d</th><th>t</th><th>t</th></tr>
<tr><td>10</td><td>4.472</td><td>1.742</td></tr>
<tr><td>20</td><td>6.325</td><td>2.899</td></tr>
<tr><td>30</td><td>7.746</td><td>3.831</td></tr>
<tr><td>40</td><td>8.944</td><td>4.633</td></tr>
<tr><td>50</td><td>10</td><td>5.348</td></tr>
</table>

</td></tr>
</table>

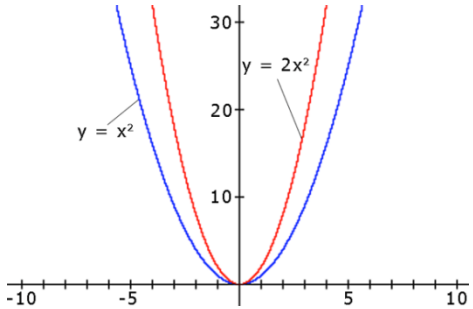
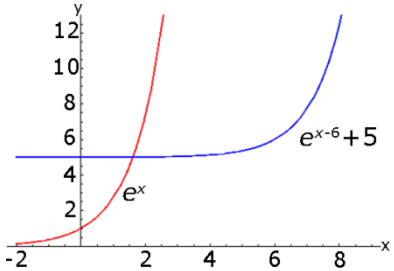
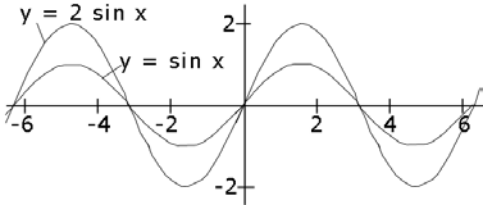
MHS Algebra 1 Mathematics Curriculum
Unit 2 Addendum

Performance Standard	Instructional Strategies
F.IF.7:	Key characteristics include but are not limited to maxima, minima, intercepts, symmetry, end behavior, and asymptotes. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to graph functions. Examples: • Describe key characteristics of the graph of $f(x) = \lvert x - 3 \rvert + 5$. • Sketch the graph and identify the key characteristics of the function described below. $F(x) = \begin{cases} x + 2 \text{ for } x \geq 0 \\ -x^2 \text{ for } x < -1 \end{cases}$  • Graph the function $f(x) = 2^x$ by creating a table of values. Identify the key characteristics of the graph. • Graph $f(x) = 2 \tan x - 1$. Describe its domain, range, intercepts, and asymptotes. Draw the graph of $f(x) = \sin x$ and $f(x) = \cos x$. What are the similarities and differences between the two graphs?

MHS Algebra 1 Mathematics Curriculum
Unit 2 Addendum

Performance Standard	Instructional Strategies
F.BF.1:	Students will analyze a given problem to determine the function expressed by identifying patterns in the function's rate of change. They will specify intervals of increase, decrease, constancy, and, if possible, relate them to the function's description in words or graphically. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model functions. Examples: • You buy a $10,000 car with an annual interest rate of 6 percent compounded annually and make monthly payments of $250. Express the amount remaining to be paid off as a function of the number of months, using a recursion equation. • A cup of coffee is initially at a temperature of 93º F. The difference between its temperature and the room temperature of 68º F decreases by 9% each minute. Write a function describing the temperature of the coffee as a function of time. • The radius of a circular oil slick after *t* hours is given in feet by $r = 10t^2 - 0.5t$, for $0 \leq t \leq 10$. Find the area of the oil slick as a function of time.
F.BF.2:	An explicit rule for the *n*th term of a sequence gives a_n as an expression in the term's position *n*; a recursive rule gives the first term of a sequence, and a recursive equation relates a_n to the preceding term(s). Both methods of presenting a sequence describe a_n as a function of *n*. Examples: • Generate the 5th-11th terms of a sequence if $A_1 = 2$ and $A_{(n+1)} = (A_n)^2 - 1$ • Use the formula: $A_n = A_1 + d(n - 1)$ where d is the common difference to generate a sequence whose first three terms are: -7, -4, and -1. • There are 2,500 fish in a pond. Each year the population decreases by 25 percent, but 1,000 fish are added to the pond at the end of the year. Find the population in five years. Also, find the long-term population. • Given the formula $A_n = 2n - 1$, find the 17th term of the sequence. What is the 9th term in the sequence: 3, 5, 7, 9, …? Given $a_1 = 4$ and $a_n = a_{n-1} + 3$, write the explicit formula.

MHS Algebra 1 Mathematics Curriculum
Unit 2 Addendum

Performance Standard	Instructional Strategies
F.BF.3:	Students will apply transformations to functions and recognize functions as even and odd. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to graph functions. Examples: • Is $f(x) = x^3 - 3x^2 + 2x + 1$ even, odd, or neither? Explain your answer orally or in written format. • Compare the shape and position of the graphs of $f(x) = x^2$ and $g(x) = 2x^2$, and explain the differences in terms of the algebraic expressions for the functions  • Describe effect of varying the parameters *a*, *h*, and *k* have on the shape and position of the graph of $f(x) = a(x-h)^2 + k$. • Compare the shape and position of the graphs of $f(x) = e^x$ to $g(x) = e^{x-6} + 5$, and explain the differences, orally or in written format, in terms of the algebraic expressions for the functions  • Describe the effect of varying the parameters *a*, *h*, and *k* on the shape and position of the graph $f(x) = ab^{(x+h)} + k$., orally or in written format. What effect do values between 0 and 1 have? What effect do negative values have? • Compare the shape and position of the graphs of $y = \sin x$ to $y = 2 \sin x$. 

MHS Algebra 1 Mathematics Curriculum
Unit 2 Addendum

Performance Standard	Instructional Strategies

F.LE.1:	Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model and compare linear and exponential functions.
	
	Examples:
	• A cell phone company has three plans. Graph the equation for each plan, and analyze the change as the number of minutes used increases. When is it beneficial to enroll in Plan 1? Plan 2? Plan 3?
	1. $59.95/month for 700 minutes and $0.25 for each additional minute,
	2. $39.95/month for 400 minutes and $0.15 for each additional minute, and
	3. $89.95/month for 1,400 minutes and $0.05 for each additional minute.
	
	• A computer store sells about 200 computers at the price of $1,000 per computer. For each $50 increase in price, about ten fewer computers are sold. How much should the computer store charge per computer in order to maximize their profit?
	
	Students can investigate functions and graphs modeling different situations involving simple and compound interest. Students can compare interest rates with different periods of compounding (monthly, daily) and compare them with the corresponding annual percentage rate. Spreadsheets and applets can be used to explore and model different interest rates and loan terms.
	Students can use graphing calculators or programs, spreadsheets, or computer algebra systems to construct linear and exponential functions.
	
	• A couple wants to buy a house in five years. They need to save a down payment of $8,000. They deposit $1,000 in a bank account earning 3.25% interest, compounded quarterly. How much will they need to save each month in order to meet their goal?
	• Sketch and analyze the graphs of the following two situations. What information can you conclude about the types of growth each type of interest has?
	  o Lee borrows $9,000 from his mother to buy a car. His mom charges him 5% interest a year, but she does not compound the interest.
	  o Lee borrows $9,000 from a bank to buy a car. The bank charges 5% interest compounded annually.
	• Calculate the future value of a given amount of money, with and without technology.
	• Calculate the present value of a certain amount of money for a given length of time in the future, with and without technology.

MHS Algebra 1 Mathematics Curriculum
Unit 2 Addendum

<table>
<tr><th>Performance Standard</th><th>Instructional Strategies</th></tr>
<tr><td>F.LE.2:</td><td>
Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to construct linear and exponential functions.

Examples:

• Determine an exponential function of the form $f(x) = ab^x$ using data points from the table. Graph the function and identify the key characteristics of the graph.
<table>
<tr><th>x</th><th>f(x)</th></tr>
<tr><td>0</td><td>1</td></tr>
<tr><td>1</td><td>3</td></tr>
<tr><td>3</td><td>27</td></tr>
</table>
• Sara’s starting salary is $32,500. Each year she receives a $700 raise. Write a sequence in explicit form to describe the situation.
</td></tr>
</table>

Unit 3 – Descriptive Statistics

Domain: Statistics and Probability (S)
Essential Question: How do data interpretations help develop informed decisions and predictions?

Mathematical Practices:

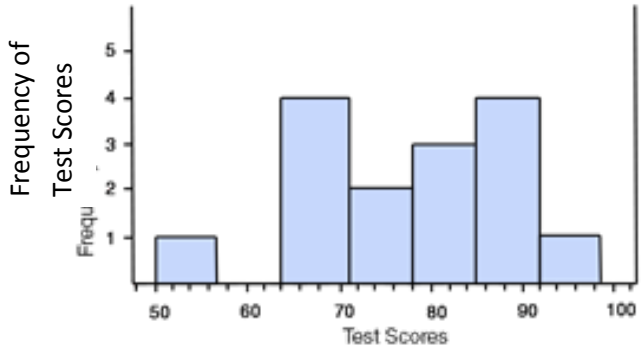
1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Summarize, interpret and represent data using various models and apply statistical reasoning to draw conclusions.** **S-ID (***)** Summarize, represent, and interpret data on a single count or measurement variable	**How…** **S.ID.1:** Represent data with plots on the real number line (dot plots, histograms, and box plots).	**Students will:** **Data Check-In Tasks** Algebra 1 Unit 3 Checkin 1.docx Algebra 1 Unit 3 Checkin 2.docx Data, Data, Everywhere Stations.	**S.ID.1** Construct dot plots, histograms and box plots for data on a real number line.

Vocabulary: context of data, differences (center, shape, spread), exponential, interquartile range, measures of center, measures of spread, mean, median, models outliers, slope, standard deviation

Algebra 1 Unit 3 – STAT Check – In 1

Problem 1: The histogram represents the scores from the last geometry test. They are graphed with a bin width of 7.



Test Scores

1) How many students are in the class? _____

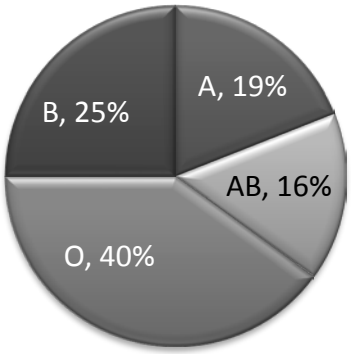
2) How many students scored above a 78? _____

3) Describe the classes performance on the test:

_____

_____

Problem 2: The pie chart below shows the percentages of blood types for a group of 200 people.



4) How many people, in this group, have blood type AB?

5) How many people, in this group, have blood types A or B?

6) Draw one conclusion about the blood types of people in this population:

_____

_____

_____

Algebra 1 Unit 3 – STAT Check – In 2

Match the data set to the correct way to graph the data:

_____	1) Eye Color	a) Histogram
_____	2) Salaries	b) Pie Chart
_____	3) Favorite Ice Cream	c) Line Graph
_____	4) Participation in Sports over 10 years	d) Bar Graph

5) Construct the appropriate graph for this data set:

Number of Televisions	Households reporting that number
0	1
1	16
2	14
3	12
4	3
5	2
6	2

a) Graph here:

b) Why did you select this type of graph? _____

_____

_____

Data, Data Everywhere! Stations Activity

Prior Learning: Students should have experience constructing graphs from data sets and drawing conclusions about the population based on the graphical representations.

Directions:
Divide students into small groups (no larger than 3 or 4). Have each group start at a station and work through each problem completely.

A time limit should be set so that the students are able to work through each station completely.

Materials Needed:
Stations worksheets – one per person
Graph Paper – one per person/per station
Colored Pencils
Calculators

Station 1

Water levels in Everglades National Park are critical to the survival of this unique region. Below is listed the water depths at the park over a month period. On a separate sheet create a graph to display this data set.

Date	Depth	Date	Depth
8/16/2000	0.23	9/1/2000	0.27
8/17/2000	0.23	9/2/2000	0.27
8/18/2000	0.22	9/3/2000	0.27
8/19/2000	0.22	9/4/2000	0.27
8/20/2000	0.22	9/5/2000	0.27
8/21/2000	0.21	9/6/2000	0.27
8/22/2000	0.21	9/7/2000	0.28
8/23/2000	0.21	9/8/2000	0.28
8/24/2000	0.2	9/9/2000	0.28
8/25/2000	0.2	9/10/2000	0.29
8/26/2000	0.2	9/11/2000	0.29
8/27/2000	0.2	9/12/2000	0.32
8/28/2000	0.21	9/13/2000	0.33
8/29/2000	0.22	9/14/2000	0.32
8/30/2000	0.26	9/15/2000	0.32
8/31/2000	0.26		

We chose to create a _____ for this data.

We chose this type of graphical representation for this data because it is

_____

_____

_____

What does this data and its graphical representation tell you about the water levels in Everglades National Park over this time period?

_____

_____

_____

Using your data and graphical representation what predictions can you make about the future of the water levels in the Everglades during this time next year or about the upcoming month?

_____

_____

_____

Station 2

Every year there are approximately 1.6 million students who enroll in colleges and universities. When entering college most of these students declare a major. Here are the data percents of the first year students and what major they declare. On a separate sheet create a graph to display this data set.

Discipline	Percent
Arts/Humanities	12.8
Biological Science	7.6
Business	17.4
Education	9.9
Engineering	8.3
Physical Science	3.1
Social Science	10.7
Professional	14.6
Technical	1.2
Other	14.1

We chose to create a _____ for this data.

We chose this type of graphical representation for this data because it is

_____

_____

_____

What does this data and its graphical representation tell you about what people choose to study when entering college?

_____

_____

_____

If you were a college dean or president how could you use your data and graphical representation to make predictions about future applicants to your institution? What kinds of classes should you prep your staff for?

_____

_____

_____

Station 3

Purchasing music online has become a very popular thing. Here are the percents of people in several age ranges who bought music online. On a separate sheet create a graph to display this data set.

Age	Percent
12 to 17	24
18 to 24	21
25 to 34	20
35 to 44	16
45 to 54	10
55 to 64	3
65 plus	1

We chose to create a _____ for this data.

We chose this type of graphical representation for this data because it is

_____

_____

_____

What does this data and its graphical representation tell you about the people who downloads music?

_____

_____

_____

If you were marketing a music downloading website how could you use your data and graphical representation to make predictions about who you market your advertisement toward?

_____

_____

_____

Station 4

Every year students all over the country take the SAT's. Here is some data from 175 students who took the SAT's at MHS last spring. The data represents student scores on the Math portion of the test. On a separate sheet create a graph to display this data set.

Score Range	Students
345 – 390	4
390 – 435	8
435 – 480	52
480 – 525	41
525 – 570	37
570 – 615	19
615 – 660	3
660 – 705	5
705 – 750	1
750 – 795	3

We chose to create a _____ for this data.

We chose this type of graphical representation for this data because it is

_____

_____

_____

What does this data and its graphical representation tell you about the students who took the SATs?

_____

_____

_____

Using your data and graphical representation what predictions can you make about the future SAT math scores for the following year? How can it be used to develop goals or focus practice of upcoming test takers?

_____

_____

_____

Unit 3 – Descriptive Statistics

Domain: Statistics and Probability (S)
Essential Question: How do data interpretations help develop informed decisions and predictions?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Summarize, interpret and represent data using various models and apply statistical reasoning to draw conclusions.** **S-ID (***)** Summarize, represent, and interpret data on a single count or measurement variable	**How…** **S.ID.2:** Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. **S.ID.3:** Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	**Students will:** **Represent and Interpret Data Activity** S.ID.2-3 Representing Data.pd	**S.ID.2:** Construct dot plots, histograms and box plots for data on a real number line. Students may use spreadsheets, graphing calculators and statistical software for calculations, summaries, and comparisons of data sets. **Examples:** • The two data sets below depict the housing prices sold in the King River area and Toby Ranch areas of Pinal County, Arizona. Based on the prices below which price range can be expected for a home purchased in Toby Ranch? In the King River area? In Pinal County? • King River area {1.2 million, 242000, 265500, 140000, 281000, 265000, 211000} • Toby Ranch homes {5million, 154000, 250000, 250000, 200000, 160000, 190000} • Given a set of test scores: 99, 96, 94, 93, 90, 88, 86, 77, 70, 68, find the mean, median and standard deviation. Explain how the values vary about the mean and median. What information does this give the teacher?

Vocabulary: context of data, differences (center, shape, spread), exponential, interquartile range, measures of center, measures of spread, mean, median, models outliers, slope, standard deviation

CONCEPT DEVELOPMENT

Mathematics Assessment Project

CLASSROOM CHALLENGES

A Formative Assessment Lesson

Representing Data 1: Using Frequency Graphs

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley

Beta Version

For more details, visit: http://map.mathshell.org
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Representing Data 1: Using Frequency Graphs

MATHEMATICAL GOALS

This lesson unit is intended to help you assess how well students:

- Are able to use frequency graphs to identify a range of measures and make sense of this data in a real-world context.
- Understand that a large number of data points allow a frequency graph to be approximated by a continuous distribution.

COMMON CORE STATE STANDARDS

This lesson relates to the following *Standards for Mathematical Content* in the *Common Core State Standards for Mathematics*:

S-ID Summarize, represent, and interpret data on a single count or measurement variable.

This lesson also relates to the following *Standards for Mathematical Practice* in the CCSS:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.

INTRODUCTION

The unit is structured in the following way:

- Before the lesson, students complete an assessment task individually that is designed to reveal their current understanding.
- A whole-class introduction provides students with guidance on how to work through the task. Students work in pairs or threes on a collaborative discussion task, matching written interpretations and graphs as they begin to link these two representations.
- Towards the end of the lesson there is a whole-class discussion.
- In a follow-up lesson, students work alone on a similar task to the assessment task.

This lesson can be taught in conjunction with the lesson *Representing Data Using Box Plots* or independently.

MATERIALS REQUIRED

Each student will need a copy of the handouts: *Cell Phones 1*, *Cell Phones 1 (revisited)*, and *Drawing Graphs*, a mini-whiteboard, a pen, and an eraser.

Each small group of students will need the following resources:

- The Card Sets: *Frequency Graphs* and *Interpretations*. Cut-up both card sets before the lesson.
- One large sheet of paper for making posters and a glue stick. The paper should be large enough to accommodate three different sets of cards.

There are some projector resources to help with whole-class discussions.

TIME NEEDED

Approximately 20 minutes before the lesson, a 65-minute lesson, and 20 minutes in a follow-up lesson (or for homework). Exact timings will depend on the needs of the class.

BEFORE THE LESSON

Assessment task: *Cell Phones 1* (20 minutes)

Give this task, in class or for homework, a few days before the formative assessment lesson. This will give you an opportunity to assess the work and to find out the kinds of difficulties students have with it. You will then be able to target your help more effectively in the lesson.

Give each student a copy of the assessment task: *Cell Phones 1*.

> *Read through the questions and try to answer them as carefully as you can.*

It is important that, as far as possible, students are allowed to answer the questions without your assistance.

Students should not worry too much if they cannot understand or do everything because, in the lesson, they will engage in a similar task that should help them. Explain to students that by the end of the next lesson they should expect to answer questions such as these confidently. This is their goal.

Cell Phones 1

Here is a frequency graph that shows the monthly spending of a group of students on their cell phones:

The graph shows:

A range of spending of about .....

I know this from the graph because .....

.....

The mode is about .....

I know this from the graph because .....

.....

The median is about .....

I know this from the graph because .....

.....

.....

Most students spend over .....

I know this from the graph because .....

.....

.....

Assessing students' responses

Collect students' responses to the task and note down what their work reveals about their current levels of understanding, and their different approaches.

We suggest that you do not score students' work. The research shows that this will be counterproductive, as it will encourage students to compare their scores and distract their attention from what they can do to improve their mathematics.

Instead, help students to make further progress by summarizing their difficulties as a series of questions. Some suggestions for these are given on the next page. These have been drawn from common difficulties observed in trials of this unit.

We suggest that you write a list of your own questions, based on your students' work, using the ideas in the *Common issues* table. We recommend that you write questions on each individual student's work. If you do not have time to do this, select a few questions that will be of help to the majority of students. These can be written on the board at the start of the lesson.

Common issues	Suggested questions and prompts
Student provides an incorrect range For example: The student writes 5 - 40. Or: The student assumes the range is the difference between the maximum and zero.	• What is the definition of the term 'range'?
Student uses the frequency readings for the measures of central tendency For example: The student writes that the mode is 48.	• What does the vertical axis represent? • What measure is the question asking for?
Student does not understand the word 'Frequency' For example: The student assumes 'Frequency' represents a percentage.	• What does the term 'frequency' mean? • What does the axis marked 'frequency' represent on the graph?
Student does not understand what median represents or how to use the graph to figure out the median For example: The student assumes the median value is equal to the mode. Or: The student assumes the median value is exactly half way between the maximum and minimum value.	• What does the median represent? • Can the mode and median values be different? • Roughly what proportion of students spends less than the median value? • Sketch on the graph the results of a different survey. In this survey the maximum and minimum values for the amount spent each month remains the same, but the number of students in the survey is half/double. What does this tell you about the area under the graph? • How can you show the median value on your graph? [Draw a vertical line to divide in half the area under the graph. The median amount is the value at the point this line intersects the *x*-axis.]
Student does not contextualize the data For example: The student states the mode is 31, the minimum and maximum values are 5 and 40 respectively, but does not refer to the context.	• What do these figures represent? • Complete this sentence "50% of students spend less than $_ _ _ each month." Explain how you arrived at the figure.

SUGGESTED LESSON OUTLINE

Throughout this lesson, encourage students to use the correct mathematical language to not just **provide** the figure for a measure, but to **place the figure in the context**. For example, rather than "The median is 20", encourage students to say "The median score is 20 out of 100."

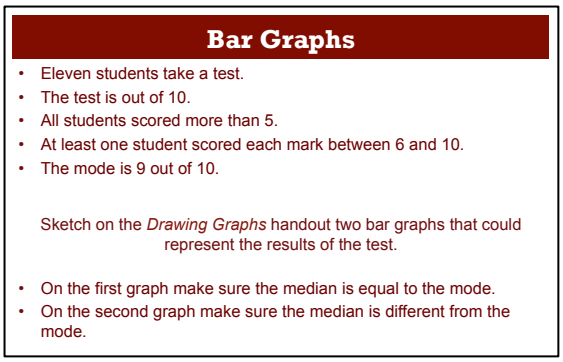
Whole-class interactive introduction: Drawing and Interpreting Graphs (20 minutes)

Give out the sheet *Drawing Graphs*. Maximize participation in the introduction by asking all students to show you their graphs once sketched.

This introduction will provide students with a model of how they should justify their matching of cards in the collaborative activity. It will also help students understand how to identify a range of measures from a graph and in particular the median value.

Often students think that the median should be the middle score of the range of scores. The first task in this introduction helps demonstrate that this is not always the case.

Show Slide P-1 of the projector resource:



Sketch on your sheet two bar graphs that could represent the test results.

On the first make sure the median is equal to the mode.

On the second the median should be different from the mode.

If students struggle, encourage them to discuss the task with a neighbor. It may help if they write down the value of the eleven scores.

After a few minutes ask students to show you their graphs and select two or three students with different graphs to justify them to the class. Ask the rest of the class if they agree with the explanations.

You may want to use Slide P-2 of the projector resource, *Mode and Median*, to support the discussion.

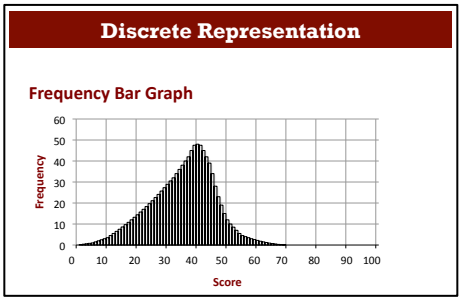
How can you check that eleven students took the test?

How many students achieved the median score or less? How do you know?

What is meant by the statement 'the median is the middle score?' [The middle score of a sorted list of scores.]

Could the median score ever be equal to the minimum or maximum score? [Yes, but the mode would not be 9 out of 10.] How do you know?

Show Slide P-3 of the projector resource:



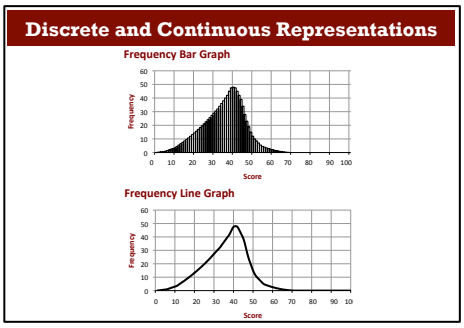
Explain to students that the bar chart represents the scores of students in a test for in which the maximum score was 100. Ask the following questions in turn:

Did anyone achieve the maximum score of 100? How can you tell?

What can you say about the test? Did students find it difficult or easy? How can you tell?

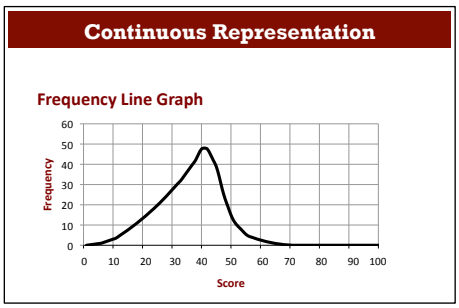
Roughly how many students took the test? [About 1,000.] How can you tell?

Show Slide P-4 of the projector resource:



When there are many bars close together the data can be represented as a continuous line, that is, a frequency graph. This makes it a little easier to read off the values.

Show Slide P-5 of the projector resource:



Students write on their mini-whiteboards all the information they can derive from the graph. After a couple of minutes ask students to show you their answers.

Ask one or two students to justify their answers. Even if their explanations are incorrect or only partially correct, ask students to write them on to the projected graph. Encourage students to challenge these interpretations and then replace them with new ones.

Depending on your class, you may want to ask students a selection of the following questions:

What is the range of scores? [About 64 out of 100.] How do you know?

What is the mode score? [42 out of 100.] How do you know?

What is the range of scores for most students? [Most people scored between 30 and 50 out of 100.]

How do you know? [A large proportion of the area under the graph is between 30 and 50 out of 100.]

What does the area under the graph represent? [The number of people in the survey.]

To confirm that students understand that the area under the graph represents the number of people in the survey you could ask them to sketch a graph on their whiteboard for another set of test results, but this time only half the number of students taking the test. The minimum and maximum scores remain the same

Now return to the original graph:

Roughly what is the median score? [37 out of 100.] How do you know?

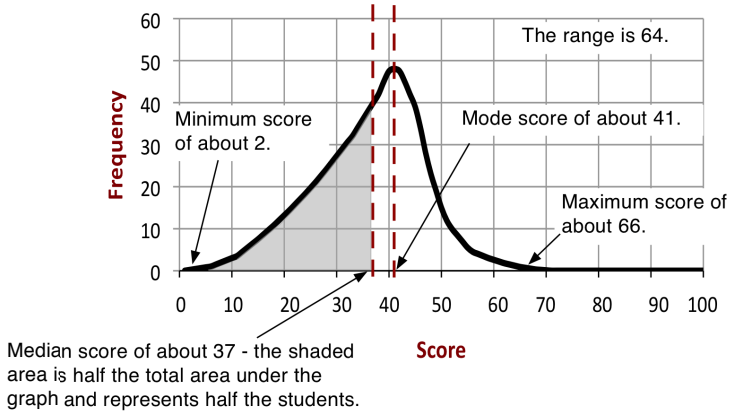
What is the range of scores for the top quarter of students?
[About 22 (66 – 44) out of 100.] How do you know?

To answer these final two questions, the area under the graph needs to be divided in half/quarters by vertical lines extending from the *x*-axis. For students to figure out these values it may help if you project the bar graph of the data.

If students find it difficult to figure out an estimate for the median, add a vertical line to the graph that intersects the *x*-axis at, say, a score of 20 out of 100. Then ask student to estimate how many children scored less than 20.

Can this be the median score? How do you know?

The graph may end up looking like this:



Collaborative activity: Matching Card Sets (20 minutes)

Ask students to work in small groups of two or three.

Give each group Card Set: *Frequency Graphs*, Card Set: *Interpretations*, and a large sheet of paper for making a poster.

> *Take turns at matching pairs of cards that you think belong together.*
>
> *Each time you do this, explain your thinking clearly and carefully. Your partner should either explain that reasoning again in his or her own words, or challenge the reasons you gave.*
>
> *Write your reasons for the match on the poster.*
>
> *Place your cards side by side on your large sheet of paper, not on top of one another, so that everyone can see them.*
>
> *You both need to be able to agree on and explain the placement of every card.*

These instructions are summarized on Slide P-6 of the projector resource, *Matching Cards*.

The purpose of this structured work is to encourage each student to engage with their partner's explanations and to take responsibility for their partner's understanding.

While students work in small groups you have two tasks: to note different student approaches to the task and to support student reasoning.

Note different student approaches to the task

In particular, notice any difficulties that students encounter, and the ways they justify and explain to each other. Do students check to see if their match is correct? Do they assume the scores go up the vertical axis? How do they understand how to use the graph to figure out the median and mode? When stating a measure, do students confuse the values on the x-axis and y-axis? Are students using the correct mathematical language? Are students using all the information on the cards or just the first sentence? What do they do if they get stuck?

You can then use this information to focus your questioning in the whole-class discussion towards the end of the lesson.

Support student reasoning

Try not to make suggestions that move students towards particular matches. Instead, ask questions to help students to reason together. You may want to use some of the questions and prompts from the *Common issues* table.

If a student struggles to get started encourage them to ask a specific question about the task. Articulating the problem in this way can sometimes offer a direction to pursue that was previously overlooked. However, if the student needs their question answered, ask another member of the group for a response.

Here are some further questions you may want to use:

> *Rewrite the description in your own words.*
>
> *Write a description of the graph.*
>
> *Tell me how you have used **all** the information [i.e. both sentences] on the Interpretation card to match it with a graph.*
>
> *How can you figure out an approximate value for the median?*

Did many students get a low/high score for this graph? How do you know?

Show me a graph that shows the median score equal to the mode score. How do you know?

Show me a graph that shows the median score different to the mode score. How do you know?

Show me a graph in which a lot of students/few students found it easy. How do you know?

Show me a graph that shows the median score equal to the minimum score. How do you know?

Which graph shows students of a similar ability? How do you know?

Make up five figures where the median is greater than the mode. Now sketch a graph of these figures.

If you find one student has matched two cards, challenge another student in the group to provide an explanation.

Danny matched these cards. Ezra, why does Danny think these two cards go together?

If you find the student is unable to answer this question, ask them to discuss the work further. Explain that you will return in a few minutes to ask a similar question.

If the whole-class is struggling on the same issue you could write a couple of questions on the board and hold a whole-class discussion.

Sharing posters (10 minutes)

As students finish matching the cards, ask one student from each group to visit another group's poster.

If you are staying at your desk, be ready to explain the reasons for your group's decisions.

If you are visiting another group, copy your matches onto a piece of paper. Go to the other group's desk and check to see which matches are different from your own. If there are differences, ask for an explanation. If you still don't agree, explain your own thinking. When you return to your own desk, you need to consider as a group whether to make any changes to your poster.

Slide P-7 of the projector resource, *Sharing Posters*, summarizes these instructions.

When students are satisfied with all their matches give them a glue stick and ask them to glue the cards onto the poster.

Whole-class discussion (15 minutes)

You may want to use transparencies of the cards or Slide 8 of the projector resource to support the discussion.

The intention is that this discussion focuses on the justification of a few examples, rather than checking students all have the correct solutions. You may want to first select a pair of cards that most groups matched correctly as this may encourage good explanations. Then select one or two matches that most groups found difficult. In trials students have had difficulty matching graphs B, C, and H.

How did you decide to match this card?

Can someone else put that into his or her own words?

Could this card be matched with another one?

After discussing two or three matches ask:

Which graph card do you think is unrealistic? Why?

Now ask students to sketch on their whiteboard a graph that shows the test results of a different class. The first piece of information about the test is that there is a in a huge range of scores.

Once students have shown you their whiteboards ask them to swap whiteboards with a neighbor and write a second piece of information about the test on their neighbors' whiteboard. This piece of information, combined with the first, should make their existing graph incorrect.

Once whiteboards are returned students will need to re-draw their graph so that it represents both pieces of information.

Ask students to show you their whiteboards. Ask a few students with differing graphs to explain why they were forced to re-draw it.

Follow-up lesson: Reviewing assessment (20 minutes)

Return the original assessment *Cell Phones 1* to the students together with a copy of *Cell Phones 1 (revisited)*. If you have not added questions to individual pieces of work, write your list of questions on the board. Students should select from this list only those questions they think are appropriate to their own work. Some students may struggle to identify which questions they should consider from this list. If this is the case it may be helpful to give students a printed version of the list of questions so that you can highlight the ones that you want them to focus on.

Look at your original responses and the questions (on the board/written on your script.) Think about what you have learned.

Now look at the new task sheet, Cell Phones 1 (revisited). Use what you have learned to answer these questions.

When you revise your work, write as if you are explaining the solutions to someone unfamiliar with this type of math.

SOLUTIONS

Assessment task: *Cell phones 1*

All answers are approximate.

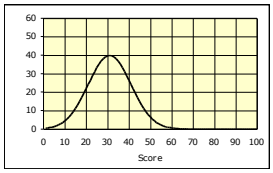
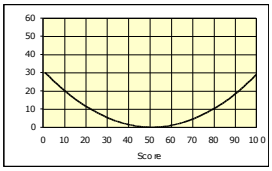
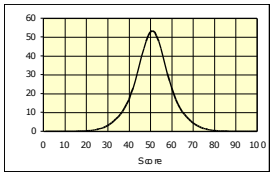
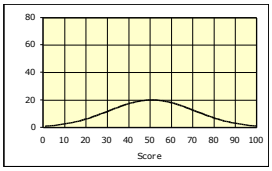
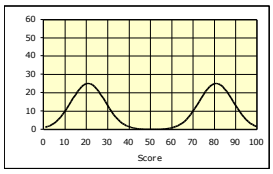
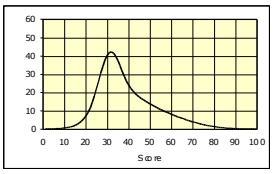
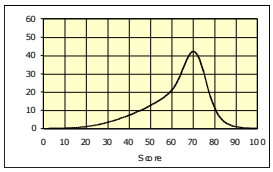
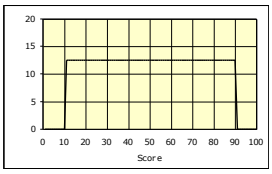
The range is about $35 a month. I know this from the graph because the range represents the difference between the maximum ($40 a month) and minimum ($5 a month) amount spent each month.

The mode is about $31 a month. I know this from the graph because the maximum frequency is about $31 a month.

The median is about $29 a month. I know this from the graph because if I drew a vertical line from this point on the *x*-axis it would divide in half the area under the graph. The area under the graph represents the number of students in the survey. This means about half the students spent less than $29 a month and half spent more than $29 a month.

Most students spend over $24 a month. I know this from the graph because the area under the graph up to $24 is a lot less than the area under the graph between $24 and $66.

Collaborative activity:

A 	8. This test was much too difficult for most people.	B 	1. This was the sort of test where you could either do everything or you couldn't get started.
C 	2. This test did not sort out the stronger students from the weaker ones. They all got similar scores.	D 	7. In this test, the median and the mode scores were the same. There was a very big range of scores.
E 	3. Two groups of students took the test. One group had studied the work for two years. The other group had only just begun.	F 	5. In this test, the median score was greater than the mode score.
G 	6. In this test, the median score was smaller than the mode score.	H 	4. This test resulted in a huge range of scores. Everyone could do something but nobody could do it all.

Assessment task: *Cell phones 1 (revisited)*

All answers are approximate.

The range is about $38 a month. I know this from the graph because the range represents the difference between the maximum ($44 a month) and minimum ($6 a month) amount spent.

The mode is about $23 a month. I know this from the graph because the maximum frequency is about $23 a month.

The median is about $18 a month. I know this from the graph because if I drew a vertical line from this point on the *x*-axis it would divide in half the area under the graph. The area under the graph represents the number of students in the survey. This means about half the students spent less than $18 a month and half spent more than $18.

Very few students spend over $30 a month. I know this from the graph because the area under the graph from $30 to $44 is a lot less than the area under the graph between $6 and $30.

Cell Phones 1

Here is a frequency graph that shows the monthly spending of a group of students on their cell phones:

The graph shows:

A range of spending of about ----------

I know this from the graph because ----------

The mode is about ----------

I know this from the graph because ----------

The median is about ----------

I know this from the graph because ----------

Most students spend over ----------

I know this from the graph because ----------

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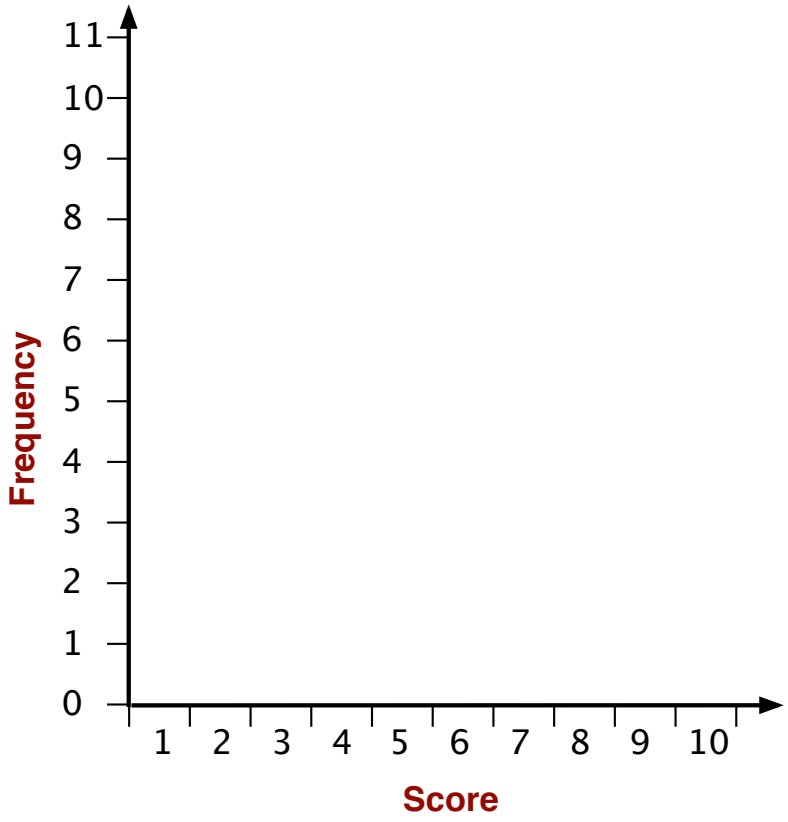
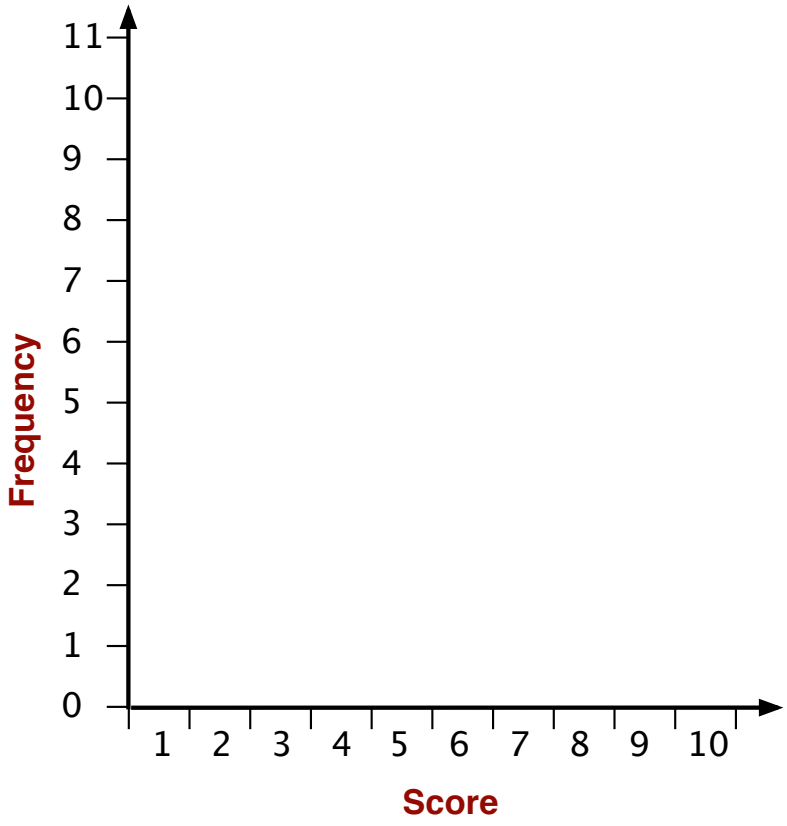
Drawing Graphs





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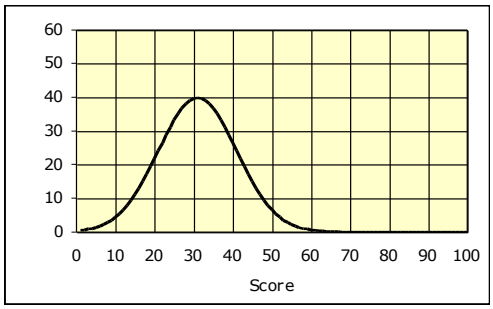
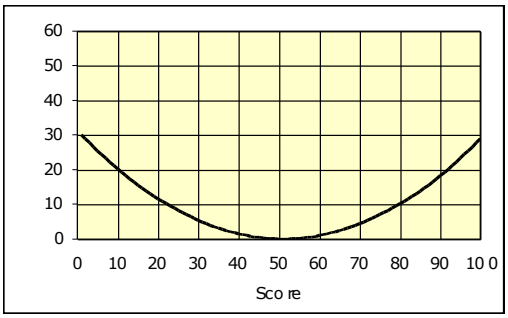
Card Set: Frequency Graphs

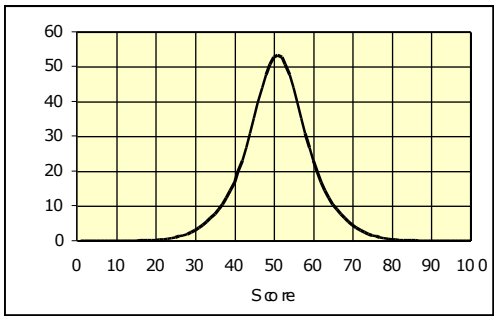
Frequency Graph A



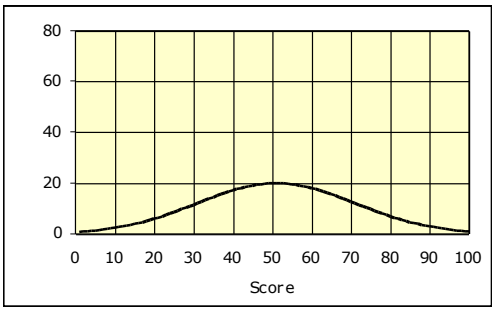
Frequency Graph B



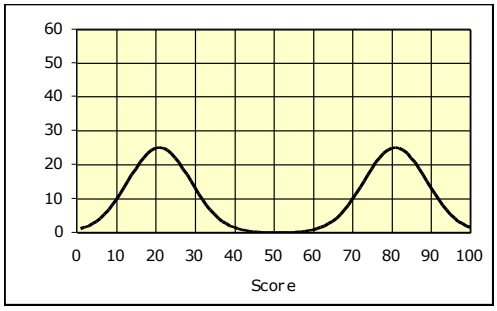
Frequency Graph C



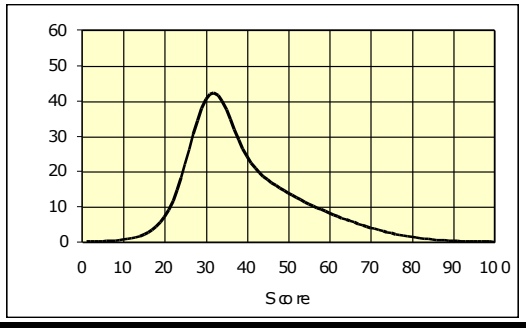
Frequency Graph D



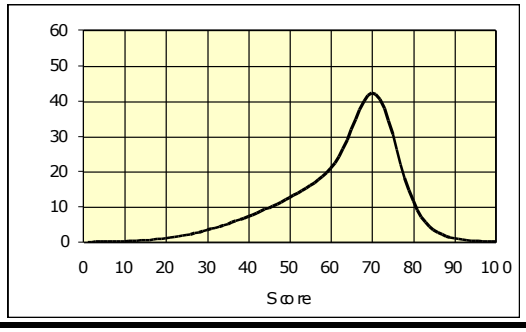
Frequency Graph E



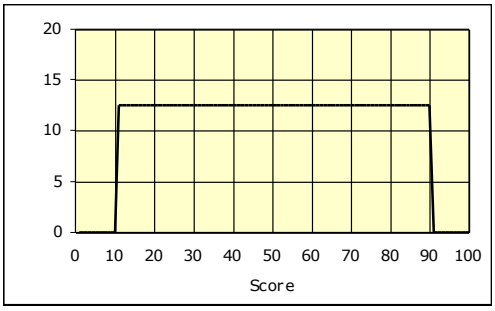
Frequency Graph F



Frequency Graph G



Frequency Graph H



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Card Set: Interpretations

1. This was the sort of test where you could either do everything or you couldn't get started.	2. This test did not sort out the stronger students from the weaker ones. They all got similar scores.
3. Two groups of students took the test. One group had studied the work for two years. The other group had only just begun.	4. This test resulted in a huge range of scores. Everyone could do something but nobody could do it all.
5. In this test, the median score was greater than the mode score.	6. In this test, the median score was smaller than the mode score.
7. In this test, the median and the mode scores were the same. There was a very big range of scores.	8. This test was much too difficult for most people.

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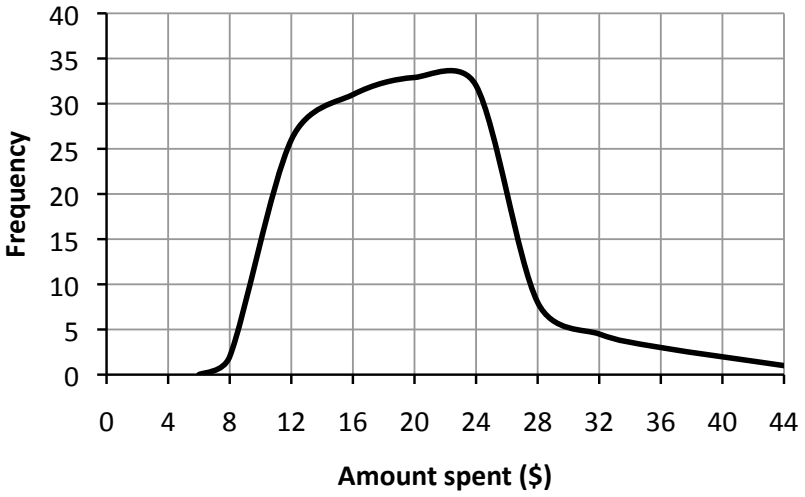
Cell Phones 1 (revisited)

Here is a frequency graph that shows the monthly spending of a group of students on their cell phones:



The graph shows:

A range of spending of .....

I know this because .....

.....

The mode is .....

I know this because .....

.....

The median is .....

I know this because .....

.....

.....

Very few students spend over .....

I know this because .....

.....

.....

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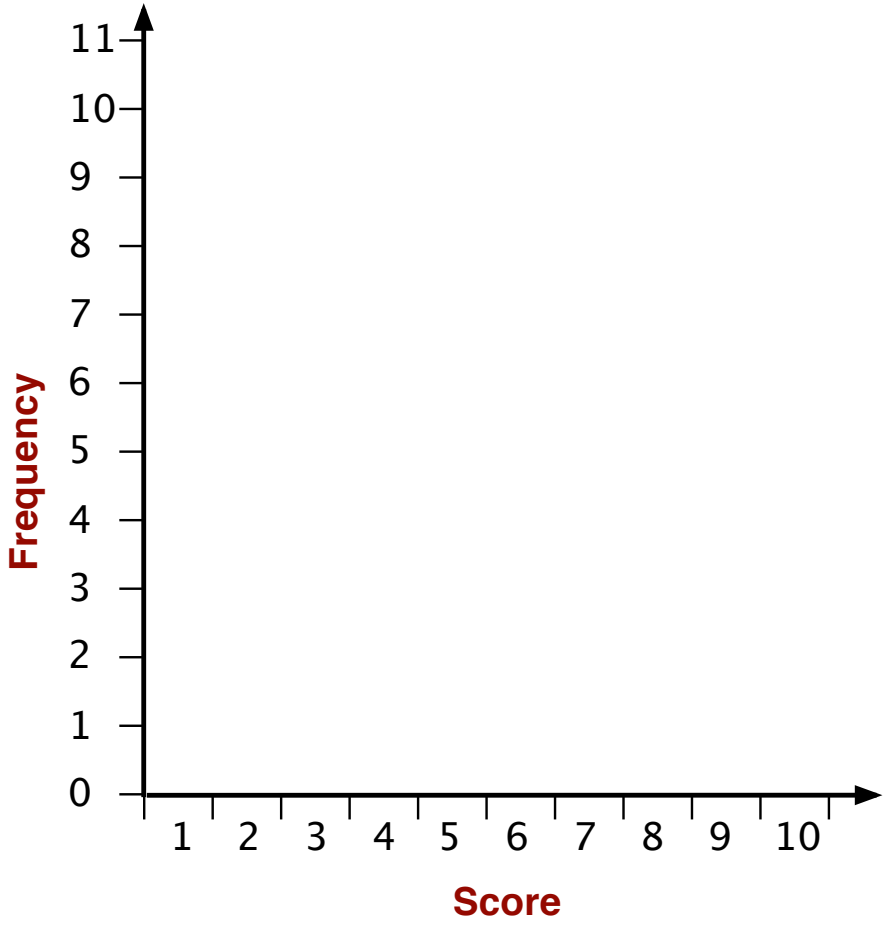
Bar Graphs

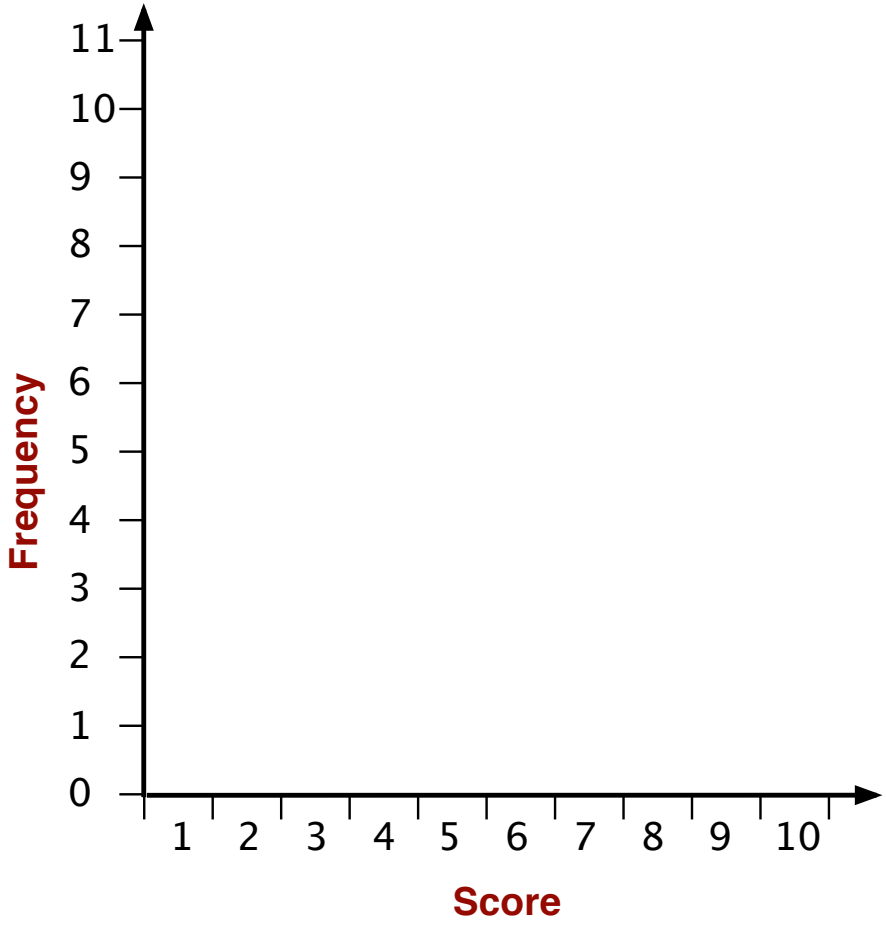
- Eleven students take a test.
- The test is out of 10.
- All students scored more than 5.
- At least one student scored each mark between 6 and 10.
- The mode is 9 out of 10.

Sketch on the *Drawing Graphs* handout two bar graphs that could represent the results of the test.

- On the first graph make sure the median is equal to the mode.
- On the second graph make sure the median is different from the mode.

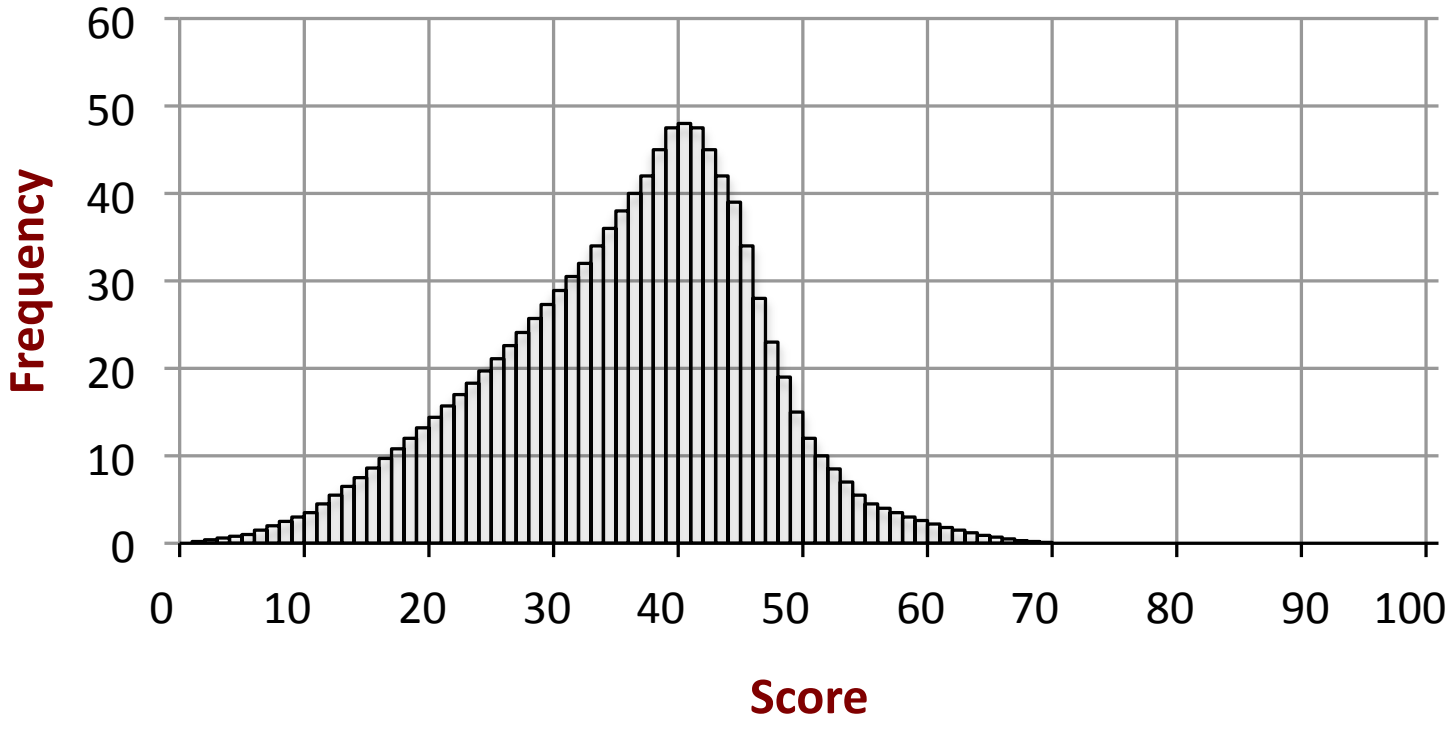
Mode and Median





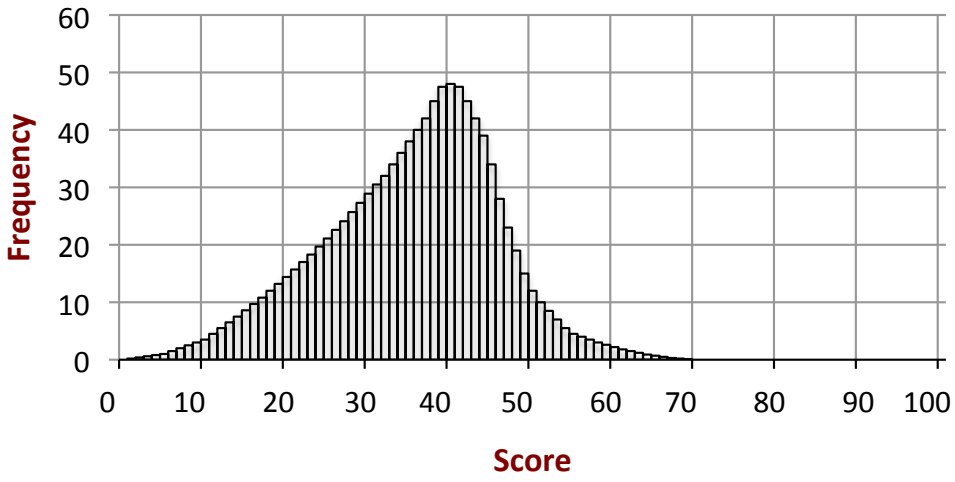
Discrete Representation

Frequency Bar Graph

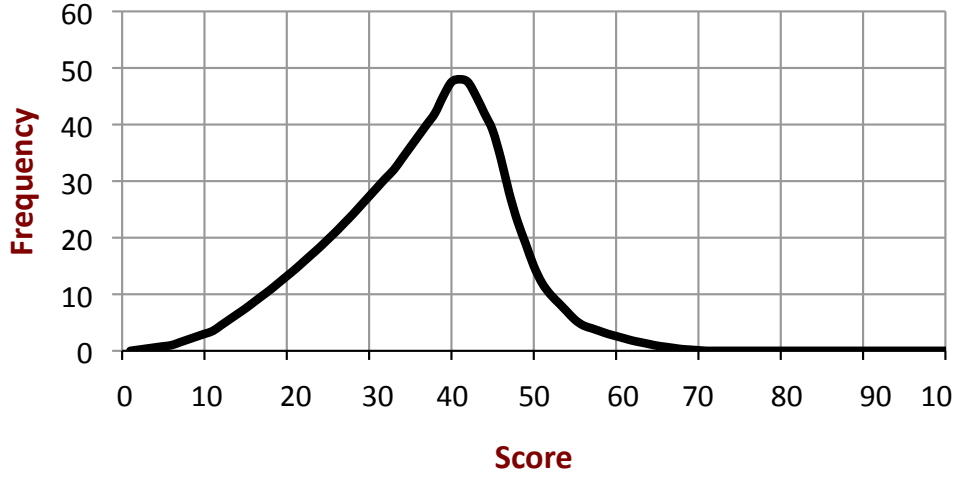


Discrete and Continuous Representations

Frequency Bar Graph

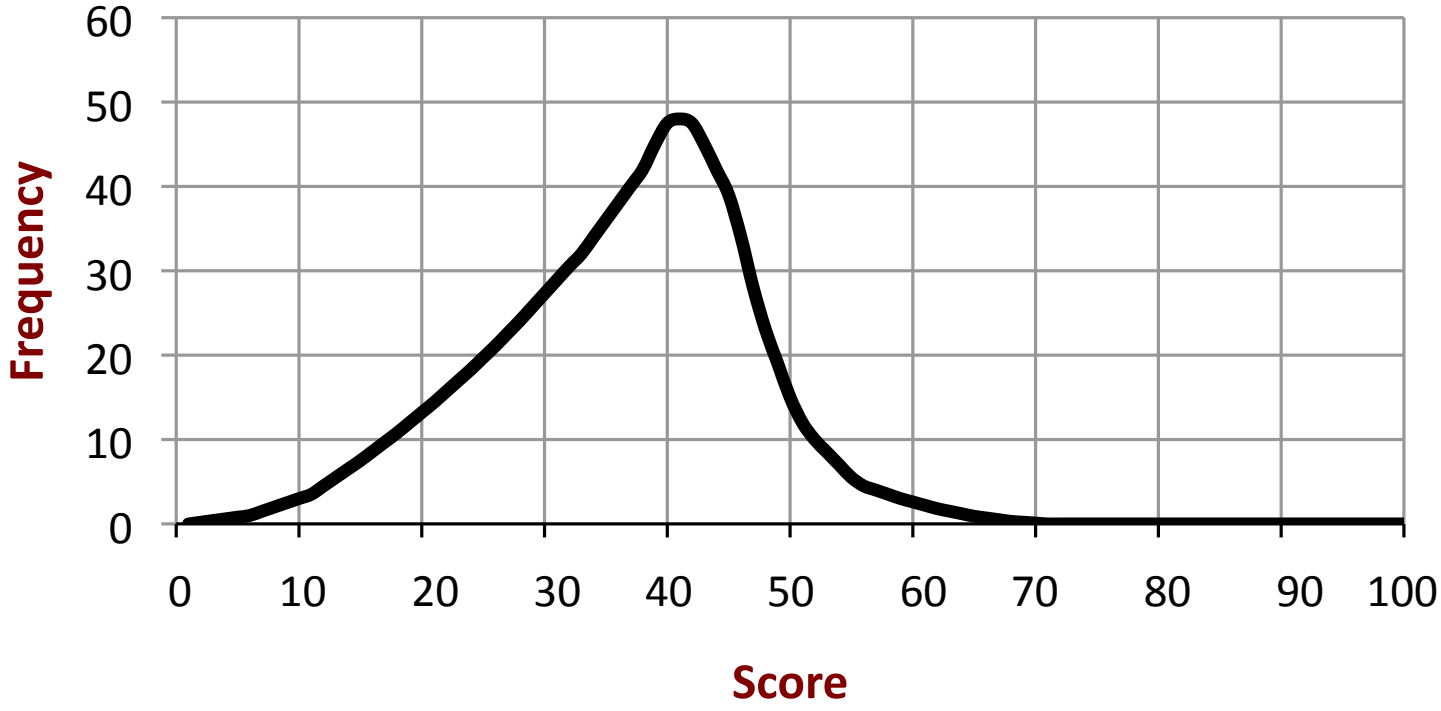


Frequency Line Graph



Continuous Representation

Frequency Line Graph



Matching Cards

1. Take turns at matching pairs of cards that you think belong together.

2. Each time you do this, explain your thinking clearly and carefully.

3. Your partner should either explain that reasoning again in his or her own words, or challenge the reasons you gave.

4. Write your reasons for the match on the poster.

5. You both need to be able to agree on and explain the placement of every card.

You may find some of ‘Word’ cards match two graphs.

This problem will be resolved as you match more cards.

Be prepared to change your mind about the matches.

Sharing Posters

1. If you are staying at your desk, be ready to explain the reasons for your group's matches.

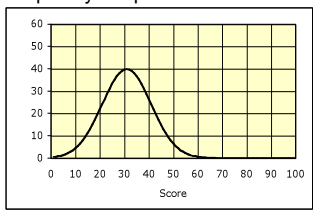
2. If you are visiting another group:
   - Copy your matches onto a piece of paper.
   - Go to another group's desk and check to see which matches are different from your own.

3. If there are differences, ask for an explanation. If you still don't agree, explain your own thinking.

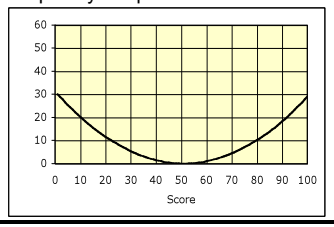
4. When you return to your own desk, you need to consider as a group whether to make any changes to your poster

Card Set: Frequency Graphs

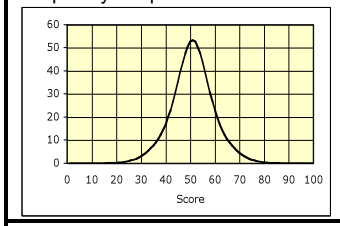
Frequency Graph A



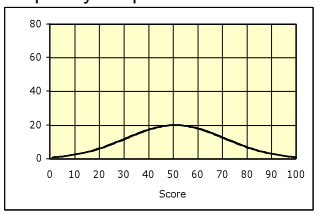
Frequency Graph B



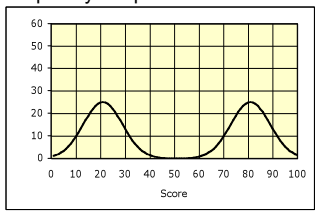
Frequency Graph C



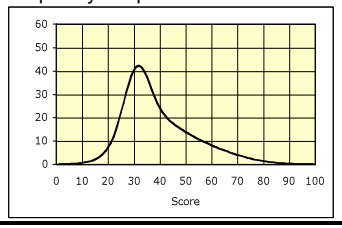
Frequency Graph D



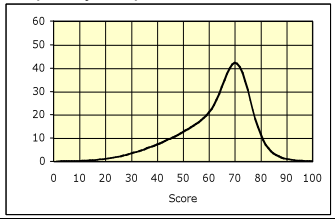
Frequency Graph E



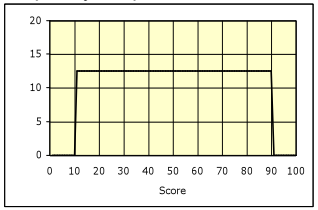
Frequency Graph F



Frequency Graph G



Frequency Graph H



Card Set: Interpretations

1. This was the sort of test where you could either do everything or you couldn't get started.	2. This test did not sort out the stronger students from the weaker ones. They all got similar scores.
3. Two groups of students took the test. One group had studied the work for two years. The other group had only just begun.	4. This test resulted in a huge range of scores. Everyone could do something but nobody could do it all.
5. In this test, the median score was greater than the mode score.	6. In this test, the median score was smaller than the mode score.
7. In this test, the median and the mode scores were the same. There was a very big range of scores.	8. This test was much too difficult for most people.

Mathematics Assessment Project

CLASSROOM CHALLENGES

This lesson was designed and developed by the
Shell Center Team
at the
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It was refined on the basis of reports from teams of observers led by
David Foster, Mary Bouck, and Diane Schaefer
based on their observation of trials in US classrooms
along with comments from teachers and other users.

This project was conceived and directed for
MARS: Mathematics Assessment Resource Service
by
Alan Schoenfeld, Hugh Burkhardt, Daniel Pead, and Malcolm Swan
and based at the University of California, Berkeley

We are grateful to the many teachers, in the UK and the US, who trialed earlier versions
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Unit 3 – Descriptive Statistics

Domain: Statistics and Probability (S)

Essential Question: How do data interpretations help develop informed decisions and predictions?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Summarize, interpret and represent data using various models and apply statistical reasoning to draw conclusions.** **S-ID (***)** Summarize, represent, and interpret data on two categorical and quantitative variables	**How…** **S.ID.5:** Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.	**Students will:** **Categorical Data Activity** S.ID5 Categorical Data.docx	**S.I.D.5:** See Addendum

Vocabulary: context of data, differences (center, shape, spread), exponential, interquartile range, measures of center, measures of spread, mean, median, models outliers, slope, standard deviation

Name: _____ Date: _____

Comparing Categorical Data

New England Patriots: From the National Football League(NFL) website, in the New England Patriots roster we obtained information on the weights and years of experience for the players on that team, as of November 7, 2005. The following table provides those data:

	Rookie	1-5 Years	6-10 Years	10+ Years	**Total**
Under 200 lbs.	2	7	3	2	14
200-300 lbs.	7	15	14	4	40
Over 300 lbs.	4	9	0	0	13
Total	13	31	17	6	67

1) How many players are on the New England Patriots roster as of November 7, 2005? _____
2) How many players are rookies? _____
3) How many players weigh between 200 and 300 pounds? _____
4) How many players are rookies that weigh between 200 and 300 pounds? _____
5) How many players have less than 6 years experience? _____
6) How many players weigh more than 200 pounds? _____
7) How many players have less than 6 years experience and weigh more than 200 pounds? _____

8) Make a general statement about the relationship between weight and years of experience?

_____

_____

_____

_____

_____

_____

_____

Unit 3 – Descriptive Statistics

Domain: Statistics and Probability (S)

Essential Question: How do data interpretations help develop informed decisions and predictions?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Summarize, interpret and represent data using various models and apply statistical reasoning to draw conclusions.** **S-ID (***)** Interpret linear models.	**How…** **S.ID.6:** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.* b. Informally assess the fit of a function by plotting and analyzing residuals. c. Fit a linear function for a scatter plot that suggests a linear association. **S.ID.7:** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. **S.ID.8:** Compute (using technology) and interpret the correlation coefficient of a linear fit.	**Students will:** **Correlation Activity** S.ID.8 Correlation using technology.doc **Statistics Performance Task** S.ID. 6-7-8 College Tuition PT.pdf	**S.ID.6: See Addendum** **S.ID.7: See Addendum** **S.ID.8: See Addendum**

Vocabulary: context of data, differences (center, shape, spread), exponential, interquartile range, measures of center, measures of spread, mean, median, models outliers, slope, standard deviation

Name: _____ Date: _____

Comparing Graduation Rates and SAT Scores

The data below compares 4 year graduation rates of 20 schools and their mean SAT Scores from 2010. Let's use this data to determine if there is a correlation between the two scores.

Before you do any work, look at the data and make a prediction.

Do you think there is any correlation? What do you think the strength of this correlation is?

_____

_____

School	2010 Graduation Rate	2010 Mean SAT Score
Manchester	75.8	1431
East Hartford	74.9	1257
Rockville	79.3	1481
New Britain	55.9	1372
Glastonbury	95.6	1691
Middletown	80.8	1395
Conard	93.5	1562
Rocky Hill	91.8	1510
Newington	86.4	1548
Windsor	80.1	1400
South Windsor	93.3	1588
East Windsor	88.9	1470
Bolton	95.5	1642
Wethersfield	86.2	1544
Southington	88.3	1557
Cromwell	95.1	1535
Bulkley	75.4	1130
Tolland	94.8	1605
Weaver	54.1	1131
Hall	93.5	1658

Now use your graphing calculator to graph the data and determine the correlation coefficient.

Describe the relationship between graduation rates and SAT Scores. What is a reasonable statement that is proven by this correlation? What is the "so what" statement – where can this be applied in the real world.

_____

_____

_____

MAT.HS.PT.4.TUITN.A.298

Sample Item ID:	MAT.HS.PT.4.TUITN.A.298
Title:	College Tuition
Grade:	HS
Primary Claim:	**Claim 4: Modeling and Data Analysis** Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.
Secondary Claim(s):	Claim 1: Concepts and Procedures Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency. Claim 2: Problem Solving Students can solve a range of well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies. Claim 3: Communicating Reasoning Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.
Primary Content Domain:	**Statistics and Probability**
Secondary Content Domain(s):	Functions, Algebra, Number and Quantity

Assessment Target(s):	4 A: Apply mathematics to solve problems arising in everyday life, society, and the workplace. 4 E: Analyze the adequacy of and make improvements to an existing model or develop a mathematical model of a real phenomenon. 4 G: Identify, analyze, and synthesize relevant external resources to pose or solve problems. 4 B: Construct, autonomously, chains of reasoning to justify mathematical models used, interpretations made, and solutions proposed for a complex problem. 4 F: Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flowcharts, or formulas). 4 D: Interpret results in the context of a situation. 1 G: Create equations that describe numbers or relationships. 1 P: Summarize, represent, and interpret data on a single- count or measurement variable. 2 B: Select and use appropriate tools strategically. 3 F: Base arguments on concrete referents such as objects, drawings, diagrams, and actions.
Standard(s):	S–ID.6, S–ID.7, S–ID.3, S–ID.2, S-ID.8, F–LE.1, F–LE.2, F–LE.5, F–IF.2, A–CED.2, N–Q.3, 8–SP.1, 8.SP.2, 7.RP.3, 7.EE.3
Mathematical Practice(s):	1, 2, 4, 5, 6, 7
DOK:	4
Item Type:	PT
Score Points:	14
Difficulty:	M
How this task addresses the "sufficient evidence" for this claim:	The student uses concepts of statistics, functions, and algebraic thinking to accomplish tasks associated with predicting the future costs of college tuition. The work is supported by calculations and explanations of reasoning.

Target-specific attributes (e.g., accessibility issues):	Accommodations may be necessary for students who have vision challenges, fine-motor-skills challenges, and language-processing challenges.
Stimulus/Source:	For articles used in prework: Article 1 http://www.usatoday.com/news/education/story/2011-10-25/public-college-costs-increase/50919598/1 Article 2 http://www.insidehighered.com/news/2010/10/28/tuition#ixzz1mbLIzz uN Article 3 http://www.changinghighereducation.com/2012/01/so-let-me- put-colleges-and-universities-on-notice-if-you-cant-stop-tuition-from-going-up-the-funding-you-get-from-taxpay.html To be used in conjunction with prework research: http://chronicle.com/article/Interactive-Tool-Tuition- Over/125043/ A simulated search will be developed in a similar fashion to the search tool provided on this Web site. The search tool will contain a subset of the data on this site. That subset of data will be from the collection of schools/institutions each student chooses in the days leading up to this activity. For data on average college tuition and fees: http://nces.ed.gov/programs/digest/d10/tables/dt10_345.asp
Notes:	Multi-part task
Task Overview:	Students will research data on college tuition over time. They will analyze their data in groups and individually to develop a model that best fits their collected data. Their models will then be used to predict future costs of college tuition. Parts C, D, and E will be the only scored portions of this task.
Teacher preparation / Resource requirements:	Teacher preparation: Up to 3 – 5 days prior to the administration of this task, students will be assigned a prework task that will be used to gather data in Part A of the task and to compare data in Part C

	of the task. The prework should be done individually, outside of school, and given to the teacher at least one day before the start of the task. The teacher will compile data for a simulated search based on the prework information. The simulated search will be performed individually during Part A of this task. In the latter part of Session 1, Part B will incorporate group work to analyze data and will require the teacher to coordinate partner/group work for this part of the task. The remaining parts of this task will be completed independently. Session 2 will involve modeling and interpreting the data analyzed during the group work. Resource requirements: Spreadsheet software and computers must be available to all students, as well as research tools to help students compile simulated data. Calculators should be available to students, either online or physically. Copies of a specific news article will be handed out and read as part of the prework activity.
Teacher Responsibilities During Administration:	Monitor individual student work and monitor group work. Provide resources as necessary.
Time Requirements:	Two sessions totaling no more than 120 minutes. Parts A and B will be completed during Session 1. Part A should be performed individually and Part B should be performed in small groups. Parts C, D, and E will be completed during Session 2. All tasks during Session 2 will be performed individually.

Prework:

In preparation for this task, teachers must assign students the following task as an individual activity at least 3 days prior to the administration of the performance task.
Teachers must hand out copies of these three articles to each student for this prework portion:

1. Marklein, Mary Beth (2011), "Tuition and fees rise more than 8% at U.S. public colleges," from *USA Today*, October 26.

Online articles:
2. Jaschik, Scott (2010), "Tuition Hikes of the Downturn," from *Inside Higher Ed*, October 28.

3. (2012), "The State of the Union on college costs," from *Changing Higher Education*, January 30.

[Note: A copy of each article is at the end of this task.]

Teacher says: Most students have plans to attend college after graduating from high school. There are many costs to consider when planning for college. The major costs are tuition and school-related fees, which are typically combined into one dollar amount. The cost of a college education is expected to increase from year to year. As a result, the yearly cost for a college education during a student's first year may be significantly different four years later when the student is ready to graduate.

During the two sessions of the upcoming performance task, you will be predicting the total costs for tuition and school-related fees, as a combined cost (which will be referred to as "tuition" throughout the task), for a college of your choice. Your assignment will include the following:

- Choose a college or university that you will use to predict the future cost of tuition. This can either be a local 2-year or 4-year institution or one that you would like to attend in the future. You must provide the name of the school and the type of institution (i.e., 2-year or 4-year, public or private, college or university).

- Find out what the current year's tuition, including school-related fees, are for the school you chose. This information can often be found by calling the school's admissions office, obtaining a current school catalog, or doing an Internet search. Be sure to get the cost for in- state students if the school is located in this state. Get the cost for out-of-state students if the school is not located in our state.

- Read the news articles "Tuition and fees rise more than 8% at U.S. public colleges," "Tuition Hikes of the Downturn," and "The State of the Union on college costs."
  [Note: Teacher must distribute copies of each of these articles.]

- Use the information you obtained about the current year's tuition at the school you chose and the information you read in the "Tuition and fees rise more than 8% at U.S. public colleges" news article to predict the cost of college tuition at your choice of schools the year you are first eligible to attend college. You should also predict the total tuition amount for the entire college education. This will be 2 years or 4 years based on the type of school you choose. Have your calculations and an explanation of how you determined your total predicted amount ready when the performance task officially begins.

[Assumptions: The method of handling the research part of the prework is based on this item writer not fully knowing what tools will be available to teachers and students as they perform this task. It is based on the assumption that students will not be allowed to do online searches in the classroom and that some sort of simulated search will need to be developed. The description below is only a suggested possibility. There may very well be an easier way to handle this research portion, such as reproducing the site or using the site itself, if possible.]

The prework from the first bullet (choosing a school) will be provided to teachers 4-5 school days before the start of the task. This is to give time for teachers to prepare a full list of schools that will be combined into one collective simulated search that all students will use
at the start of the performance task.

To do this, teachers will compile the complete list of schools provided by the students. They will enter each school into the search tool provided below.

http://chronicle.com/article/Interactive-Tool-Tuition-Over/125043/

Screen shots will be made of each school's tuition data. The screen shots will then be combined to form the simulated search tool to be used by the students when they begin Session 1 of the task.

[Notes: Ideally, the simulated search should be laid out in a similar fashion to the search tool provided on this Web site listed below. The simulated search will be a subset of all schools found on this site.

http://chronicle.com/article/Interactive-Tool-Tuition-Over/125043/

A sample screen shot is below. (It can be enlarged.) This is the school whose data are used in the sample response.

A simulated search should be created to have only the data from the collective list of schools provided by the students. Students will locate the school they specifically chose, and ideally have these two tables displayed: "1999-2010 In-state tuition & fees" and "1999-2010 Out-
of-state tuition & fees."

Additional information:
1. If the simulated search can be computer based, scrolling over the bars from the bar graph will list both in- and out-of-state tuition and fees for the particular school.

2. If the simulation will not be available via computer, the output should be adjusted from what is shown in the screen shot below. (The screen shot below shows "2010 in-state tuition
& fees" displayed on top, followed by the "in-state tuition and fees" table.) The output

should be adjusted to show the "1999-2010 In-state tuition and fees" on top and the "1999-2010 Out-of-state tuition and fees" underneath.

3. The data on this Web site is based on tuition and fees only. No additional expenses are reflected unless specifically noted.

4. This is the data students will use in *Part A* of the task.]

Session 1

[Note: Session 1 of the performance task consists of two *Parts*: *A* and *B*. *Part A* should be performed individually. *Part B* should be performed in pairs or small groups. The teacher should allow for the majority of Session 1 to be devoted to group work.]

College Tuition

Your Assignment:

Based on your research during the last few days, you may have realized that the cost of a college education in the United States can be expensive. During this performance task, you will use a spreadsheet and your knowledge of functions and statistics to predict the future cost of college tuition.

Steps you will be following:

To accomplish this, you will use a spreadsheet to help perform the following:

1. Gather data on the past year's tuition amounts.
2. Analyze the data and choose a model type that will best predict the future tuition total.
3. Develop a model equation based on the model type chosen.
4. Predict the total tuition amount for a 2-year or a 4-year college education in the near future.
5. Compare the predicted total tuition amount using the model equation with the total predicted tuition amount you calculated prior to the start of this task.
6. Compare tuition amounts at the college you chose with the average tuition amounts of all public 4-year colleges in the United States.
7. Predict the total tuition amount at a 2-year or a 4-year college education in the distant future.

Part A

Past Year's Tuition

During the past few days, each of you chose a specific college or university to research. In order to predict the total tuition amount at that college or university, you must first research past year's tuition amounts for that school.

You will use a computer to search for these data specific to the school you chose. Your search will provide you with the combined cost for tuition and school-related fees at your school over the past several years. Gather these data and enter them into a spreadsheet. The data must include the tuition and school-related fees, as one total dollar amount, for the past 10 years.

[Note: With the data provided on the simulated searches in this example, that will be for the years 2001–2010.]

Part B

Choosing a Model

After you have collected your data and entered it into the spreadsheet, get into pairs or groups of 3 or 4. In your group, you will analyze each team member's data by determining the following:

- what the data look like graphically
- what outliers, if any, exist
- what model type, either linear or nonlinear, best fits the data

As a group, decide which model type will be used to determine the function (model equation) that will predict each member's future tuition amounts at his/her chosen school. The model types may or may not be the same for all group members.

Model Types

Member 1: _____

Member 2: _____

Member 3: _____

Member 4: _____

Look for similarities and differences in each group member's data. Discuss some reasons why the data cause the model types for each group member to be the same or different.

[Note: Allow 5-10 minutes at the end of Session 1 for a whole class discussion about what was discovered during the group work. Students should discuss the reasons they came up with for why the model types in their individual groups, and the class as a whole, may or may not be the same.]

End of Session 1

Session 2

[Assumptions: All work involving the use of spreadsheet is based on the assumption that students know how to enter data, how to use the chart wizard (or other graphing utility) to create graphs, and how to enter formulas—particularly for linear and exponential regressions, mean, and standard deviation.]

[Note: Session 2 of the performance task consists of three parts: *C*, *D*, and *E*. All parts should be performed individually. *Part C* will make use of the prework involving student's predicted tuition amounts based on the "Tuition and fees rise more than 8% at U.S. public colleges" news article they read and the current year's tuition amount for their chosen school. *Part E* will require students to analyze their work based on information they read in the news articles "Tuition Hikes of the Downturn" and "The State of the Union on college costs."]

Part C

Predicting Future Tuition

[Note: Have students take out the predicted tuition amounts they calculated as part of their prework assignment. Ask the students to summarize what they learned from the "Tuition and fees rise more than 8% at U.S. public colleges" news article they read as part of the prework and how the article guided them in calculating their predicted tuition amounts.]

When you did your previous group work for this performance task, you each determined the model type that fit your data best. Now, use your spreadsheet to determine the model equation that will be used to predict the tuition amount, as a single dollar amount, at the school you chose.

[Note: Using actual 4-digit years (2001, 2002, 2003, etc.) as opposed to whole-number years (1, 2, 3, etc.) will result in very different model equations. The equation using the actual 4-digit years will not give tuition amounts appropriate to this problem. Teachers should instruct students to either use whole-number years or try both types of year inputs and make their own judgments as to the appropriate model equation to use.]

1. Write your model equation here. Show or explain how you found your answer.

2. Use your spreadsheet and your model equation to predict the total 2-year or 4-year tuition amount for the school you chose. Your prediction should begin with the school year that you are first eligible to attend college. Write the tuition amounts below, as well as the total for all years. Be sure to include each year.

Next, compare these total predicted tuition amounts:

- the prediction based on your model equation and spreadsheet data from above
- the prediction you made after reading the “Tuition and fees rise more than 8% at U.S. public colleges” news article prior to starting this task, using the current year’s tuition amount from your particular school

3. Are these predicted amounts similar or different? Explain why these amounts are similar or different. What does this suggest about the rate of increase for both predicted calculations?

Part D

Comparing Individual Tuition and Average Tuition

The following table shows actual data for the average tuition costs for all 4-year public universities in the United States.

Annual Tuition Average of Public 4-Year Colleges

School Year	Yearly Amount ($)
2000-2001	3501
2001-2002	3735
2002-2003	4046
2003-2004	4587
2004-2005	5027
2005-2006	5351
2006-2007	5666
2007-2008	5943
2008-2009	6312
2009-2010	6695

Use your spreadsheet to compare the data you found on your college's tuition amounts over a 10-year period with the data for the average tuition amounts at 4-year public colleges.

Suppose the data for the average tuition amounts for all 4-year public colleges were used to create a new model equation. Also, suppose this model equation was used to predict the future college tuition amounts.

4. Which model equation would have a higher correlation factor:

- the model equation created from the average tuition amounts; or
- the model equation created from the tuition amounts at your chosen school?

Explain why that model equation has a higher correlation factor. What does this suggest about how reliable each model is for predicting future college tuition amounts? Explain your reasoning. You may use a combination of diagrams, mathematical expressions/equations, and words.

Part E

Predicting Tuition for the Next Generation

In this last part of the task, you will predict the total tuition amount someone in a future generation will be expected to pay.

Use your spreadsheet and the model equation you determined for your college to predict the total tuition amount for a family that has a child born 10 years from now. You should assume that this child will:

- attend the college you chose,
- begin college at age 18, and
- attend for the full length of time (either 2 or 4 years).

5. Show how you determined the predicted total college tuition amount for this person. You may use a combination of diagrams, mathematical expressions/equations, and words.

Consider the two articles, “Tuition Hikes of the Downturn” and “The State of the Union on college costs,” you read prior to starting this task.

6. Use these articles to help justify why predicting college tuition costs too far into the future, beyond a few years, might not be reliable. Cite information from each article that supports your reasoning.

[Note: Students should have both articles available during this portion of the task.]

End of Session 2

Sample Top-Score Response:

Prework (Not Scored) Should include:
Name of school, type of school, and current full-year tuition amount
E.g.,
UMASS Boston, 4-year University, $11,407 for 2011-2012 in-state tuition and fees
Source: http://www.umb.edu/bursar/tuition_and_fees/

The predicted cost for college tuition should apply a yearly increase close to 8.3% for each year's tuition amount.
E.g.,
Start college for school year 2013-2014
First year's predicted tuition amount: $11,407 × 1.083 × 1.083 = $13,379

The sum of 2 or 4 consecutive years' projected tuition amounts should be made, approximately 1-3 years from the current year.
E.g.,
First year predicted amount: $13,379
Second year: $13,379 × 1.083 = $14,489
Third year: $14,489 × 1.083 = $15,692
Fourth year: $15,692 × 1.083 = $16,994
Total = $13,379 + $14,489 + $15,692 + $16,994 = **$60,554**

Part A (Not Scored)

The spreadsheet should show data in two columns. The first column should include the most recent 10 years shown on the tuition data site. The second column should include the corresponding yearly tuition amounts for the college chosen by the student. For example:

Year		UMASS Boston In-state Tuition
1	2001-2002	5170
2	2002-2003	6229
3	2003-2004	8142
4	2004-2005	9112
5	2005-2006	9028
6	2006-2007	9143
7	2007-2008	9060
8	2008-2009	9353
9	2009-2010	10,611
10	2010-2011	10,611

Part B (Not Scored)
Model types (linear or nonlinear – exponential or quadratic) should be listed for each group member.

Some notes or reflections should portray some similarities and some differences among the data in the group. The students should provide reasonable explanations why the data have these similarities and differences.

Part C (Scored)

1. The student determines the correct linear (or nonlinear) model that best fits their specific data. The model is written either as an equation or as a function. The student should show the formula and data they used in the spreadsheet in order to determine their model equation. The student is able to interpret the output from the formula correctly in order to write the

model equation.

E.g., using data from part A
Formula = "=LINEST(C2:C11,A2:A11)" Output = 519.133, 5790.667
Model equation: $f(x) = 519.133x + 5790.667$

2. The student correctly applies the model equation they determined for years in the near future. The student substitutes the appropriate 2 or 4 school years for x into the model equation and gets the predicted college tuition amount, y. The student adds the amounts for each year's output, y, to determine the total predicted college tuition amount. For example:

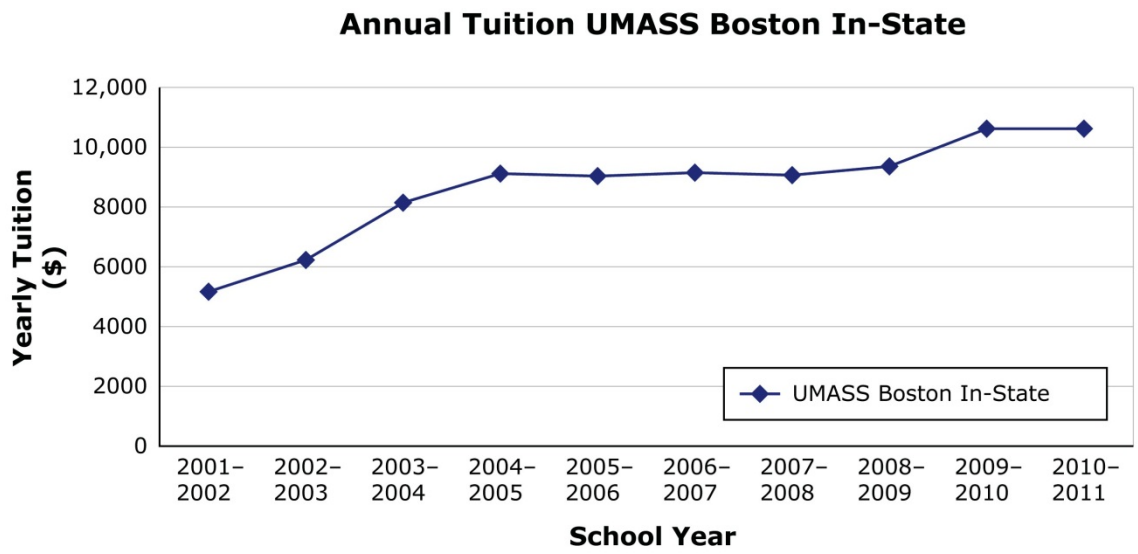
Year		UMASS Boston In-state Tuition
1	2001–2002	5170
2	2002–2003	6229
3	2003–2004	8142
4	2004–2005	9112
5	2005–2006	9028
6	2006–2007	9143
7	2007–2008	9060
8	2008–2009	9353
9	2009–2010	10,611
10	2010–2011	10,611
	slope	**y-intercept**
Linear model	519.133	5790.667
11	2011–2012	$11,501
12	2012–2013	$12,020
13	2013–2014	$12,539
14	2015–2016	$13,059
15	2016–2017	$13,578
16	2017–2018	$14,097
	Total	$53,273

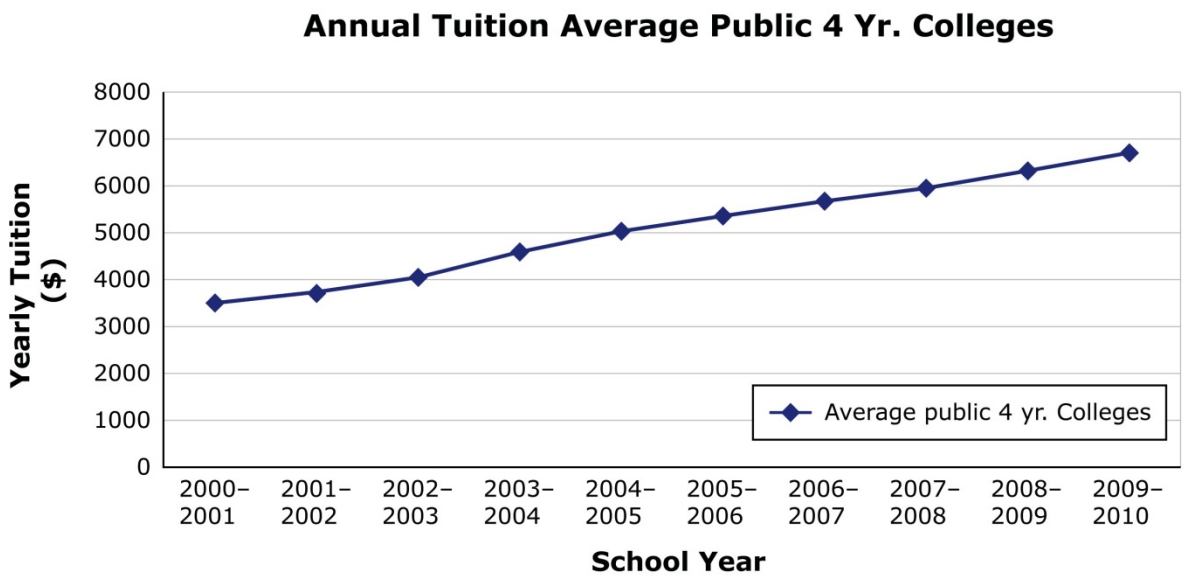
3. The student notices a similarity or difference between the two predicted total amounts and is able to reasonably explain why the similarity or difference exists. If the predicted total using the model equation is less than the predicted total based on the article, the student explains that the average yearly increase must be less than 8.3%. If the predicted total using the model equation is more than the predicted total based on the article, the student relates that the average yearly increase must be greater than 8.3%.

For example:
Total predicted tuition based on model equation: $53,272
Total predicted tuition based on current year's tuition and data from article: $60,554
These predicted amounts differ by a significant amount, $7282. This is most likely because the rate at which the model predicts tuition to increase each year is less than the 8.3%
average mentioned in the article.

Part D (Scored)

4. The student gives a reasonable explanation of why the average tuition model equation (most likely) has the higher correlation factor. The student relates the almost perfectly linear relationship of the average data to having a line of best fit that produces a model equation very close to the actual data points, creating a high correlation factor. The data for the chosen school, however (most likely), do not have as close a linear relationship, so the line that best fits that data will have a (slightly) lower correlation factor. For example:





The student offers a reasonable explanation as to how the reliability of each model and the correlation factor associated with each model are related. The student recognizes that the higher the correlation factor, the more reliable the model is expected to be.

For example:
The model equation for the average tuition data is $f(x) = 362.612x + 3091.933$. When I input the years 1-10 into this equation and correlate these outputs to the actual averages, I get a correlation factor of 0.99739. This is a very high correlation factor, which is expected since the data on the graph show an almost linear relationship with the given data. This high correlation factor indicates that the model for the average tuition data is very reliable.

When I input the years 1-10 into the model equation I found for my college and correlate these outputs to the actual averages, I get a correlation factor of 0.90441. This makes sense that the correlation factor is somewhat lower because the data for UMASS' tuition is not quite as linear as the data for the average tuition. While this is not as high a correlation factor as the average tuitions' model, it still is fairly high and indicates that the model equation to predict the tuition for my college is still fairly reliable.

Part E (Scored)

5. The student correctly applies the model equation from part C. The student recognizes that the starting value of *x* must be 28 (10 + 18) more than the value of *x* for the current year. The student substitutes the appropriate 2 or 4 school years for *x* into the model equation and gets the predicted college tuition amount, *y*. The student adds the amounts for each year's output, *y*, to determine the total predicted college tuition amount for someone in the next generation. For example:

28 years from now

40	2040–2041	$26,556
41	2041–2042	$27,075
42	2042–2043	$27,594
43	2043–2044	$28,113
	Total	$109,338

I determined that this person would be attending college 28 years from now, beginning in the year 2040. I used the 4-year period beginning in that year to determine the total tuition using the model equation for UMASS found in part C.

6. The student gives a reasonable explanation to justify why predicting college tuition costs too far into the future might not be reliable. The student cites details relating to how the varying percentage amount increases by decade shown in the Annual Average Tuition Increases (Inflation-Adjusted) by Sector table [in article 2] indicates that a single percent increase, as given in the model equation, cannot be relied on. The student relates the impact that the economy plays in driving tuition rates up [in articles 2 and 3], and that without a clear prediction of the future of the economy, it is hard to predict how college tuition rates will increase. The student discusses that differences exist in average tuition increases based on the type of school it is (public vs. private). The factors that influence these different percent increases are discussed along with the possibility that this trend may not always be the case [in article 2]. The student cites concerns outlined in President Obama's State of the Union address [in article 3] that government may need to take some control in the future by limiting the amount of government aid colleges receive if tuition increases continue climbing at their current rate. The student relates this to the possibility that tuition increases may start to decline, thus making their model unreliable several years into the future.

Scoring Notes:
Each question in the scored parts is evaluated individually. The total number of points is determined by adding the points assigned for each question.

Scoring Rubric:

Scoring Rubric for ***Part C***:

Part C, Question 1: Responses to this item will receive 0-2 points, based on the following:

2 points: The student shows a thorough understanding of how to use a spreadsheet to determine a regression function and interpret the result of the output. The student uses the correct regression formula in the spreadsheet, references the correct data values for the function, and interprets each output number from the formula to write the correct model equation (e.g., A linear regression formula outputs two numbers. The first number represents the slope of the function and the second number represents the *y*-intercept).

1 point: The student shows partial understanding of how to use a spreadsheet to determine a regression function and interpret the result of the output. The student uses the correct regression formula in the spreadsheet, references the correct data values for the function, but misinterprets the output numbers from the formula. **OR** The student uses the correct regression formula in the spreadsheet, but references the incorrect data values for the function. However, the student is able to interpret the output numbers from the formula correctly.

0 points: The student shows inconsistent understanding of how to use a spreadsheet to determine a regression function and interpret the result of the output. The student does not use the correct regression formula in the spreadsheet, and does not interpret the output numbers from the formula correctly to write a model equation.

Part C, Question 2: Responses to this item will receive 0-2 points, based on the following:

2 points: The student shows a thorough understanding of how to use a spreadsheet to apply a regression function to find a total predicted tuition amount. The student projects out the starting year and ending year correctly for when they plan to attend the college. The student uses those years in the model equation to find the predicted tuition amounts for those years and then sums the 2- or 4-year amounts for one total.

1 point: The student shows partial understanding of how to use a spreadsheet to apply a regression function to find a total predicted tuition amount. The student projects out the starting year and ending year correctly for when they plan to attend the college. The student uses those years in the model equation to find the predicted tuition amounts for those

years but does not find their total sum. **OR** The student projects out the incorrect starting year and ending year but applies the model equation correctly to find predicted tuition amounts for those years. The student sums the amounts for one total.

0 points: The student shows inconsistent understanding of how to use a spreadsheet to apply a regression function to find a total predicted tuition amount. The student may or may not project out the starting year and ending year correctly. However, the student applies
the model equation incorrectly and does not find or incorrectly finds the sum the 2- or 4- year amounts.

Part C, Question 3: Responses to this item will receive 0-2 points, based on the following:

2 points: The student shows a thorough understanding of how to compare predicted tuition amounts using more than one method for predicting. The student correctly identifies whether the two predicted tuition sums are similar or different. The student relates the rate of increase mentioned in the article to the rate of increase used in the model equation.

1 point: The student shows partial understanding of how to compare predicted tuition amounts using more than one method for predicting. The student correctly identifies whether the two predicted tuition sums are similar or different but does not relate the rate of increase mentioned in the article to the rate of increase used in the model equation.

0 points: The student shows inconsistent understanding of how to compare predicted tuition amounts using more than one method for predicting. The student incorrectly identifies whether the two predicted tuition sums are similar or different.

Scoring Rubric for ***Part D***:

Part D, Question 4: Responses to this item will receive 0-3 points, based on the following:

3 points: The student shows a thorough understanding of how to analyze data in terms of correlation and uses that knowledge to make judgments about the reliability of models. The student identifies which model has a higher correlation factor and provides an accurate explanation as to why it is higher. The student reasonably relates the correlation factor to the reliability of each model. The student provides a complete and accurate explanation for all aspects of this part using diagrams, expressions/equations, and/or words.

2 points: The student shows some understanding of how to analyze data in terms of correlation and uses that knowledge to make judgments about the reliability of models. The student identifies which model has a higher correlation factor but does not provide a complete explanation as to why it is higher. The student reasonably relates the correlation factor to the

reliability of each model. The student provides a complete and accurate explanation for most aspects of this part using diagrams, expressions/equations, and/or words.

1 point: The student shows partial understanding of how to analyze data in terms of correlation and uses that knowledge to make judgments about the reliability of models. The student identifies which model has a higher correlation factor and provides an accurate explanation as to why it is higher. The student incompletely or inaccurately relates the correlation factor to the reliability of each model. The student provides a complete and accurate explanation for some aspects of this part using diagrams, expressions/equations, and/or words.

0 points: The student shows inconsistent understanding of how to analyze data in terms of correlation and uses that knowledge to make judgments about the reliability of models. The student may or may not identify which model has a higher correlation factor but provides no explanation or an incorrect explanation as to why it is higher. The student incompletely or inaccurately relates the correlation factor to the reliability of each model. The student provides no complete or accurate explanation for any aspect of this part using diagrams, expressions/equations, and/or words.

Scoring Rubric for ***Part E***:

Part E, Question 5: Responses to this item will receive 0-2 points, based on the following:

2 points: The student shows a thorough understanding of how to apply the model equation to find a total predicted tuition amount in the distant future. The student projects out the starting year and ending year correctly. The student uses the appropriate years in the model equation to find the predicted sum of the tuition for a student attending college in the future.

1 point: The student shows partial understanding of how to apply the model equation to find a total predicted tuition amount in the distant future. The student projects out the starting year and ending year correctly but applies the model equation incorrectly.

0 points: The student shows inconsistent understanding of how to apply the model equation to find a total predicted tuition amount in the distant future. The student does not project out the starting year and ending year correctly. The student does not apply the model equation correctly to find the predicted sum of the tuition for a student attending college in the future.

Part E, Question 6: Responses to this item will receive 0-3 points, based on the following:

3 points: The student shows a thorough understanding of how to interpret information regarding the future of college tuition rate increases presented in news articles and justifies conclusions based on the analysis. The student gives a complete and reasonable explanation as to why predicting college tuition costs too far into the future might not be reliable. The student supports his or her reasoning with at least 3 statements coming from both related articles.

2 points: The student shows some understanding of how to interpret information regarding the future of college tuition rate increases presented in news articles and justifies
conclusions based on the analysis. The student gives a reasonable explanation as to why predicting college tuition costs too far into the future might not be reliable. The student supports his or her reasoning with at least 2 statements coming from both related articles.

1 point: The student shows partial understanding of how to interpret information regarding the future of college tuition rate increases presented in news articles and justifies conclusions based on the analysis. The student gives an incomplete or partially correct explanation as to why predicting college tuition costs too far into the future might not be reliable. The student supports his or her reasoning with at least 2 statements coming from both related articles. Or the student supports his or her reasoning with statements coming from only one article.

0 points: The student shows inconsistent understanding of how to interpret information regarding the future of college tuition rate increases presented in news articles and justifies conclusions based on the analysis. The student gives an incorrect explanation as to why predicting college tuition costs too far into the future might not be reliable. The student may or may not support his or her reasoning with statements from both related articles.

Article # 1

http://www.usatoday.com/news/education/story/2011-10-25/public-college-costs- increase/50919598/1

Tuition and fees rise more than 8% at U.S. public colleges

By Mary Beth Marklein, USA TODAY

Updated 10/26/2011 2:12 AM

Tuition and fees at America's public colleges rose more than 8% this year as a weakened economy and severe cuts in state funding took their toll, a report out today says.

By Jacquelyn Martin, AP

Gan Golan of Los Angeles, dressed as the "Master of Degrees," holds a ball and chain representing his college loan debt during an "Occupy D.C." protest Oct. 6.

Public four-year universities charged residents an average of $8,244, up 8.3% from last year, while public two-year schools charged an average of $2,963, up 8.7%, says the report by the non-profit College Board. About 80% of the nation's undergraduates attend public institutions.

That increase is more than double the inflation rate of 3.6% between July 2010 and July 2011. Family earnings dropped across all income levels. And state funding per student declined by 4% in 2010, the latest year available, and 23% over the past decade, the report says.

College costs

Average estimated costs* for full-time undergraduates in 2011-12, before grant aid**: Public two-year commuter

student: $15,286

Public four-year, in-state, lives on campus: $21,447

Public four-year, out-of-state, lives on campus: $33,973

Private non-profit four-year, lives on campus: $42,224

*Costs include tuition and fees, room and board, books and supplies, transportation and other expenses.

**About one-third of full-time undergraduates receive no grant aid.

Source: College Board

Molly Corbett Broad, president of the American Council on Education, called the findings "sadly familiar," and said the drop in state support was particularly troubling. "It has become all too common for state legislatures to dip into the pockets of students and families to balance state budgets," she says.

The tuition and fee hike is not the worst of the decade — that occurred in 2004, when sticker prices rose 11% beyond inflation from the previous year.

The report says there may be some good news: a rise in federal student aid — including tax credits and deductions — is blunting the impact for most families. "At a time when students and families are ill-equipped to manage additional expenses, student financial aid is more important than ever," report author Sandy Baum says.

Net price — the published price minus grants and tax breaks — at public four-year colleges averaged $2,490, the report found.

About two-thirds of undergraduates receive grant aid, which averaged $6,539 last year. Average federal loans averaged $4,907. Borrowing by students and parents increased about 2% from 2009-10 to 2010-11.

Borrowing from private sources declined for the third straight year. In Denver today, President Obama will announce a plan through which students can consolidate their debt and reduce their interest rates. The plan also will allow borrowers to cap their student loan payments at 10% of discretionary income.

Article # 2

http://www.insidehighered.com/news/2010/10/28/tuition#ixzz1mbLIzzuN

Inside Higher Ed

Tuition Hikes of the Downturn

October 28, 2010 - 3:00am

By Scott Jaschik

Tuition is up (no surprise) and this year the percentage increases for public and private four-year colleges and universities are higher than they were last year. Generally, the percentage increases at public institutions are larger than those at privates (which are more expensive to start with). Those trends are standard for tight economic times, when states cut budgets and try to make up for shortfalls with larger tuition increases, and when many private colleges worry that sticker shock will scare away families and so tend to moderate price increases.

Across the board, the increases exceed the inflation rate of about 1.2 percent for the last year, which, while low, was higher than the slightly negative rate of the year before.

Those are the key findings from this year's annual survey on college prices (and a companion survey on student aid) being released today by the College Board. In many respects, the data extend trends that were evident last year as well. Here are the overall figures for the 2010-11 academic year:

Tuition and Fees by Sector

Sector	2010–11 Tuition and Fees	One-Year Dollar Increase	One-Year % Increase	Previous Year's % Increase
Private, nonprofit four-year colleges	$27,293	$1,164	4.5%	4.4%
Public four-year colleges, in-state residents	$7,605	$555	7.9%	6.5%
Public four-year colleges, out-of-state residents	$19,595	$1,111	6.0%	6.2%
Community colleges	$2,713	$155	6.0%	7.3%
For-profit colleges	$13,935	$679	5.1%	6.5%

For room and board, public increases also outpaced the privates, and privates are also more expensive. The average public college rate is going up by 4.6 percent, to $8,535, and the average private rate is going up by 3.9 percent, to $9,700. Those figures are for four-year institutions only, as the pool of community colleges and for-profit colleges charging for room and board remains small.

As is the case every year, College Board officials stress that the data show that most colleges -- however much their prices frustrate students and families -- are not in the mid-$50,000 range that attracts so much attention. Total expenses for a private four-year institution are, on average, just under $37,000 a year. But because the most famous private institutions tend to be well above that average, many people assume tuition rates are even higher than they are. (At Harvard University, an undergraduate's total costs this year, typical for those at elite private research universities and liberal arts colleges, are estimated by the university to be between $53,950 and $56,750.)

Many of the data in the report focus on the impact of state budget shortfalls on public colleges. For instance, in comparing inflation-adjusted average tuition increases from the last three decades, the College Board finds that over that time, the rate of increase has dropped for private four-year institutions and gone up for public four-year institutions. Further, while the rate of increase at private institutions was greater than that of publics in the 1980s, it is now smaller.

Annual Average Tuition Increases (Inflation-Adjusted) by Sector

Sector	1980–1 to 1990–1	1990–1 to 2000–1	2000–1 to 2010–1
Private four-year	5.1%	2.6%	3.0%
Public four-year	4.2%	3.3%	5.6%
Community colleges	3.9%	3.2%	2.7%

The College Board's report on student aid notes that the past two years -- which have seen significant increases in tuition at many public colleges and universities and growing economic pressures on many families -- have seen a rapid expansion in aid packages.

From 2008-9 to 2009-10, grant aid per full-time equivalent undergraduate increased by about 22 percent (or $1,073) and federal loans increased by 9 percent (about $408). Particularly notable, the College Board report said, was the increase in the

maximum Pell Grant of 16 percent in constant dollars in 2009-10, the largest one-year increase in program history. The total Pell budget reached $28.2 billion, divided among 7.7 million students.

Sandy Baum, a policy analyst for the College Board and co-author of the reports being issued, said that the tuition figures "were not very surprising," given the state of the economy. "I don't think anybody thought public tuition would go up only 2 percent this year."

She urged educators and policy-makers to pay more attention to the long-term issues raised by this year's data. She noted, for example, that the impact of tuition increases on low-income students has been mitigated in part by the strong support for the growth in Pell Grants -- growth that probably will not be matched in the years ahead. "No matter what kind of Congress we get, the idea that Pell Grants will keep growing at this rate is unlikely," she said.

Baum said that in many ways she sees the tuition trends posing more of a threat ahead to public higher education than to private colleges. She said that some private institutions -- those that are being forced to give so much aid to attract students that they can't balance their books -- are in danger. But she said that the basic financial model for most privates, in which some students pay enough to subsidize others, is sound.

For public higher education, however, she said she feared that "the basic model may no longer be sustainable." While states are likely to restore some support for higher education as the economy improves, she said, it seems unlikely that enough support will be provided to maintain tuition at affordable levels. She said she anticipates public colleges having to consider more radical changes in how they provide education, ideally using means that cut costs. She noted that while technology has to date not cut costs in providing higher education, that may not be the case in the future.

If new models fail to provide more students with quality education, she said, "we could lose public higher education, and that would be a huge social failure."

Article # 3

http://www.changinghighereducation.com/2012/01/so-let-me-put-colleges-and- universities-on-notice-if-you-cant-stop-tuition-from-going-up-the-funding-you-get-from- taxpay.html

The State of the Union on college costs

So let me put colleges and universities on notice: If you can't stop tuition from going up, the funding you get from taxpayers will go down. Higher education can't be a luxury. It is an economic imperative that every family in America should be able to afford.

Barak Obama, State of Union 2012

Does this speech signal that the time has finally arrived when the government - which pays a good part of the bill - will step in to limit the rapid and seemingly never ending growth of tuition? In normal times, the answer would likely be "yes" given that politicians from both sides of the aisle have been introducing bills that would cap tuition in one way or another for almost a decade. Thus, we might expect to see a quick moving bipartisan effort.

These, of course, are not times when bipartisan efforts go very far, so Obama's statements will probably push Republicans into fierce opposition to the idea. The response of Representative Virginia Foxx, the North Carolina Republican who is chairwoman of the House Higher Education subcommittee, is probably a pretty good representation of what we will now hear from the Republican side:

The president is saying that people can't afford to go to college anymore, and that just simply is not true. Tuition is too high at most schools, but it isn't the job of the federal government to punish those schools. It's very arbitrary, and the president sounds like a dictator.

So this probably won't be the tipping point for this issue. But before the higher education community breathes a sigh of relief, its members should note that a President of the United States views the issue as important enough, with enough broad voter appeal, to put it into a State of the Union address, and he is continuing to speak about it at public events. It would be surprising if we didn't hear a lot more over the next two years about the relationship between tuition increases and taxpayer support. And, despite the negative initial overall response of Representative Foxx, it should be noted that she agreed that tuition is too high at most schools - hardly the position that makes a strong ally in this matter.

The reported responses from the academic community to Obama's speech, sadly, fall pretty much as one would anticipate - The current system is close to perfect, and any constraints (fiscal or administrative) will lead to declines in educational outcomes. This is indeed the likely outcome if educational institutions try to handle the constraints without changing their basic approach.

However this speech makes it increasingly clear that the reality must be faced - it is simply not possible for higher education costs to increase at 3% above inflation forever, and the end of the period of rapid increases is getting closer. Educational leaders that refuse to come to grips with this reality are ensuring that the negative outcomes they describe will indeed occur.

It is highly likely that the changes that will be required will involve things that most people in traditional higher education find undesirable because they break with comfortable traditional standards of "how things should be done". But the economic

realities of the United States (and most of the rest of the world) are such that "undesirable" actions have been, are, and will be required of almost every segment in order to transition to new, viable configurations. Does higher education have the leadership to rise to the challenge of this kind of transformative change, or will it simply sink into mediocrity while defending the *status quo*?

Unit 3 – Descriptive Statistics

Domain: Statistics and Probability (S)

Essential Question: How do data interpretations help develop informed decisions and predictions?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Summarize, interpret and represent data using various models and apply statistical reasoning to draw conclusions.** **S-ID (***)** Interpret linear models.	**How…** **S.ID.9:** Distinguish between correlation and causation.	**Students will:** **Correlation-Causation Investigation** S.ID.9 Corr vs Causation.pdf	**S.ID.9:** Some data leads observers to believe that there is a cause and effect relationship when a strong relationship is observed. Students should be careful not to assume that correlation implies causation. The determination that one thing causes another requires a controlled randomized experiment. Example: Diane did a study for a health class about the effects of a student's end-of-year math test scores on height. Based on a graph of her data, she found that there was a direct relationship between students' math scores and height. She concluded that "doing well on your end-of-course math tests makes you tall." Is this conclusion justified? Explain any flaws in Diane's reasoning.

Vocabulary: context of data, differences (center, shape, spread), exponential, interquartile range, measures of center, measures of spread, mean, median, models outliers, slope, standard deviation

TargetStrategies®
Aligned Mathematics Strategies Arkansas Student Learning Expectations
Algebra I

ASLE Expectation: AR04MA1050512
DIP.5.AI.12
Compare various methods of reporting data to make inferences or predictions: Recognize when arguments based on data confuse correlation with causation (fully develop weak and strong correlation)

Focus Objective: The student will recognize when arguments based on data confuse correlation with causation.

Level: Analysis

Strand: Data Interpretation and Probability

Prerequisite Skills:

- graph in all four quadrants of the Cartesian plane (MA.19)
- create appropriate graphical representations of data, including histograms, stem-and-leaf plots, box-and-whisker plots, and scatter plots
- make conjectures about possible relationships between two characteristics of a sample on the basis of scatter plots of the data

(Coding refers to an applicable TargetFundamentals™ lesson.).

Related Expectations:

AR04MA1050501 DIP.5.AI.1	Compare various methods of reporting data to make inferences or predictions: Construct and use scatter plots and line of best fit to make inferences in real life situations
AR04MA1050505 DIP.5.AI.5	Compare various methods of reporting data to make inferences or predictions: Use two or more graphs (i.e., box-and-whisker, histograms, scatter plots) to compare data sets

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AR04MA1050511 Compare various methods of reporting data to make inferences or
DIP.5.AI.11 predictions: Explain how sampling methods, bias, and phrasing of questions in data collection impact the conclusions

INSTRUCTIONAL PREPARATION

Materials:
- copies of a coordinate grid (one per student)

Duplicate the following (one per student unless otherwise indicated):
- Vocabulary reference sheet
- Correlation vs. Causation reference sheet
- Correlate the Data worksheet
- Data Correlation worksheet

Prepare a transparency of the following:
- Vocabulary reference sheet
- Correlation vs. Causation reference sheet
- Correlate the Data worksheet

Display the Focus Questions:
- What is the difference between correlation and causation?
- What are the three types of correlation?
- How do you recognize when an argument implies causation?

INSTRUCTION

1. To begin the lesson, give each student a coordinate grid and ask the students to plot the following points: (3, 1), (4, 2), (1, 2), (5, 3), (4, 6). Give the students enough time to plot the points, and then ask the following questions:

    - What type of pattern do you notice in the relationship between the x- and y-values of the graph? (Answers should include

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that there is not an apparent pattern.)

- How confident would you be in predicting a y-value given an x-value based on this graph? (Answers may vary but should include that it would not be easy to accurately predict the value of y given x.)

Explain to the students that, often, two sets of data taken from the same individuals are plotted on a coordinate plane to create a scatter plot, much like the graph they just plotted. Tell them that the pattern that the data points form and the degree to which the data reflects a particular pattern (e.g., a line) can be useful in making predictions and in drawing conclusions about the relationship between variables. Tell the class that in this lesson they will be looking at a specific relationship between variables, called correlation.

2. Distribute copies of the Vocabulary reference sheet and display the transparency. Review the terms correlation and regression line. Correlation relates two pieces of data and explains the strength of their linear relationship as being positive, negative, or none at all. You might
want to briefly distinguish between correlation, which establishes a relationship between data (e.g., as x increases, y increases), and causation (e.g., x increasing causes y to increase).

   Explain to the students that they will discuss in more detail the relationship between correlation and causation later in the lesson. You might also want to discuss with the class that the strength of the correlation of two variables can be described using terms such as “strong” and “weak.” Point on the reference sheet to the examples of positive and negative correlation, explaining that since these points demonstrate a pattern that is very close to linear, they can be described as having a strong correlation, while data points that only very roughly approximate a linear pattern can be described as having a weak correlation. Explain that a regression line is a line that estimates the relationship between two variables or pieces of data. Ask the following question:

   - What do you notice about the line on the scatter plot? (Answers may vary but should include the line runs down the middle of the points on the graph.)

   Tell the students that a regression line could also be referred to as a “trend line” or “line of best fit.” This line typically runs down the middle of the data set, with about half the points not on the line lying either above or below it. Explain to the students that this line could be used to predict the value of one variable if the other variable is known. Answer any questions students may have.

3. Distribute copies of the Correlation vs. Causation reference sheet and display the transparency. Read aloud the boxed information explaining correlation, while the class reads along silently. Explain to the students that a scatter plot consists of a coordinate graph with many data points plotted. The term scatter plot comes from the “scattered” look of the “plot” on the grid.

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Tell the students that most scatter plots are created in quadrant I of a coordinate grid, because both variables there are positive. Explain that real-world data is often measured using positive units. Continue reading about the three types of correlations—positive, negative, and none—on the reference sheet.

Direct the students' attention to the "Positive" correlation graph. Show the students how the data points flow in a pattern from lower left to the upper right portion of the grid. Explain that this is considered a positive correlation because as the x-values increase, so do the
y-values on the graph. Identify point (2, 2) and point (6, 5) on the graph. Show the students how the x-value increased from 2 to 6 and the y-value increased from 2 to 5. Using this information, we can see there is a positive relationship between the data in which, generally, when one increases, so does the other. Ask the following question:

- Using the data plots on the graph, how can we identify it is a positive correlation? (The data is moving in an uphill manner, or in a positive direction, which can associate with a positive correlation.)

Explain that if a line of best fit was drawn through the data points, it would have a positive slope, which indicates a positive correlation. Answer any questions students may have. Direct the students' attention to the "Negative" correlation graph. Show the students how the data plots flow in a pattern from upper left to the lower right portion of the grid. Explain that this is considered a negative correlation because as the x-values increase, the y-values decrease. You might want to point out that not all points decrease in their y-value as x-increases (for instance, the point (2, 9) is a vertical increase from the point (1, 7)). Emphasize to the class that they are analyzing an overall trend in the graph, a decrease from left to right, which could be confirmed by drawing a line of best fit through the points. Answer any questions students may have about negative correlations. Direct the students' attention to the "None" correlation graph. Show the students that the data on this grid does not flow in any pattern. These plots are scattered all over the graph, and in no way do they form any kind of up-hill or down-hill pattern. Explain to the students that when the data forms no pattern, it is considered to be no correlation. Several different lines of best fit could be applied to the data, and each would represent the data points poorly. Answer any questions students may have. Then direct the students' attention to the "Linear Regression Line" graph. Ask the students the following question:

- What type of correlation does this graph represent? (Positive correlation, because the points generally rise to the right)

Explain to the students that the data forms a positive correlation, but a regression line is a line that estimates the relationship between the two variables. Direct the students to the line on the graph, and point out how the line is drawn down the middle of the data points. Explain that this line represents the trend of the data and can be used to predict outcomes. In this case, the line is going up-hill in a positive direction, directly relating to the positive correlation. Tell the students that the line will always

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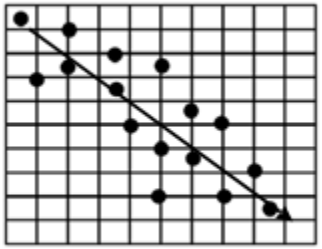
resemble the data and go in the same direction as the data points. Answer any questions students may have. Ask the students to draw a linear regression line on the negative and no correlation graphs on the reference sheet. Give the students enough time to draw the line, and then go over the following answers:

Negative



None: line cannot be drawn because there is no pattern to follow

Explain to the students that it would not be useful to draw a regression line on the graph of the no correlation data, because the data does not form a pattern or flow in the same direction. The data is scattered, and there is no apparent relationship between the variables. Answer any questions students may have.

4. Direct the students' attention to the box that defines causation. Explain to the students that causation means that a change in one variable produces a change in the other. However, in the case of scatter plots, you cannot assume that a change in one variable caused a change in the other. Causation is a way to say one result was a direct result of the other, e.g., a plant grew taller because it received more sunlight. Tell the students that a correlation between two variables does not indicate a causal relationship between them. We are not able to make a conclusion about causation strictly from the plot because there may be other factors outside of the study that cause results to vary, which would not be indicated in the graph. Answer any questions students may have. Ask the following questions:

   - A study was taken researching the weight of a person and the amount of time the person spends in the gym. The results indicated that as the amount of time in the gym increased, the weight decreased. What type of correlation is this? (Negative, because as the time increased, the weight decreased)
   - The researchers said spending more time in the gym causes a loss in weight. Based on the data, can this be concluded? Explain. (You cannot form a conclusion about the data. It is only a correlation because there may be other factors that cause weight loss, such as food intake, being sick, or genetics.)

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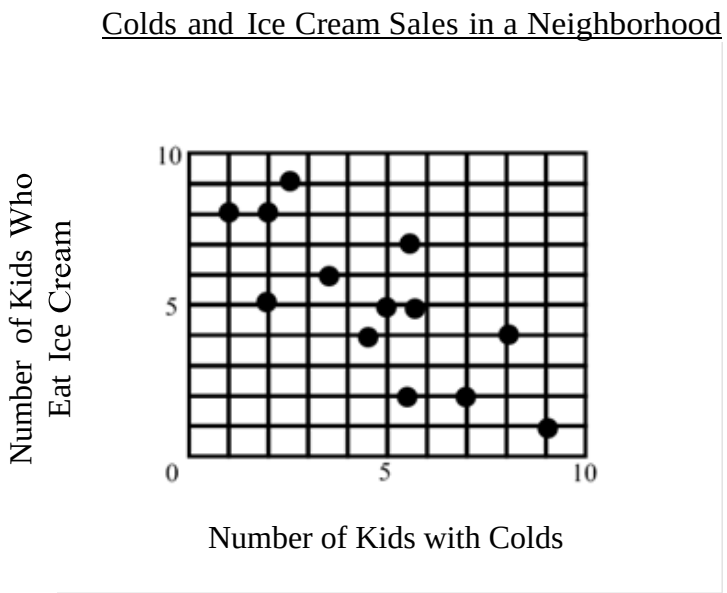
5. Display the transparency of the second page of the Correlation vs. Causation reference sheet. Direct the students' attention to the example. Explain to the students why it is unreasonable to conclude that higher temperatures cause more shuttle launches. There are many outside factors, including the three listed, that could impact the launch. In the case of this example, it can only be considered a coincidence that there more shuttle launches took place when the temperature was higher. Continue by reading aloud the information in the box at the bottom of page 2, which provides an argument supported by the graph, as well as arguments that cannot be supported by it. Answer any questions students may have.

Sketch the following example on the classroom board:



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Ask the students the following questions:

- What type of correlation does the graph represent? (Negative)

- True or False:
  - More kids with colds causes less kids who eat ice cream? (False. You cannot assume that a change in one variable produced the change in the other.)
  - There is a positive correlation between the data? (False. It is negative.)
  - There is a correlation but not a causation between the data? (True)
- What factors could affect this study, making it not reasonable to make conclusions? (Answers will vary but may include the time of year, o outside temperature, type of ice cream, etc.)

You might want to explain that in this instance, it is likely that a lurking variable, in this case temperature, might be producing an apparent relationship between the variables when one does not actually exist. When the temperature decreases during the fall and winter, the frequency with which kids get colds tends to increase, and ice cream consumption decreases because of falling temperatures. Emphasize that this example indicates that a strong correlation does not at all imply that there is a causal relationship between the variables. Answer any questions students may have.

6. Display the focus questions. Give the students time to think about them. Facilitate a discussion that includes the following points:

   - Correlation is a way to describe the linear relationship between two variables.
   - Scatter plots that display a positive correlation have data that flows in a positive, or up-hill, pattern.
   - Scatter plots that display a negative correlation have data that flows in a negative, or down-hill, pattern.
   - A scatter plot that displays no correlation has data that doesn't flow in any significant pattern.
   - Causation is when a change in one variable causes a change in another.
   - Scatter plots can be used to infer correlation, but not causation, between two variables.

7. Distribute copies of the Correlate the Data worksheet, and display the transparency. Read the directions aloud and answer any questions students have about the worksheet. Assign the worksheet to be completed individually. When enough time has been allowed to complete the assignment, review the answers with the class, using the Teacher's Answer Key.

8. To conclude the lesson, instruct the students to put away their worksheet and reference sheets. Have them answer the focus

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questions in their math journal or on a separate sheet of paper. They should include examples of positive, negative, and no correlation, as well as an explanation of causation.

ASSESSMENT

Distribute copies of the Data Correlation worksheet to be completed individually.

ALTERNATIVE INSTRUCTION

Review with the students how to plot coordinates on a coordinate grid. Introduce the three different types of correlations, including their definitions and example graphs. Make sure the students can identify scatter plots that demonstrate a strong positive correlation, strong negative correlation, and a weak correlation. Explain what linear regression means, and have the students draw a regression line on the positive and negative graphs, but not on a no-correlation graph.

Place the students into groups of three or four, and hand each student a coordinate grid. Write on the classroom board a list of the following 12 coordinates: (1, 9), (1, 7), (3, 7), (3, 5), (4, 3),
(5, 3), (5, 5), (6, 4), (7, 2), (7, 5), (8, 3), (9, 1). Have each group plot the points on their graphs. Allow the students enough time to finish plotting the data. Next, have the students draw a linear regression line and describe the correlation between the variables on the graph.

Have the students label the x-axis "revolutions per minute" and the y-axis "miles per gallon." Explain the definition of causation and why conclusions cannot be made from a graph. Refer back to the students, graph, and ask the following questions:

- What type of correlation does the graph represent? (Negative)
- Can the following arguments be supported and true?
  - There is a negative correlation between RPM and MPG. (Yes)
  - Higher RPM's cause higher MPG. (No, higher RPM is associated with lower MPG.)
  - There is a positive correlation between RPM and MPG. (No, it is negative.)
  - There is a causation between RPM and MPG. (No, that isn't possible due to outside factors that could affect the data.)

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Finish by having a class discussion about the data and what arguments can be supported from the data. Explain to the students how certain factors can affect the data and therefore prevent this example from having a conclusion from the data. These factors may include:

- Type of gas being used
- Outside weather conditions
- Type of driving occurring, e.g., racing vs. casual

ALTERNATIVE ASSESSMENT

Prepare additional assessment questions in the same format as the Data Correlation worksheet.

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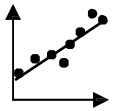
Vocabulary

Correlation
A trend between two sets of data; can be positive, negative, or no correlation

Positive Negative No Correlation

Regression line
A line, segment, or ray drawn on a scatter plot to estimate the relationship between two sets of data



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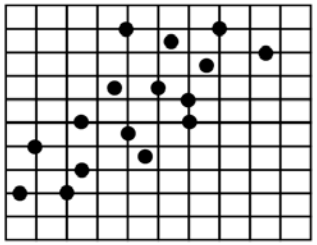
Correlation vs. Causation

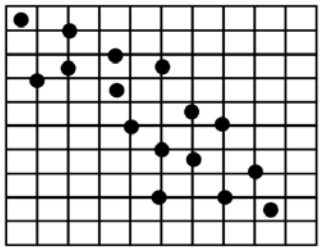
Correlation is used in statistics to represent the strength and direction of a linear relationship between two random variables. A scatter plot is a graphical representation of data that shows different types of correlations.

Positive



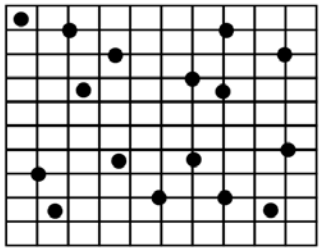
Data points display a positive pattern. As x-values increase the y-values also increase.

Negative



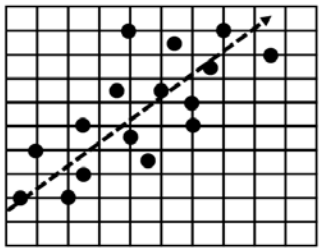
Data points display a negative pattern. As x-values increase, y-values decrease.

None



Data points do not produce an apparent pattern.

Linear Regression Line



A line that estimates the relationship between two variables; also called the “line of best fit”

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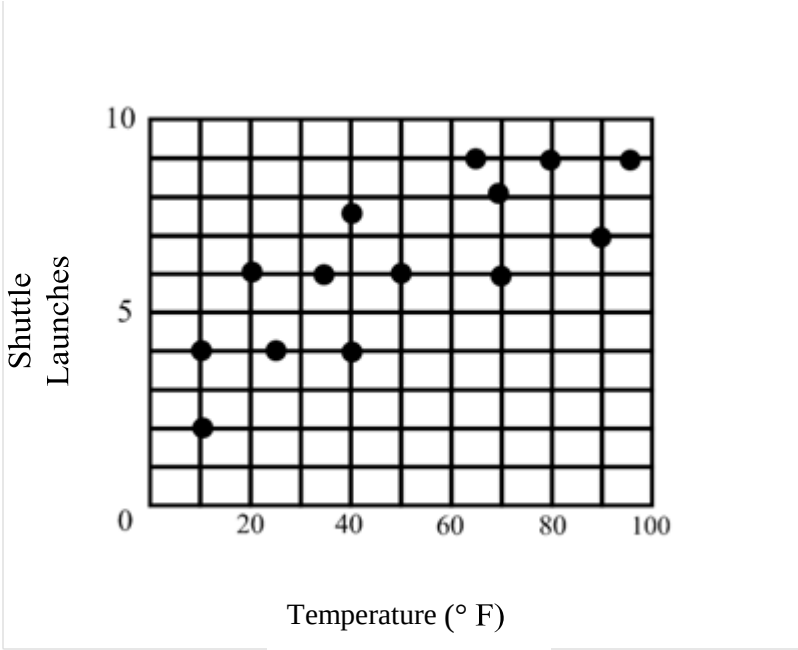
Correlation vs. Causation

Causation is a link between variables so that a change in one variable is believed to produce the change in the other variable. A correlation between two variables does not imply a causation. It is possible that a common, outside factor, called a confound, might produce the relationship indicated by the strong correlation.

Example:

A research study was conducted investigating the number of space shuttle launches a year and the average daily temperature in Orlando, Florida. The data was placed on the following scatter plot.



The data shows a strong positive trend in which the plots form a pattern from low to high. This is considered a positive correlation. The variables do relate and indicate that when the temperature was higher, more shuttle launches occurred. However, it is a fallacy to conclude that higher temperature causes more shuttle launches. There may have been factors that led to the

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correlation, such as:

- NASA likes to launch during the spring and summer to miss the hurricane season
- The launches were for yearly space station maintenance and happen at the same time
- New astronauts finish their training in summer and are ready to launch

Using the data, the following argument can be supported:

1. There is a positive correlation between the temperature and shuttle launches.

There are also arguments that can not be supported by the data, including:

1. Higher temperatures cause more launches.
2. There is a negative correlation between higher temperatures and more launches.

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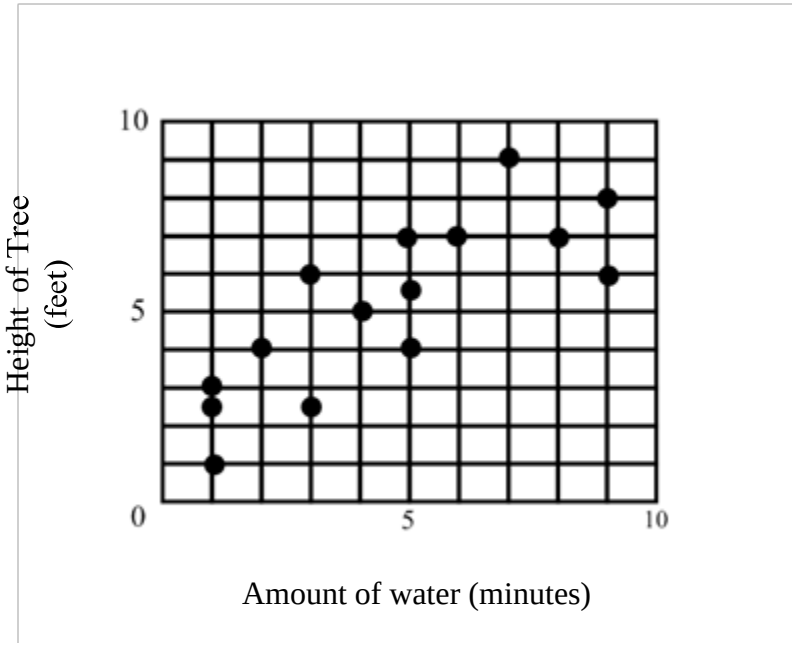
Name_____

Correlate the Data

Directions: Read each problem carefully and solve.

The following graph depicts the relationship between the heights of trees and amount of time they are watered per day.



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1. What type of correlation does this data represent?

2. What does the correlation show about the relationship between watering time and height of a tree?

3. Is it possible to conclude that the more water you give a tree, the taller it grows? Explain why or why not.

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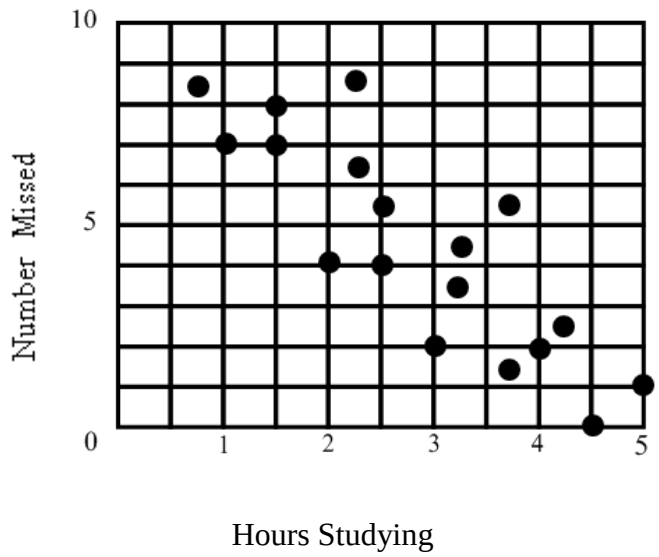
Name_____

Data Correlation

Directions: Read each question. Circle the letter that contains the correct answer to the question or complete the problem in the space provided.

The following graph depicts the results of a class on a test measuring the number of problems missed with the number of hours studied.



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1. The data represents which type of correlation?

   A. Positive
   B. Negative
   C. Weak
   D. None

2. Which statement can be concluded about the data?

   A. Studying more hours causes fewer missed problems.
   B. There is a negative correlation between the number of hours studied and the number of questions missed.
   C. Studying causes great test scores.
   D. There is causation but not a correlation between hours studied and the number of questions missed.

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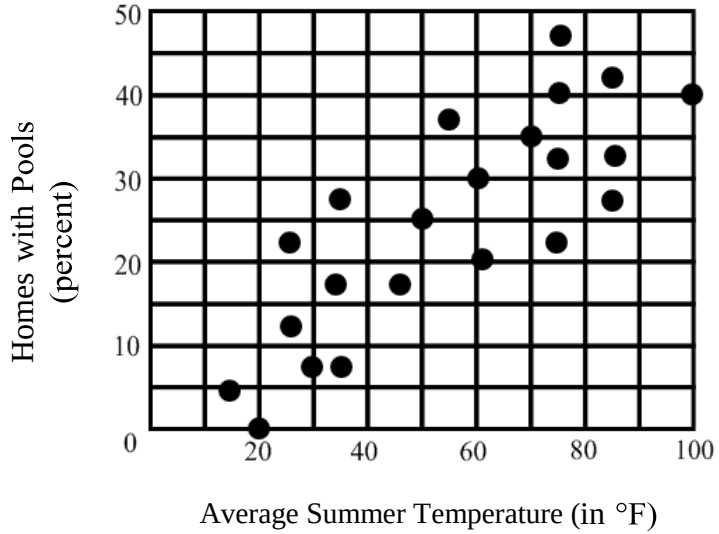
A survey of 100 families in each of 22 cities was taken to measure the average summer temperature and the percentage of residents that had pools in their yards. The results are shown below.



3. The data represents which type of correlation?

   A. Positive
   B. Negative
   C. Weak
   D. Zero

4. Which statement can be concluded about the data?

   A. There is a causation between the number of homes with pools and the average summer temperature.
   B. There is no correlation between the number of homes with pools and the average summer temperature.
   C. Hot temperatures cause people to build more pools.
   D. There is a correlation but not necessarily causation between the number of homes with pools and the average summer temperature.

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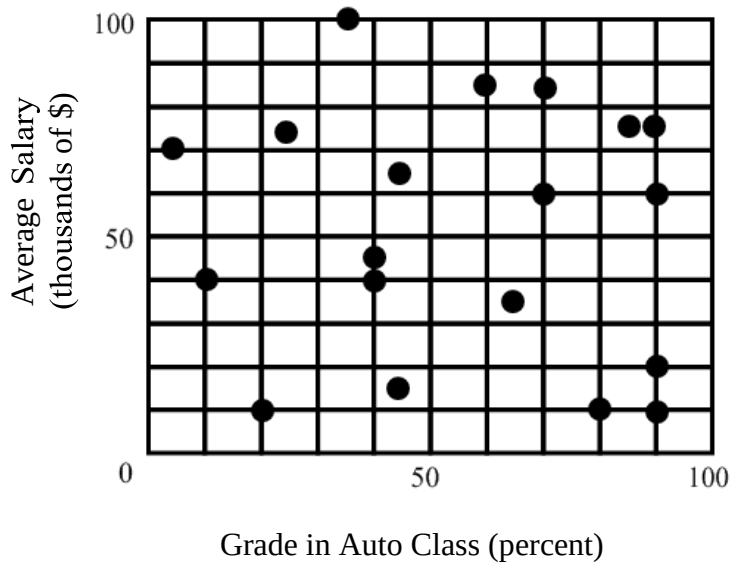
5. Jason concluded that more pools are caused by hotter weather. This is incorrect for what reason?

A. The data shows no correlation.
B. The relationship could be related to outside factors, such as the cost of building pools.
C. There is a causation but no correlation between the variables.
D. The graph shows that higher temperatures cause fewer pools to be built.

At a ten year reunion, the teachers took a survey of the students who took automotive class their senior year. They compared the students' current average salary and their grade in auto class during their senior year. The results were plotted on the following graph.



6. The data represents which type of correlation?

A. Positive
B. Negative
C. None
D. Can't be determined

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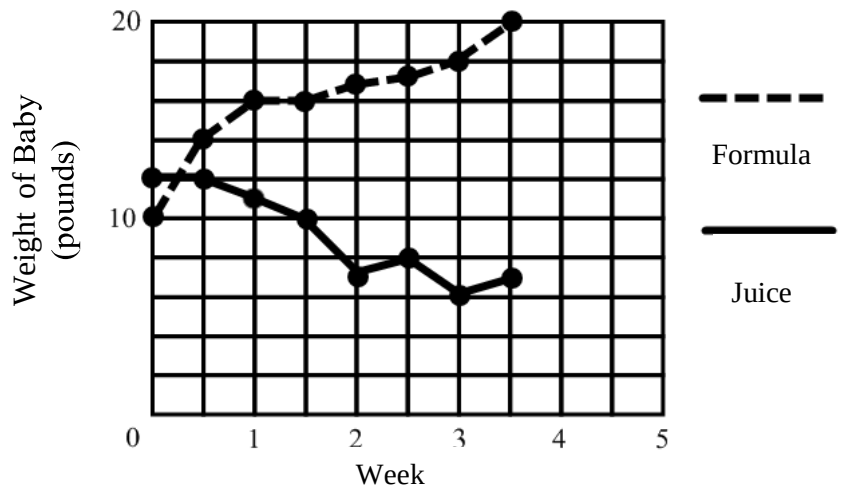
7. Which statement can be concluded about the data?

A. Students with a higher salary had lower grades in auto class.
B. There is no correlation between the average salary and grade in auto class.
C. There is a negative correlation.
D. There is a correlation but not necessarily causation between the average salary and grade in auto class.

8. Which of the following statements about the data is correct?

A. There is a causation but no correlation.
B. There is a correlation but no causation.
C. There is both correlation and causation.
D. There is neither correlation nor causation.

Sam did research on the weight of two newborn babies, one who was fed formula and one who was fed juice. The results are represented on the following graph.



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9. Describe the relationship between the diet and weight of each baby. Then explain what the graph seems to indicate about the relationship between diet and weight.

10. Explain the meaning of the following expression: “Correlation does not imply causation.” Provide a real-world example to support your explanation.

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TEACHER'S ANSWER KEY

Correlate the Data

1. Positive correlation
2. The data shows there is a correlation between the amount of time in minutes a tree is watered and the height of the tree. The relationship shows that, in general, trees that are watered for longer periods of time are taller than those that are watered for less time.
3. No. Some types of trees grow taller than others, and some trees require more water. This only shows a correlation in this data set.

Data Correlation

1. B
2. B
3. A
4. D
5. B
6. C
7. B
8. D
9. 0-2 points: 2 points for stating that there appears to be a positive correlation between the number of weeks of formula consumption and weight, and there appears to be a negative correlation between the number of weeks of juice consumption and weight. This infers that a baby consuming formula over time will weigh more than a baby consuming juice; 1 point for interpreting the correlations correctly, but no explanation of the relationship it implies between diet and weight; or 0 points for an incorrect answer or for no answer.
10. 0-2 points: 2 points for explaining that correlation reflects the relationship between two variables, and that this relationship does not imply one causes the other because there are outside factors, called confounds, that could affect the results of the data, and for including an example of a scenario, including two variables that show a correlation, and an explanation that there is no causation between the variables due to a possible outside factor;
    1 point for correctly explaining "correlation does not imply causation" or providing a real- world example; or 0 points for an incorrect explanation and an incorrect or missing example or for no response.

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MHS Algebra 1 Mathematics Curriculum

Unit 3 Addendum

Performance Standard	Instructional Strategies
S.I.D.5:	**Two-way Frequency Table** A two-way frequency table is shown below displaying the relationship between age and baldness. We took a sample of 100 male subjects, and determined who is or is not bald. We also recorded the age of the male subjects by categories. **Two-way Frequency Table** Bald | Age: Younger than 45, 45 or older | Total No: 35, 11, 46 Yes: 24, 30, 54 Total: 59, 41, 100 The *total* row and *total* column entries in the table above report the marginal frequencies, while entries in the body of the table are the joint frequencies. **Two-way Relative Frequency Table** The relative frequencies in the body of the table are called conditional relative frequencies. **Two-way Relative Frequency Table** Bald | Age: Younger than 45, 45 or older | Total No: 0.35, 0.11, 0.46 Yes: 0.24, 0.30, 0.54 Total: 0.59, 0.41, 1.00

Unit 4 – Expressions and Equations

Domain: Algebra (A)

Essential Question: How do equations and inequalities help model non-linear relationships?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

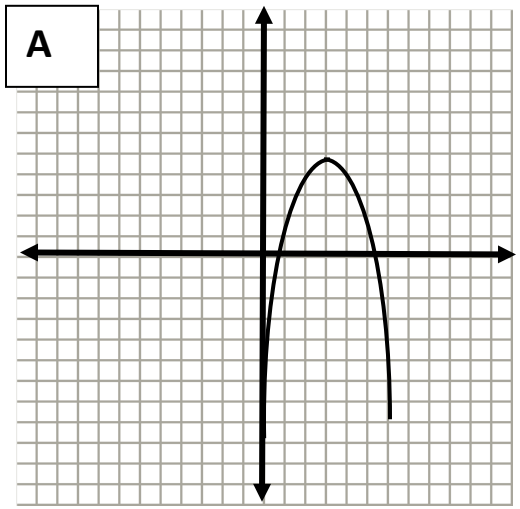
Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Extend reasoning of linear relationships to evaluate a variety of non-linear models.** **A-SSE(***)** Interpret the structure of expressions	**How…** **A.SSE.1:** Interpret expressions that represent a quantity in terms of its context a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.* **A.SSE.2:** Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*	**Students will:** **Comparing Activity** A.SSE.1 - Comparing Quadratics (CCSS).dc **Factoring Practice** A.SSE.2 - Factoring and solving equations **Power Activity and Exit** A.SSE.2 - Powers Activity (CCSS).doc A.SSE.2 - Powers Activity Exit (CCSS).d	**A.SSE.2:** Students should extract the greatest common factor (whether a constant, a variable, or a combination of each). If the remaining expression is quadratic, students should factor the expression further. **Example:** Factor $x^3 - 2x^2 - 35x$

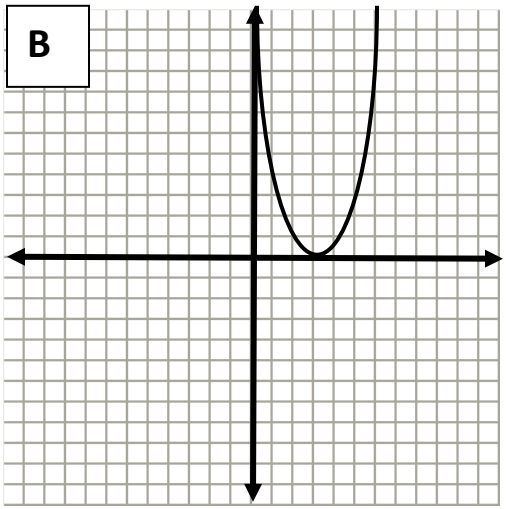
Vocabulary: approximately, equations in two of more variables, exactly, system of linear equations

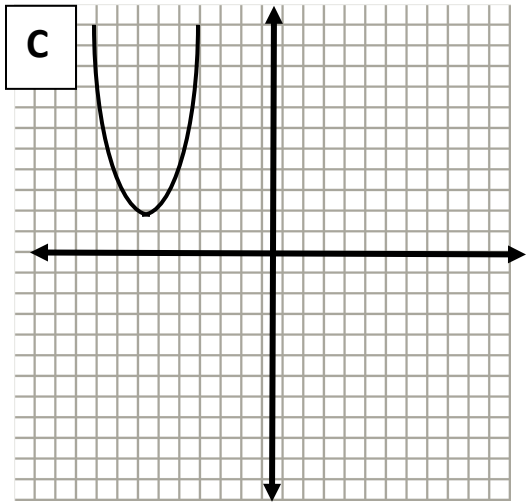
Name: _____ **Date:** _____

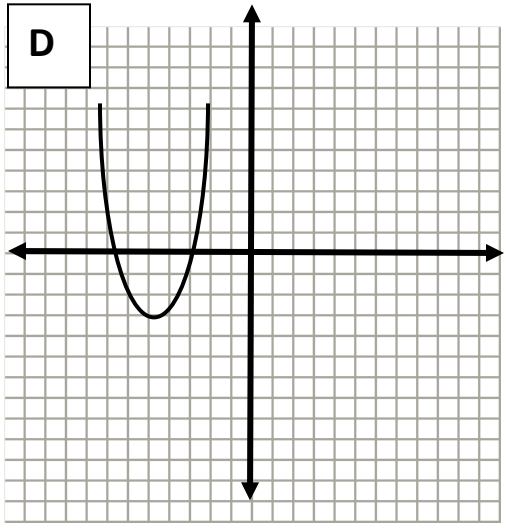
Comparing Quadratics

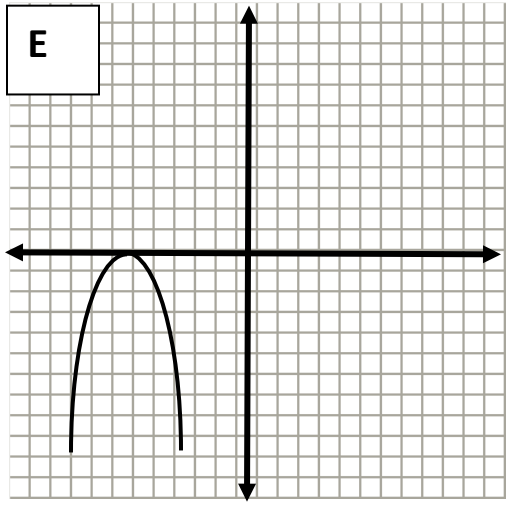
PART 1

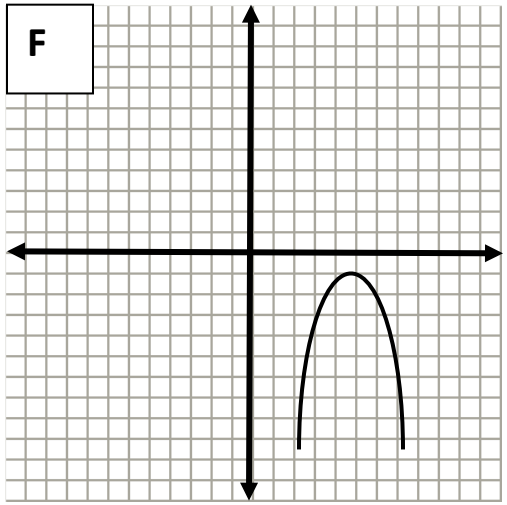












Discussion questions:

1) For each graph where does it cross the x-axis? y – axis?

A	B
C	D
E	F

2) For each graph, what is the maximum or minimum?

A	B
C	D
E	F

3) Group the 6 graphs as many different ways as you can. Each group must have at least 2 members.

Comparing Quadratics

PART 2

Use your graphing calculator to graph each of the following equations.

1) $y = -x^2 - 8x - 17$

x-int	y-int	Max/min

2) $y = (x - 3)^2 + 4$

x-int	y-int	Max/min

3) $y = 3x^2 + 6x + 1$

x-int	y-int	Max/min

4) $y = -x^2 - 4x - 4$

x-int	y-int	Max/min

5) $y = x^2 - 2x + 1$

x-int	y-int	Max/min

6) $y = -(x + 4)^2$

x-int	y-int	Max/min

Discussion questions:

1) Is there any relationship between the equations and the intercepts and max/min values?

2) What in the equation causes the graph to open up or down?

3) Are there any other observations that you can make based on the graph and its equation?

Name_____Date _____ Period: _____

Factoring and Solving Equations

Question 1: (1 point) Cynthia had to factor completely $12x+18$. Her answer was $2(6x+9)$.
Explain what error she made.

_____

_____

_____

Question 2: (1 point) The solutions for the equation $x^2+bx+c=0$ are $x=-1$ and $x=-7$. Which of the following would be the factored form of the equation?

a. $(x+7)(x-1)=0$ b. $(x-7)(x-1)=0$ c. $(x+7)(x+1)=0$ d. $(x-7)(x+1)=0$

Question 3: (1 point each). Factor each expression completely.

a. $6m^2-9m$ b. x^2-4 c. $a^2-9a-36$ d. $m^2-16m+64$

Question 4: (1 point each). Solve each equation.

a. $7x(x+6)=0$ b. $(x+3)(x-7)=0$

Question 5: (2 points each). Solve each equation.

a. $x^2+10x+25=0$ b. $x^2+5x=24$ c. $x^2-16=0$

Name _____ ***Date***_____ _____

Powers Activity

Instructions:

- **You are given 4 monomials on the chart below.**
- **Your task is to find the expressions on the following page that are equivalent to each expression. (Hint: There will be 3 for each.)**
- **Write the three expressions that match UNDER each monomial on the chart.**
- **Then create one of your own in the last block.**

Example:

x^8	*Given monomial*
$(x^4)^2$	*3 expressions that "match"*
$x^5 \bullet x^3$	
$\frac{x^{10}}{x^2}$	
$x^2 \bullet x^2 \bullet x^4$	*One you create yourself*

1.	**2.**	**3.**	**4.**
x^2y^4	$-24x^4$	$8x^6y^3$	x^{-10}

$\frac{1}{x^{10}}$	$(xy^2)^2$	$-24x \cdot x^3$	$(2x^2y)^3$
$\frac{x^{10}y^9}{x^8y^5}$	$-6(2x^2)^2$	$\frac{1}{4}x^2y \cdot 32x^4y^2$	$\frac{48x^6}{-2x^2}$
$\left(x^2\right)^{-5}$	$\frac{56x^{22}y^{99}z^0}{7x^{16}y^{96}}$	$3xy^2 \cdot \frac{1}{3}xy^2$	$x^{-7} \cdot x^{-3}$

Name_____ Date_____ Period_____

Powers Activity Exit

Question 1: (3 points) Give an example of a fourth degree trinomial.

Example: _____

Explain why your example is a fourth degree trinomial.

_____

_____

_____

Question 2: (1 point each) Simplify each expression.

a) $(5m^{7})(6m^{-4})$

b) $\dfrac{10y^{6}}{2y^{4}}$

c) $(2x^{3})^{5}$

d) $\dfrac{-5x^{3}y^{5}}{15x^{7}y^{2}}$

e) $(-3m^{4}n^{6})^{3}$

Question 3: (2 points each) Simplify each expression.

a) $(6x^3y^7)(-2x^3y^9z^2)(2y^2z^6)$

Unit 4 – Expressions and Equations

Domain: Algebra (A)

Essential Question: How do equations and inequalities help model non-linear relationships?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

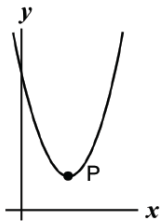
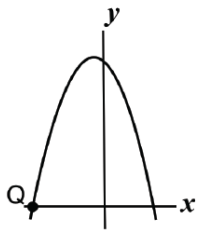
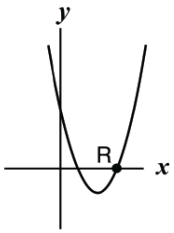
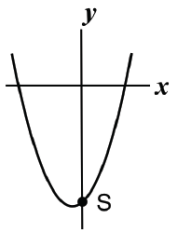
Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Extend reasoning of linear relationships to evaluate a variety of non-linear models.** **A-SSE3(***)** Write expressions in equivalent forms to solve problems.	**How…** **A.SSE.3:** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*	**Students will:** **Quadratic Partner Activity** A.SSE.3 - Quadratics Dominos (CCSS).docx	**A.SSE.3:** Students will use the properties of operations to create equivalent expressions. **Examples:** • Express $2(x^3 - 3x^2 + x - 6) - (x - 3)(x + 4)$ in factored form and use your answer to say for what values of x the expression is zero. • Write the expression below as a constant times a power of x and use your answer to decide whether the expression gets larger or smaller as x gets larger. o $\frac{(2x^3)^2(3x^4)}{(x^2)^3}$

Vocabulary: approximately, equations in two of more variables, exactly, system of linear equations

Quadratic Functions

1. Here are 4 equations of quadratic functions and 4 sketches of the graphs of quadratic functions.

A. $y = x^2 - 6x + 8$	B. $y = (x-6)(x+8)$	C. $y = (x-6)^2 + 8$	D. $y = -(x+8)(x-6)$

1. 	2. 	3. 	4. 

a. Match the equation to its graph and explain your decision.

Equation A ***matches*** Graph ....., because

Equation B ***matches*** Graph ....., because

Equation C ***matches*** Graph ....., because

Equation D ***matches*** Graph ....., because

---

b. Write the co ordinates of the points: P (....,....) Q(....,....) R (....,....) S (....,....)

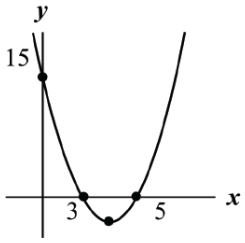
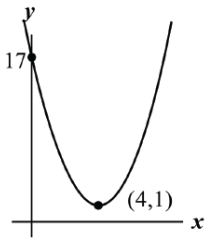
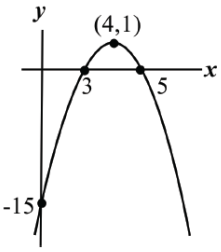
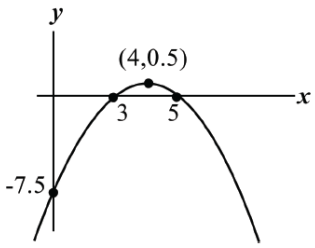
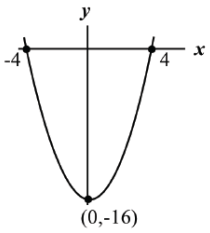
2. The graph of a quadratic function has a y intercept at (0,5) and a minimum at (3, −4).

a. Write the equation of its curve.

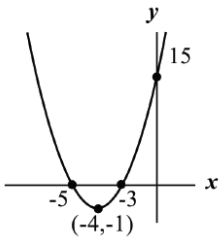
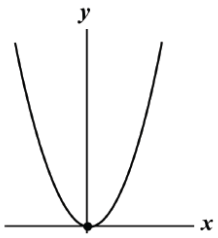
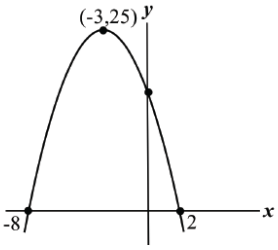
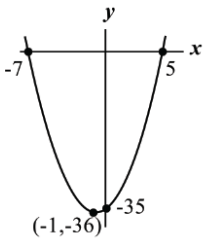
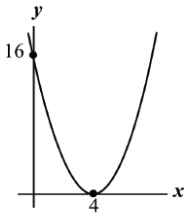
---

b. Write the coordinates of the root(s) of this quadratic function.

Domino Cards: 1

A $y = x^2 + 2x - 35$ $y =$ ..... $y =$	
B $y = x^2 + 8x$..... $y =$ ..... $y = (x+4)^2 - 1$	
C $y = x^2 - 8x$..... $y = (x-4)(x-4)$ $y =$	
D $y = -x^2 + 8x$..... $y =$ ..... $y = -(x-4)^2 + 1$	
E $y = -x^2 - 6x + 16$ $y = -(x+8)(x-2)$ $y = -(x+3)^2 + 25$	

Domino Cards: 2

F $y = x^2$..... $y = (x-4)(x+4)$ $y =$	
G $y = x^2 - 8x$..... No roots $y =$	
H $y = x^2 - 8x + 15$ $y = (x-3)(x-5)$ $y = (x-4)^2 - 1$	
I $y = -\frac{1}{2}x^2 + 4x$..... $y = -\frac{(x-3)(x-5)}{2}$ $y =$	
J $y = x^2$ $y =$ ..... $y =$	

Unit 4 – Expressions and Equations

Domain: Algebra (A)
Essential Question: How do equations and inequalities help model non-linear relationships?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Extend reasoning of linear relationships to evaluate a variety of non-linear models.** **A-APR(***)** Perform arithmetic operations on polynomials	**How…** **A.APR.1:** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	**Students will:** **Polynomial Check** A.APR.1 - Polynomial Check In (CCSS).doc	

Vocabulary: approximately, equations in two of more variables, exactly, system of linear equations

Name_____ Date_____ Period: _____

Polynomial Check-In

Question 1: (1 point)

Tyler added the following polynomials $(5x^2 - 4)$ and $(-3x + 5)$. His answer was $(2x^2 + 1)$.

Explain what error he made.

_____

_____

Question 2: (1 point) The sides of a rectangle measure $(x + 3)$ by $(x - 2)$. Which of the following is the area of the rectangle?

a. $x^2 - x + 6$ b. $x^2 + x - 6$ c. $x^2 + 5x + 6$ d. $x^2 - 5x - 6$

Question 3: (2 points each) Add or subtract the following. Simplify all solutions.

a. $(-x^2 + 4x - 12) + (-3x^2 - 2x + 8)$ b. $(10x^2 - 14) - (3x^2 + 4x - 11)$

Question 4: (2 points each) Multiply the following. Simplify all solutions.

a. $-3x^2(-4x^3 + 2x^2 - 3x + 8)$ b. $(x + 2)(x - 9)$ c. $(x + 4)^2$

MHS Algebra 1 Mathematics Curriculum

Unit 4 – Expressions and Equations

Domain: Algebra (A)

Essential Question: How do equations and inequalities help model non-linear relationships?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. **Use appropriate tools strategically.**
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Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Extend reasoning of linear relationships to evaluate a variety of non-linear models.** **A-CED(***)** Create equations that describe numbers or relationships	**How…** **A.CED.1:** Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* **A.CED.2:** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. **A.CED.4:** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*	**Students will:** **Team Relay Activity** A.CED.1 - Parking Lot (CCSS).docx **Systems Stations** A.CED.2 - Creating and Graphing Station **Complete the Square Intro** A.CED.4 - Domino Intro.pptx	**A.CED.1:** See Unit Addendum **A.CED.4:** See Unit Addendum

Vocabulary: approximately, equations in two of more variables, exactly, system of linear equations

Revised: 8/21/2012

Parking Lot: A Relay Activity

Teacher Directions:

Divide students into groups of no more than three students. Each group will work through the activity one stage at a time. It is suggested you set a time limit for the first stage so that groups struggling with factoring do not get stuck and don't get to the applications problems. Please note that on each stage, accuracy is a must. Students should not move the next stage without successfully completing the previous one. On the first stage, feel free to differentiate for time and ability. Stage 4 is an extension activity and provided for the group that is moving through the material quickly.

Stage 1 – Solve each equation correctly. Have your solutions verified by the teacher and then move on to the next stage.

1. $x^2 - 11x + 30 = 0$

2. $x^2 + 13x + 36 = 0$

3. $x^2 + 5x = 24$

4. $5x^2 + 28x + 32 = 0$

5. $3x^2 + 17x + 10 = 0$

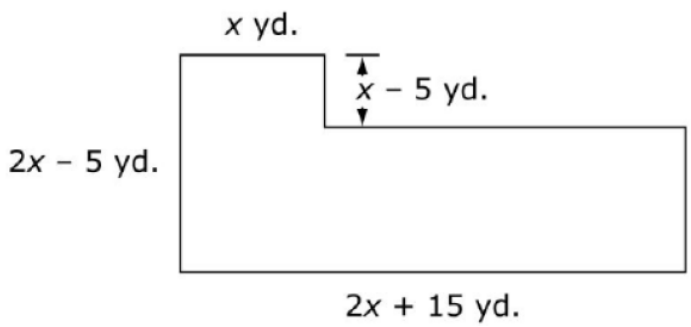
6. $7x^2 - 10x = 8$

7. $18x^2 + 10x - 8 = 0$

8. $14x^2 - 28x = -12$

Stage 2: The Parking Lot

A town council plans to build a public parking lot. The outline below represents the proposed shape of the parking. The area of the parking lot is 3000 square yards.



1) Write an expression for the area, in square yards, of the proposed parking lot. Explain the reasoning you used to find the expression.

2) Since the area is 3000 square yards, find the value of x.

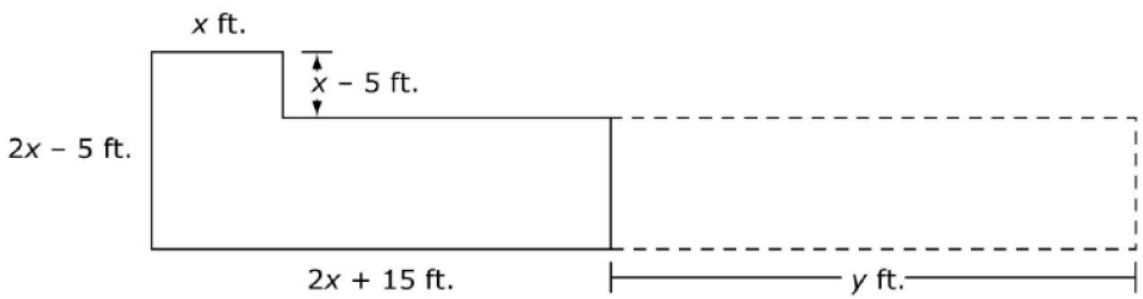
Stage 3:

The town council has decided to double the area of the parking lot. They created a plan to do this. The plan increases the length of the base of the parking lot by y feet, as shown in the diagram below.

1) Write an expression in terms of x to represent the value of y, in feet.

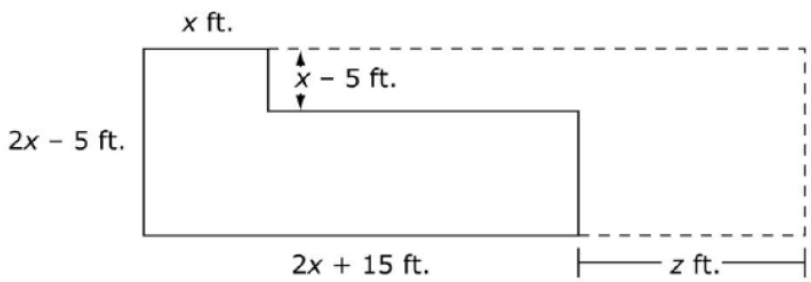
2) The addition will create an extra 3000 square yards of parking, solve the equation to find the length (y) we are adding to the parking lot.

BEWARE: The units have changed to feet, but the area is in yards!



Stage 4:

A second plan for doubling the area changes the shape of the parking lot to a rectangle as shown in the diagram. Can the value of z be represented as a polynomial with integer coefficients? Justify your reasoning.



Names: _____ **Date:** _____

Creating and Graphing Equations **Station 1**

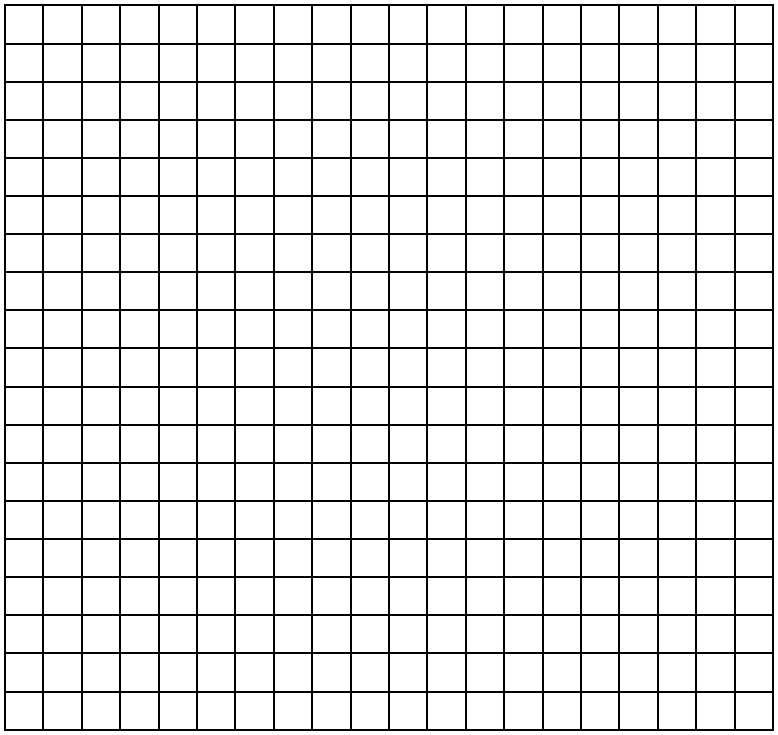
Directions:
A) Create an equation based on the given problem.
B) Then graph your equation on the given axes with appropriate labels and scales.

1) Mr. T and Mr. T have been running a lawn service this summer and offered this deal to all of their customers. Create an equation that describes what a person would expect to pay Mr. T and Mr. T.

T & T Lawn service
$15 per hour of service plus an additional $20 for equipment maintenance

A) Equation: _____

B) Graph



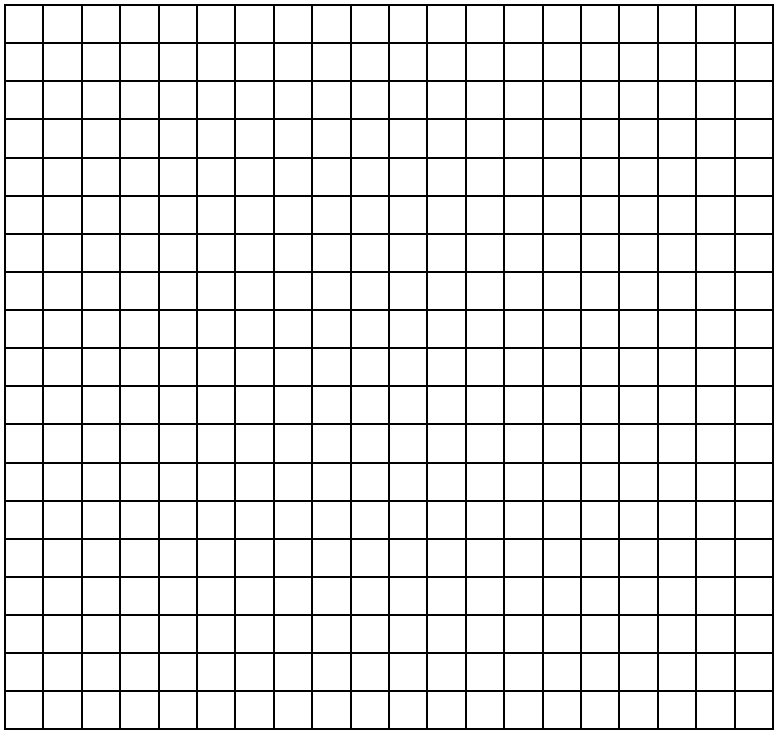
Names: _____ **Date: _____**

Creating and Graphing Equations **Station 2**

Directions:
A) Create an equation based on the given problem.
B) Then graph your equation on the given axes with appropriate labels and scales.

2) You and your family have been looking to build a deck off the back of your house this year. Your family wants the length of the deck to be 5ft longer than 2 times the width. Create an equation that describes the area of your deck.

   A) Equation: _____
   B) Graph



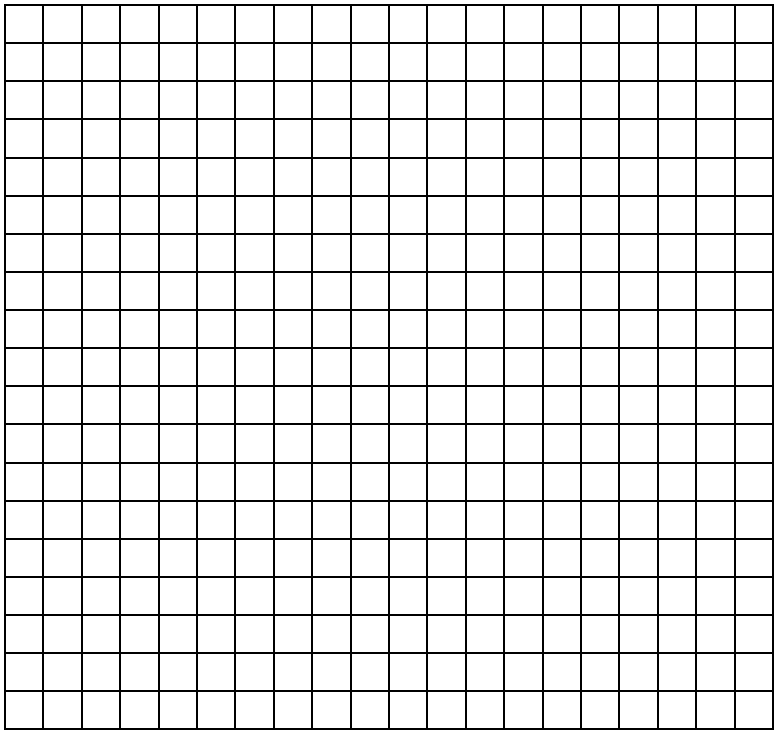
Names: _____ **Date: _____**

Creating and Graphing Equations **Station 3**

Directions:
A) Create an equation based on the given problem.
B) Then graph your equation on the given axes with appropriate labels and scales.

3) In your science class you are testing the growth of a bacterial culture. You find that one single bacterium triples every hour. Create an equation that describes the number of bacteria present at any given time.

   A) Equation: _____
   B) Graph



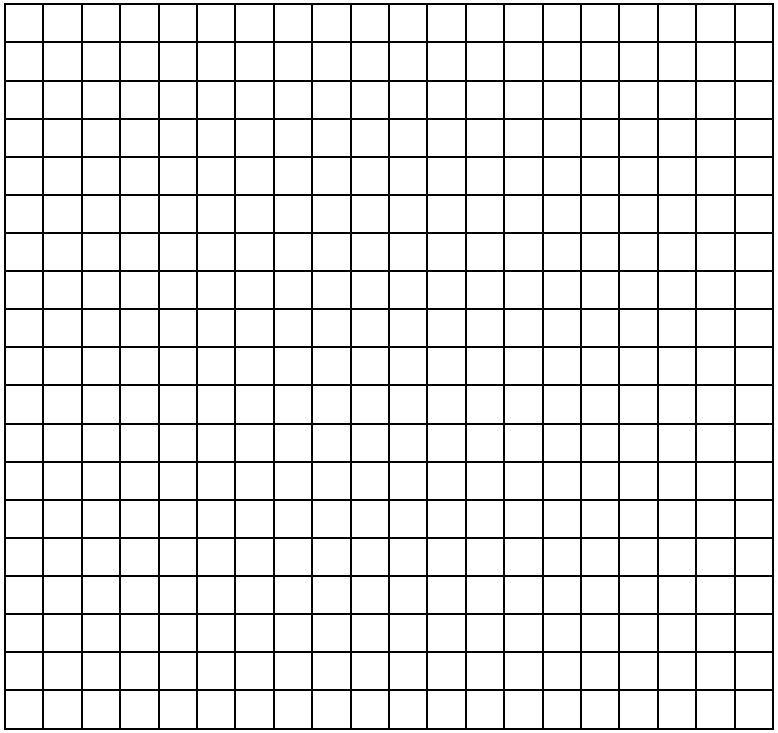
Names: _____ **Date:** _____

Creating and Graphing Equations **Station 4**

4) A firework is launched in the air from the ground, following a parabolic path, with an initial upward velocity of 128 ft/s^2. The height of this firework is represented by the equation
$y = -16t^2 + 128t$.

A second firework is launched in the air from the ground, following a linear path. The rocket starts 15ft off the ground and continually rises at a rate of 25 ft/s. The height of this firework is represented by the equation $y = 25x + 15$.

Graph these 2 equations on the same grid below.



A) What is represented by the points where the two graphs intersect?

_____
_____
_____
_____

B) Which one of the graphs is a better representation of the path of the firework? Why?

_____
_____
_____
_____

From Standard Form To Completing the Square

You can change a quadratic equation from standard form to vertex form by completing the square.

STANDARD $\longrightarrow$ $y = x^2 - 6x + 4$

$$y - 4 + ? = (x^2 - 6x + ?)$$

$$y - 4 + 9 = (x^2 - 6x + 9)$$

$$y + 5 = (x^2 - 6x + 9)$$

$$y + 5 = (x+3)^2$$

VERTEX $\longrightarrow$ $y = (x+3)^2 - 5$

From Standard Form To Completing the Square

Let's try another one together:

STANDARD $\longrightarrow$ $y = 2x^2 + 12x - 4$

$$y + 4 = 2x^2 + 12x$$

$$y + 4 + ? = 2(x^2 + 6x + ?)$$

$$y + 4 + 18 = 2(x^2 + 6x + 9)$$

$$y + 22 = 2(x^2 + 6x + 9)$$

$$y + 22 = 2(x + 3)(x + 3)$$

$$y + 22 = 2(x + 3)^2$$

VERTEX $\longrightarrow$ $y = 2(x + 3)^2 - 22$

From Standard Form To Completing the Square

Now, you try these on your own:

1) $y = x^2 + 12x + 34$

2) $y = 2x^2 + 4x + 6$

Unit 4 – Expressions and Equations

Domain: Algebra (A)

Essential Question: How do equations and inequalities help model non-linear relationships?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
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8. **Look for and express regularity in repeated reasoning.**

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Extend reasoning of linear relationships to evaluate a variety of non-linear models.** **A-REI(***)** Solve equations and inequalities in one variable **A-REI(**)** Solve systems of equations	**How…** **A.REI.4:** Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b. **A.REI.7:** Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.*	**Students will:** **Solving Quads Group Activity** A.REI.4 - Solving Quadratic Equations (**Nature of Roots Exit** A.REI.4 - Exit - Nature of Roots (CCS **Linear-Non-linear System** A.REI.7 - Highway systems (CCSS).docx	**A.REI.4:** See Unit Addendum **A.REI.7:** **Example:** Two friends are driving to the Grand Canyon in separate cars. Suzette has been there before and knows the way but Andrea does not. During the trip Andrea gets ahead of Suzette and pulls over to wait for her. Suzette is traveling at a constant rate of 65 miles per hour. Andrea sees Suzette drive past. To catch up, Andrea accelerates at a constant rate. The distance in miles (d) that her car travels as a function of time in hours (t) since Suzette's car passed is given by $d = 3500t^2$. Write and solve a system of equations to determine how long it takes for Andrea to catch up with Suzette

Vocabulary: approximately, equations in two of more variables, exactly, system of linear equations

Solving Quadratic Equations: using the most appropriate method activity

Prior Learning: Students should have experience solving quadratic equations using the following methods

1) Factoring
2) Completing the square
3) The Quadratic Formula

Directions:

Each group will be given the same equation, but must solve it using different methods. Randomly assign a method to each group. They must then present their method to the class and evaluate the appropriateness of their method. The group will be given poster paper on which they write out the solution and graph the equation.

This should be done three times, once with each equation. Each time the group should be given a different method.

It is important to note that the emphasis of this lesson is not on the solving, but rather the appropriateness of the method. The class should reflect on each method and when it was used the best.

Materials Needed:

Equation cards – one per group
Method card – one per group
Presentation paper – three per group
Markers, colored pencils etc.
Calculators

Equations:

1) $x^2 + x + 5 = 0$ 2) $x^2 - 12x + 45 = 0$ 3) $x^2 + x - 12 = 0$

Equation Cards

1) $x^2 + x + 5 = 0$	1) $x^2 + x + 5 = 0$
1) $x^2 + x + 5 = 0$	1) $x^2 + x + 5 = 0$
2) $x^2 -12x + 45 = 0$	2) $x^2 -12x + 45 = 0$
2) $x^2 -12x + 45 = 0$	2) $x^2 -12x + 45 = 0$
3) $x^2 + x -12 = 0$	3) $x^2 + x -12 = 0$
3) $x^2 + x -12 = 0$	3) $x^2 + x -12 = 0$

Methods Cards

1) Factoring	1) Factoring
1) Factoring	1) Factoring
2) Completing the square	2) Completing the square
2) Completing the square	2) Completing the square
3) The Quadratic Formula	3) The Quadratic Formula
3) The Quadratic Formula	3) The Quadratic Formula

Name: _____ **Date:** _____

Exit Check: Methods of Solving Quadratic Equations

Solve the following quadratic equation using any method you would like. You must provide a reason you selected that method:

Solve: $x^2 - 4x - 12 = 0$

Method Selected: _____

Why you chose that method:

_____

_____

_____

Name: _____ **Date:** _____

Exit Check: Methods of Solving Quadratic Equations

Solve the following quadratic equation using any method you would like. You must provide a reason you selected that method:

Solve: $x^2 - 4x - 12 = 0$

Method Selected: _____

Why you chose that method:

_____

_____

_____

Name: _____ **Date: _____**

Exit Slip – Nature of Roots

Directions:

1) Determine the nature of the roots of each equation. (real or complex)
2) Determine how many roots each equation has.
3) Find all of the solutions of each equation, using the most appropriate method.

A. $2x^2 + 5 = 2x$

Nature of Roots -	Number of roots -	Solutions -

B. B. $x^2 + 6x - 10 = 0$

Nature of Roots -	Number of roots -	Solutions -

C. $3x^2 + 5x = 12$

Nature of Roots -	Number of roots -	Solutions -

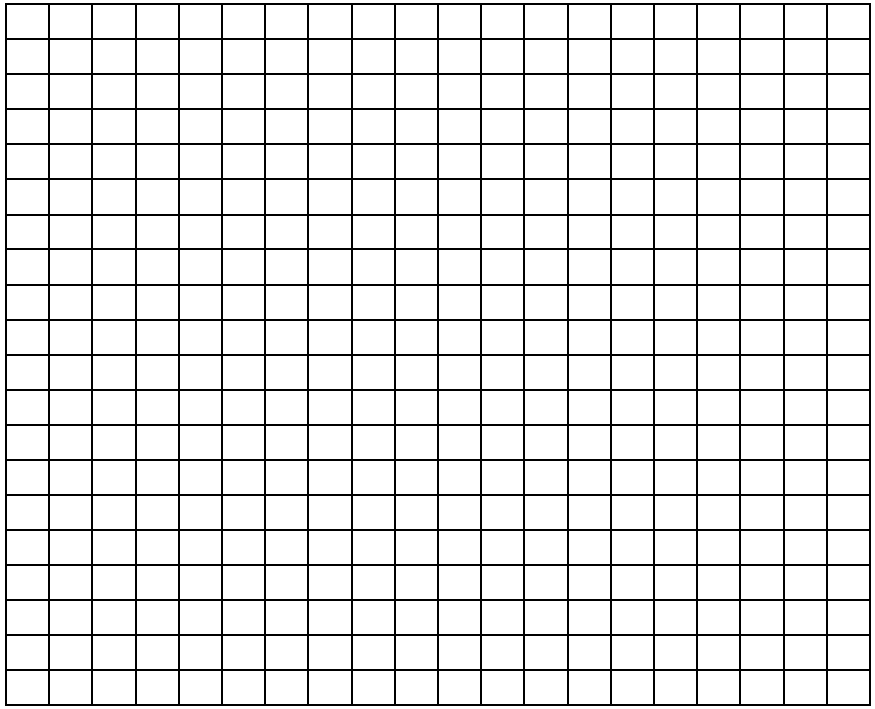
Name: _____ **Date: _____**

Highway System

City B has a highway system that includes a parabolic beltway highway around the city. This has caused transportation problems for emergency services. Ambulances are stationed at the hospital and they have difficulty at rush hour with fast access to the hospital due to traffic. The equation $y = -x^2 + 11x + 40$ is a model of the beltway. The transportation authority has proposed an ambulance access road through the beltway for emergency vehicles. The equation $y = 3x + 20$ models the proposed emergency services road.

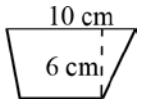
Assume that the new road will intersect the beltway at points A and B; point B being the location of the city hospital. Due to construction constraints the transportation authority will only fund the project if the distance on the new road is less than 40 miles.

Will the authority fund the construction of the new road? Justify your answer.
Represent both roadways graphically on a coordinate plane.



MHS Algebra 1 Mathematics Curriculum

Unit 4 Addendum

Performance Standard	Instructional Strategies
A.CED.1:	Equations can represent real world and mathematical problems. Include equations and inequalities that arise when comparing the values of two different functions, such as one describing linear growth and one describing exponential growth. Examples: • Given that the following trapezoid has area 54 cm^2, set up an equation to find the length of the base, and solve the equation.  • Lava coming from the eruption of a volcano follows a parabolic path. The height *h* in feet of a piece of lava *t* seconds after it is ejected from the volcano is given by $h(t) = -t^2 + 16t + 936.$ After how many seconds does the lava reach its maximum height of 1000 feet?
A.CED.4:	Examples: • The Pythagorean Theorem expresses the relation between the legs a and b of a right triangle and its hypotenuse c with the equation $a^2 + b^2 = c^2$. o Why might the theorem need to be solved for c? o Solve the equation for c and write a problem situation where this form of the equation might be useful. • Solve $V = \frac{4}{3}\pi r^3$ for radius *r*. • Motion can be described by the formula below, where *t* = time elapsed, *u*=initial velocity, *a* = acceleration, and *s* = distance traveled $s = ut + \frac{1}{2}at^2$ o Why might the equation need to be rewritten in terms of *a*? Rewrite the equation in terms of *a*.

MHS Algebra 1 Mathematics Curriculum

Unit 4 Addendum

<table>
<tr><th>Performance Standard</th><th>Instructional Strategies</th></tr>
<tr><td>A.REI.4:</td><td>
Students should solve by factoring, completing the square, and using the quadratic formula. The zero product property is used to explain why the factors are set equal to zero. Students should relate the value of the discriminant to the type of root to expect. A natural extension would be to relate the type of solutions to $ax^2 + bx + c = 0$ to the behavior of the graph of $y = ax^2 + bx + c$.
<table>
<tr><th>Value of Discriminant</th><th>Nature of Roots</th><th>Nature of Graph</th></tr>
<tr><td>$b^2 - 4ac = 0$</td><td>1 real roots</td><td>intersects x-axis once</td></tr>
<tr><td>$b^2 - 4ac > 0$</td><td>2 real roots</td><td>intersects x-axis twice</td></tr>
<tr><td>$b^2 - 4ac < 0$</td><td>2 complex roots</td><td>does not intersect x-axis</td></tr>
</table>
Are the roots of $2x^2 + 5 = 2x$ real or complex? How many roots does it have? Find all solutions of the equation.
What is the nature of the roots of $x^2 + 6x + 10 = 0$? Solve the equation using the quadratic formula and completing the square. How are the two methods related?
</td></tr>
</table>

Unit 5 – Quadratic Functions and Modeling

Domain: Number and Quantity (N)
Essential Question: How do quadratic functions help model and solve problems?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Interpret, analyze and compare linear and quadratic models using varied representations.** **N-RN(*)** Use properties of rational and irrational numbers	**How…** **N.RN.3:** Explain why the sum or product of two rational numbers are rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational	**Students will:** **Properties of Rational and Irrational Numbers – Exit Check** • Given a set of numbers, categorize them as rational or irrational • Perform given operations on rational and irrational numbers, classifying the results as rational or irrational and summarize the closedness of the rational and irrational numbers	**N.RN.3:** See Addendum

Vocabulary: axis of symmetry, end behavior, extreme values, intercepts, intervals, maximum/minimum values, polynomials, quadratic expression, quadratic functions, relative maximums and minimums, symmetries, zeros

Unit 5 – Quadratic Functions and Modeling

Domain: <u>Functions</u> (F)

Essential Question: How do quadratic functions help model and solve problems?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Interpret, analyze and compare linear and quadratic models using varied representations.** **F-IF(***)** Interpret functions that arise in applications in terms of context	**How…** **F.IF.4**: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* **F.IF.5:** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* **F.IF.6:** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	**Students will:** **Objects in Motion – Group Work** Suppose a soccer ball is kicked directly u F.IF.4,5,6 Graphing quadratic functions gi	**F.IF.4:** See Addendum **F.IF.5:** Students may explain orally, or in written format, the existing relationships. **F.IF.6:** See Addendum

<u>Vocabulary:</u> axis of symmetry, end behavior, extreme values, intercepts, intervals, maximum/minimum values, polynomials, quadratic expression, quadratic functions, relative maximums and minimums, symmetries, zeros

Name _____ Date _____

Suppose a soccer ball is kicked directly upward at a speed of 32 feet per second. The algebraic rule that describes the height of the soccer ball as a function of time is $h(t) = -16t^2 + 32t + 48$, where t represents the number of seconds since the kick and $h(t)$ represents the ball's height (in feet) above the ground.

Determine the answers to the following questions and sketch the graph.

a. How high doe the soccer ball go?

b. How long does it take to go that High?

c. When will the ball reach the ground again?

1. To celebrate the 4th of July, the town of West Central has hired a pyrotechnic company to launch fireworks from an 80 ft tower in town. The company can launch the fireworks at an initial upward velocity of 64 ft per sec. The company estimates the fireworks height by the function $h(t)=-16t^2+64t+80$

- Your task is to work as a group to graph the path of the object using the function given.
- Remember you need the following:

  - x-intercepts/zeros - y-intercept -A.O.S. – vertex -at least 5 points

- Your group will also make a brief presentation to the class discussing the equation, and overall graph, and submit a written description as well. (They may be the same thing) Be sure to describe:
  - The intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries.
  - The relationship of the domain to the function along with what an appropriate domain might be, and why.

You will be graded based on the following:

	3	2	1	0
Clarity of Graph	Graph is clear and neat. All appropriate labels are included	Graph is clear & neat. Labels are missing or incorrect	Time & effort was not put into graph	No Attempt
"5 Points"	All 5 of the points are correct and appropriately marked	3 or 4 of the points are correct & appropriately marked	Only 1 or 2 of the points are correct & appropriately marked	No Attempt
Solving Process	Appropriate processes are used accurately. Work demonstrates full understanding	Minor errors are made in appropriate process, with a general understand-ing shown	Inappropriate process chosen, showing lack of understanding	No Attempt
Presentation	Specific & accurate description of the graph and meaning of points is present-ed. All members participate & share	General, yet accurate description is given. Most of points are given meaning. Most members share	Semi-accurate description but meaning of points is vague. Only 1-2 members share	No Attempt
Written Submission	All 3 of the following are discussed 1. Intercepts,max/min, symmetry 2. Intervals of inc/dec and pos/neg 3. Appropriate domain	2 of the 3 are discussed 1. Intercepts,max/min, symmetry 2. Intervals of inc/dec and pos/neg 3. Appropriate domain	1 of the 3 are discussed 1. Intercepts,max/min, symmetry 2. Intervals of inc/dec and pos/neg 3. Appropriate domain	No Attempt

- Calculate and interpret the average rate of change of the function between 1 and 3 seconds.

2. To help create the new Tiger Woods video game, EA sports has a few golfers drive virtual golf balls. It is averaged that the balls at driven with an initial upward velocity of 48 ft per sec. EA sports estimates the golf ball height by the following function $h(t)=-16t^2+48t$

- Your task is to work as a group to graph the path of the object using the function given.
- Remember you need the following:

  - zeros/roots - y-intercept -A.O.S. – vertex -at least 5 points

- Your group will also make a brief presentation to the class discussing the equation, and overall graph, and submit a written description as well. (They may be the same thing) Be sure to describe:
  - The intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries.
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- Calculate and interpret the average rate of change of the function between 0 and 1.5 seconds.

3. A circus acrobat is shot out of a cannon with an initial upward velocity of 50 ft per sec. When the acrobat leaves the cannon they are 4 ft above the ground. The height of the acrobat is estimated by the function $h(t)=-16t^2+50t+4$

- Your task is to work as a group to graph the path of the object using the function given.
- Remember you need the following:

  - zeros/roots - y-intercept -A.O.S. – vertex -at least 5 points

- Your group will also make a brief presentation to the class discussing the equation, and overall graph, and submit a written description as well. (They may be the same thing) Be sure to describe:
  - The intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries.
  - The relationship of the domain to the function along with what an appropriate domain might be, and why.

You will be graded based on the following:

	3	2	1	0
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Solving Process	Appropriate processes are used accurately. Work demonstrates full understanding	Minor errors are made in appropriate process, with a general understand-ing shown	Inappropriate process chosen, showing lack of understanding	No Attempt
Presentation	Specific & accurate description of the graph and meaning of points is present-ed. All members participate & share	General, yet accurate description is given. Most of points are given meaning. Most members share	Semi-accurate description but meaning of points is vague. Only 1-2 members share	No Attempt
Written Submission	All 3 of the following are discussed 1. Intercepts,max/min, symmetry 2. Intervals of inc/dec and pos/neg 3. Appropriate domain	2 of the 3 are discussed 1. Intercepts,max/min, symmetry 2. Intervals of inc/dec and pos/neg 3. Appropriate domain	1 of the 3 are discussed 1. Intercepts,max/min, symmetry 2. Intervals of inc/dec and pos/neg 3. Appropriate domain	No Attempt

- Calculate and interpret the average rate of change of the function between 3 and 4.5 seconds.

4. A diver springs from the edge of a 80 ft cliff with an initial upward velocity of 8 ft per sec. The height of this diver can be approximated by the function $h(t)=-16t^2+8t+80$

- Your task is to work as a group to graph the path of the object using the function given.
- Remember you need the following:

  - zeros/roots - y-intercept -A.O.S. – vertex -at least 5 points

- Your group will also make a brief presentation to the class discussing the equation, and overall graph, and submit a written description as well. (They may be the same thing) Be sure to describe:
  - The intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries.
  - The relationship of the domain to the function along with what an appropriate domain might be, and why.

You will be graded based on the following:

	3	2	1	0
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- Calculate and interpret the average rate of change of the function between 2.5 and 3 seconds.

5. Galileo dropped a cannonball from the top of the Leaning Tower of Pisa, which is 185 ft above the ground, to prove that mass effected fall time. The height of this ball is approximated by the function $h(t)=-16t^2+185$

- Your task is to work as a group to graph the path of the object using the function given.
- Remember you need the following:

  - zeros/roots - y-intercept -A.O.S. – vertex -at least 5 points

- Your group will also make a brief presentation to the class discussing the equation, and overall graph, and submit a written description as well. (They may be the same thing) Be sure to describe:
  - The intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries.
  - The relationship of the domain to the function along with what an appropriate domain might be, and why.

You will be graded based on the following:

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- Calculate and interpret the average rate of change of the function between 0.5 and 1.25 seconds.

Unit 5 – Quadratic Functions and Modeling

Domain: Functions (F)
Essential Question: How do quadratic functions help model and solve problems?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Interpret, analyze and compare linear and quadratic models using varied representations.** **F-IF(***)** Analyze functions using different representations	**How…** **F.IF.7:** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. **F.IF.8:** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential functions. *Identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.*	**Students will:** **Function Representation Analysis** F.IF.7 (Unit 5).pdf **Function Exit Check** F.IF.8 (Unit 5).pdf	**F.IF.7:** See Addendum

Vocabulary: axis of symmetry, end behavior, extreme values, intercepts, intervals, maximum/minimum values, polynomials, quadratic expression, quadratic functions, relative maximums and minimums, symmetries, zeros

HS Math C1 TM

Claim 1: Concepts and Procedures Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.	
Content Domain: **Functions**	
Target M [m]: Analyze functions using different representations. Tasks for this target will ask students to graph functions (linear, quadratic, square root, cube root, piecewise-defined, polynomial, exponential and logarithmic) by hand or using technology and compare properties of two functions represented in different ways. Some tasks will focus on understanding equivalent forms that can be used to explain properties of functions, and may be associated with 9-12 Algebra Target H.	
Standards:	F-IF.7, F-IF.8, F-IF.9
DOK Target(s):	1, 2
Evidence Required:	1. The student graphs linear, quadratic, square root, cube root, piecewise-defined, polynomial, exponential, and/or logarithmic functions without the use of technology. 2. The student graphs linear, quadratic, square root, cube root, piecewise-defined, polynomial, exponential, and/or logarithmic functions with the use of technology. 3. The student compares properties of two functions represented in different ways (e.g., as equations, tables, graphs, or written descriptions). 4. The student recognizes and determines equivalent forms of functions that can be used to explain properties of the function.
Allowable Item Types*:	SR, CR, TE

Task Models:	1. SR (DOK 1, 2) **Prompt Features:** The student is prompted to identify the graph(s) of linear, quadratic, square root, cube root, piecewise-defined, polynomial, exponential, and/or logarithmic function(s). **Stimulus 1:** The student is presented with a function equation. **Stimulus 2:** The student is presented with a written description that may or may not discuss properties or key features of the function. **Stimulus 3:** The student is presented with a table of values representing a function. 1. CR (DOK 2) **Prompt Features:** The student is prompted to draw graph(s) of linear, quadratic, square root, cube root, piecewise-defined, polynomial, exponential, and/or logarithmic function(s) without the use of technology. **Stimulus 1:** The student is presented with a function equation. **Stimulus 2:** The student is presented with a written description that may or may not discuss properties or key features of the function. **Stimulus 3:** The student is presented with a table of values representing a function. 2. TE (DOK 2) **Prompt Features:** The student is prompted to generate graphs of linear, quadratic, square root, cube root, piecewise-defined, polynomial, exponential, and/or logarithmic function(s). **Stimulus 1:** The student is presented with a function equation. **Stimulus 2:** The student is presented with a written description that may or may not discuss properties or key features of the function. **Stimulus 3:** The student is presented with a table of values representing a function. **Interaction:** The student uses a graphing tool to enter data points and/or a function equation and draw either a line or a curve to represent the graph of the function

	3. SR (DOK 1, 2) **Prompt Features 1:** The student is prompted to identify the relationship(s), common property(ies), or key feature(s) shared between two functions. **Prompt Features 2:** The student is prompted to identify, from a set, which two functions share common feature(s) or which function has feature(s) that compare in a specific way to another given function (e.g., which two functions have no zeros). **Stimulus:** The student is presented with two (or more) functions represented in different ways, either as a table of values, a graph, a function equation, or a written description. *Functions include: linear, quadratic, square root, cube root, piecewise-defined, polynomial, exponential, and logarithmic functions.* 3. CR (DOK 2) **Prompt Features:** The student is prompted to describe or determine the relationship(s), common property(ies), or key feature(s) shared between two functions. **Stimulus:** The student is presented with two functions represented in different ways, either as a table of values, a graph, a function equation, or a written description. *Functions include: linear, quadratic, square root, cube root, piecewise-defined, polynomial, exponential, and logarithmic functions.* 4. SR (DOK 1, 2) **Prompt Features 1:** The student is prompted to select the equivalent form of a given function that best identifies or explains key feature(s) of the function [e.g., The form $f(x) = (x+2)(3x-5)$ would be best to find the roots, or x-intercepts, of the function $f(x) = 3x^2 + x - 10$; The form $f(x) = -(x-1)^2 + 2$ would be best to find the vertex of the function $f(x) = -x^2 + 2x + 1$; both forms of this latter example would indicate that the graph of the functions opens downward, therefore creating a maximum point at its vertex. **Stimulus 1:** The student is presented with a property or key feature of a function. **Stimulus 2:** The student is presented with a function. **Prompt Features 2:** The student is prompted to identify the property(ies) or key feature(s) of a function that can be found within the function when written in given form(s). **Stimulus:** The student is presented with one or more functions.

	4. CR (DOK 2) **Prompt Features:** The student is prompted to construct the equivalent form of a given function that best identifies or explains key feature(s) of the function [e.g., The form $f(x) = (x+2)(3x-5)$ would be best to find the roots, or x-intercepts, of the function $f(x) = 3x^2 + x - 10$; The form $f(x) = -(x-1)^2 + 2$ would be best to find the vertex of the function $f(x) = -x^2 + 2x + 1$; both forms of this latter example would indicate the graph of the functions opens downward, therefore creating a maximum point at its vertex. **Stimulus 1:** The student is presented with a single function written in a given form. **Stimulus 2:** The student is presented with a function and given property(ies) or key feature(s) of the function.
Allowable Stimulus Materials:	linear functions, quadratic functions, square-root functions, cube-root functions, piecewise-defined functions, polynomial functions, exponential functions, logarithmic functions, table of values, description of a function, description of a property or key feature of a function
Allowable Disciplinary Vocabulary:	quadratic, square root, cube root, piecewise-defined, polynomial, exponential, logarithmic, x-intercept, y-intercept, interval, increasing interval, decreasing interval, relative maximum, relative minimum, symmetry, axis of symmetry, end behavior of a graph, limit, periodicity
Allowable Tools:	graph paper, graphing calculator, computer-simulated graphing calculator
Target-Specific Attributes:	Properties and key features include: x- and y-intercepts; intervals where the function is increasing, decreasing, positive, or negative; zeros; relative maximums and minimums; symmetries; end behavior; and periodicity.
Nontargeted Constructs:	
Accessibility Concerns:	Blind and low-vision students may have difficulty with aspects of this target related to drawing, reading, and interpreting graphs.
Sample Items:	MAT.HS.CR.1.00FIF.M.274, MAT.HS.CR.1.00FIF.M.613, MAT.HS.SR.1.00FIF.M.612

*SR = selected-response item; CR = constructed-response item; TE = technology-enhanced item; ER = extended-response item; PT = performance task

MAT.HS.CR.1.00FIF.M.274

Sample Item ID:	MAT.HS.CR.1.00FIF.M.274
Grade:	HS
Claim(s):	**Claim 1: Concepts and Procedures** Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.
Assessment Target(s):	1 M: Analyze functions using different representations.
Content Domain:	Functions
Standard(s):	F-IF.8
Mathematical Practice(s):	1, 7
DOK:	1
Item Type:	CR
Score Points:	1
Difficulty:	L
Key:	$y = \frac{2}{3}x + \frac{1}{3}$
Stimulus/Source:	
Target-specific attributes (e.g., accessibility issues):	
Notes:	

Write the function $y - 3 = \frac{2}{3}(x - 4)$ in the equivalent form **most** appropriate for identifying the slope and y-intercept of the function.

Scoring Rubric for Multi-part Items:

Correct responses to this item receive 1 point.

1 point for the correct form $y = \frac{2}{3}x + \frac{1}{3}$

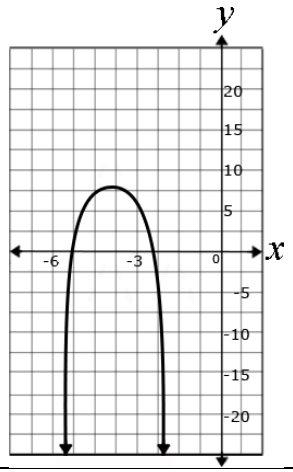
Unit 5 – Quadratic Functions and Modeling

Domain: <u>Functions</u> (F)

Essential Question: How do quadratic functions help model and solve problems?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Interpret, analyze and compare linear and quadratic models using varied representations.** **F-IF(***)** Analyze functions using different representations	**How…** **F.IF.9:** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*	**Students will:** **Function Comparison** Sorting Functions.docx	**F.IF.9:** Example: • Examine the functions below. Which function has the larger maximum? How do you know? $f(x) = -2x^2 - 8x + 20$ 

Vocabulary: axis of symmetry, end behavior, extreme values, intercepts, intervals, maximum/minimum values, polynomials, quadratic expression, quadratic functions, relative maximums and minimums, symmetries, zeros

Sorting Functions

On the next page are four graphs, four equations, four tables, and four rules.

Your task is to match each graph with an equation, a table and a rule.

1. Write your answers in the following table.

Graph	Equation	Table	Rule
A			
B			
C			
D			

2. Explain how you matched each of the four graphs to its equation.

Graph A _____

_____

_____

Graph B _____

_____

_____

Graph C _____

_____

_____

Graph D _____

_____

_____

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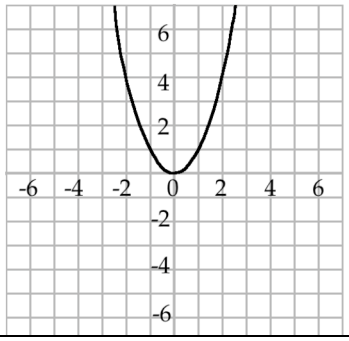
Graph A



Equation A

$xy = 2$

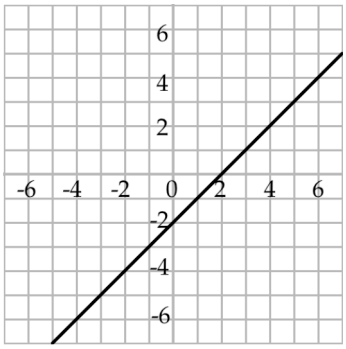
Table A

x	-2	-1	0	1	2	3
y	-4	-3	-2	-1	0	1

Rule A

y is the same as *x* multiplied by *x*

Graph B



Equation B

$y^2 = x$

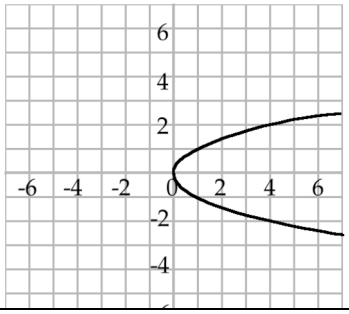
Table B

x	-2	-1	0	1	2	3
y	4	1	0	1	4	9

Rule B

x multiplied by *y* is equal to 2

Graph C



Equation C

$y = x^2$

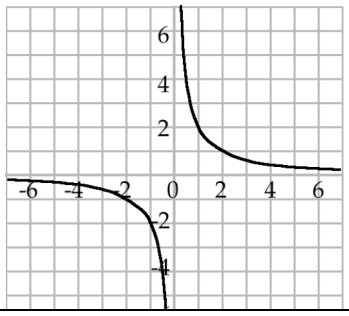
Table C

x	0	1	4	9	16
y	0	±1	±2	±3	±4

Rule C

y is 2 less than *x*

Graph D



Equation D

$y = x - 2$

Table D

x	-2	-1	0	1	2	4
y	-1	-2	±∞	2	1	0.5

Rule D

x is the same as *y* multiplied by *y*

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Unit 5 – Quadratic Functions and Modeling

Domain: Functions (F)
Essential Question: How do quadratic functions help model and solve problems?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Interpret, analyze and compare linear and quadratic models using varied representations.** **F-BF(***)** Build a function that models a relationship between two quantities	**How…** **F.BF.1:** Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*	**Students will:** **Build a Function – Group Work** • Create a function to model a given situation such as compound interest or percent change • Create a new function from a given function represented graphically or algebraically	**F.BF.1:** See Addendum

Vocabulary: axis of symmetry, end behavior, extreme values, intercepts, intervals, maximum/minimum values, polynomials, quadratic expression, quadratic functions, relative maximums and minimums, symmetries, zeros

Unit 5 – Quadratic Functions and Modeling

Domain: Functions (F)

Essential Question: How do quadratic functions help model and solve problems?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

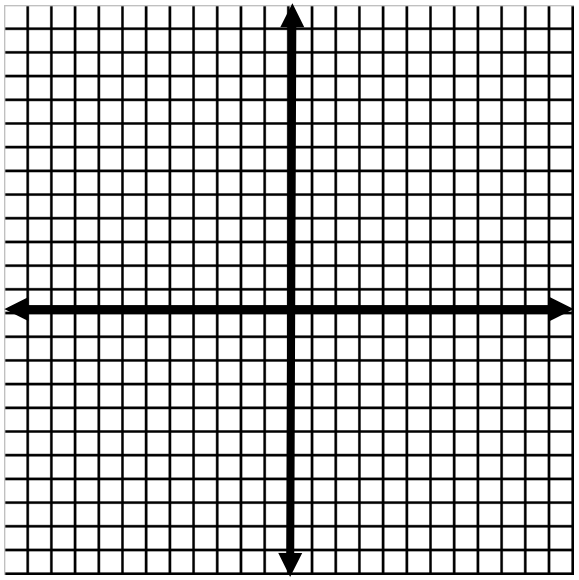
Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Interpret, analyze and compare linear and quadratic models using varied representations.** **F-BF(*)** Build new functions from existing functions	**How…** **F.BF.3:** Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.* **F.BF.4:** Find inverse functions. a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. *For example, $f(x) = 2x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$.*	**Students will:** **Graphing Calculator Activity – Function Parameters** F-BF.3 Func Fam Shifting Graphs.docx . F-BF.3 Function Family Summary.doc **Functions and their Inverses** F-BF.4 Inverse Functions.docx	**F.BF.3:** See Addendum **F.BF.4:** Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model functions. **Examples:** • For the function $h(x) = (x - 2)^3$, defined on the domain of all real numbers, find the inverse function if it exists or explain why it doesn't exist. • Graph $h(x)$ and $h^{-1}(x)$ and explain how they relate to each other graphically. • Find a domain for $f(x) = 3x^2 + 12x - 8$ on which it has an inverse. Explain why it is necessary to restrict the domain of the function.

Vocabulary: axis of symmetry, end behavior, extreme values, intercepts, intervals, maximum/minimum values, polynomials, quadratic expression, quadratic functions, relative maximums and minimums, symmetries, zeros

On each of the following pages, use the given graph of the parent function to graph the 3 other functions given. Be sure to label at least 3 points, and state the domain and range of each function.

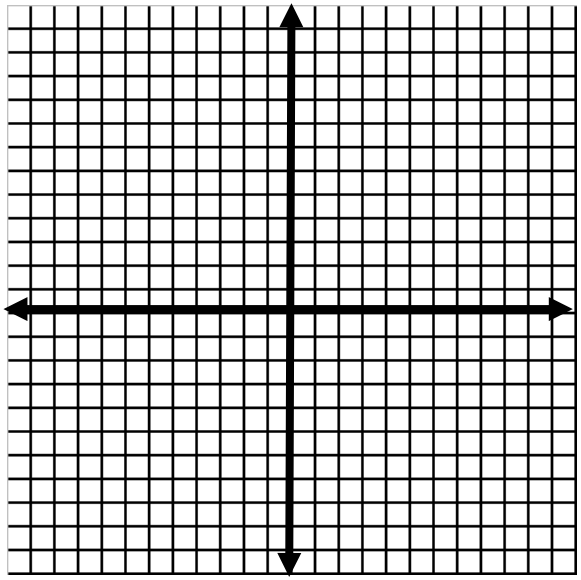
The Absolute Value Family

1. f(x) = $|x|$



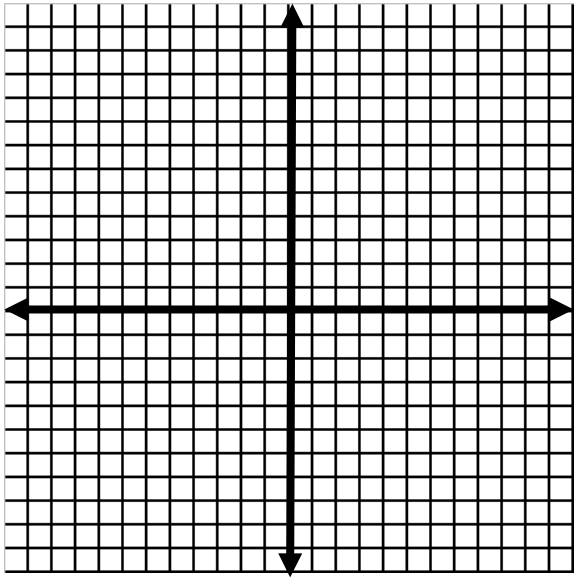
Description:

2. f(x) = $|x - 4|$



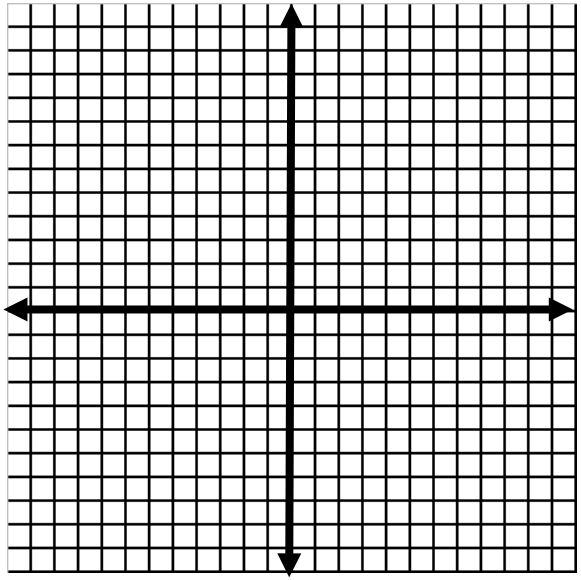
Description:

3. f(x) = $-|x|$



Description:

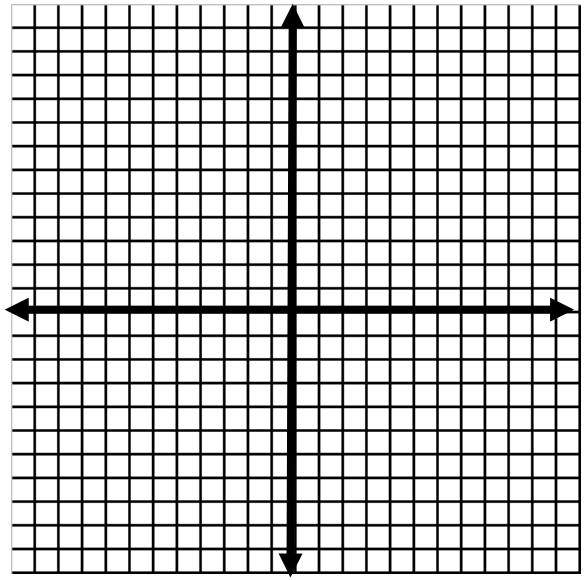
4. f(x) = $|x| - 5$



Description:

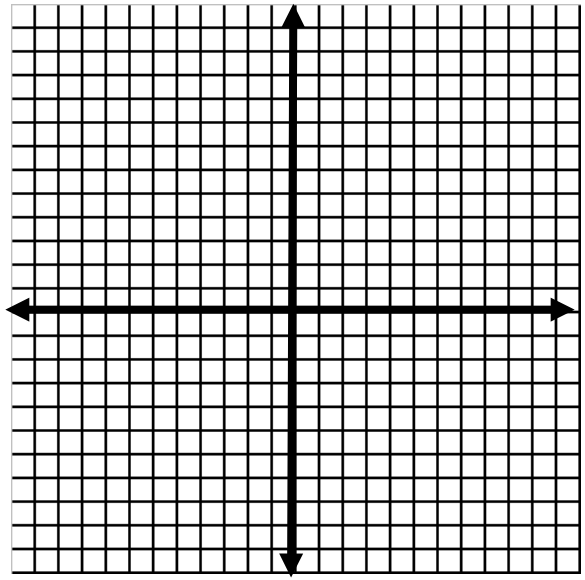
The Square Root Family

1. f(x) = $\sqrt{x}$



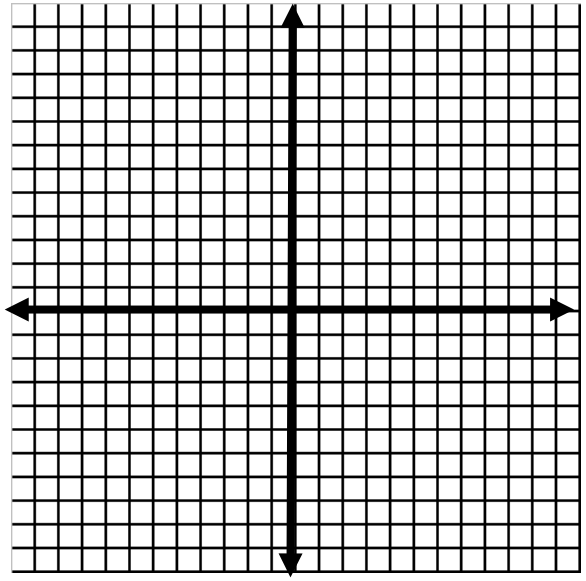
Description:

2. f(x) = $\sqrt{x}$ + 3



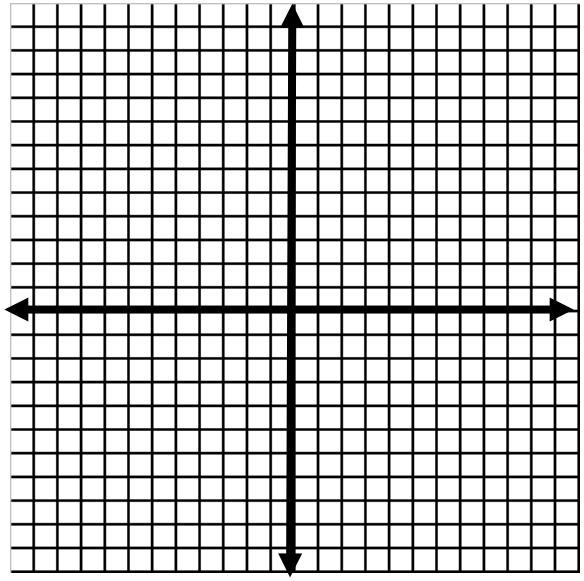
Description:

3. f(x) = $\sqrt{x-2}$



Description:

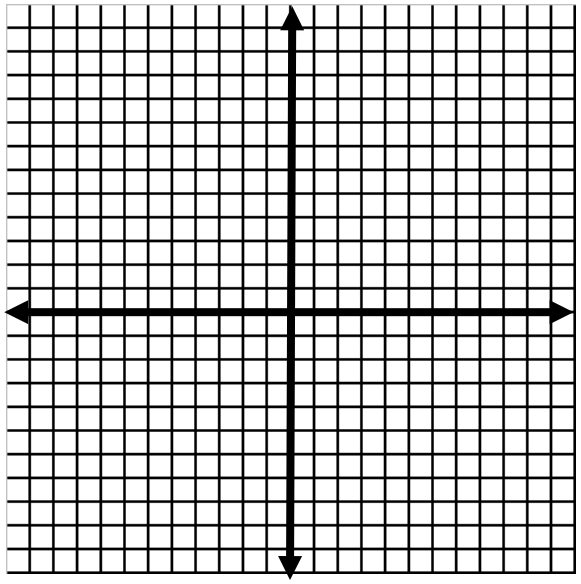
4. f(x) = $\sqrt{3x}$



Description:

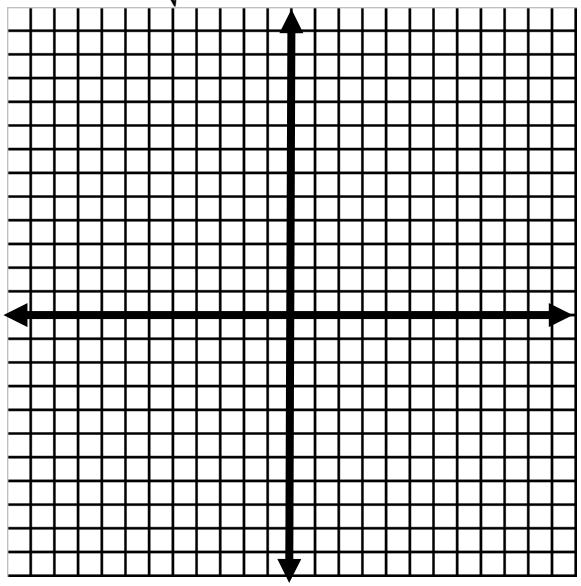
The Cube Root Family

1. f(x) = $\sqrt[3]{x}$



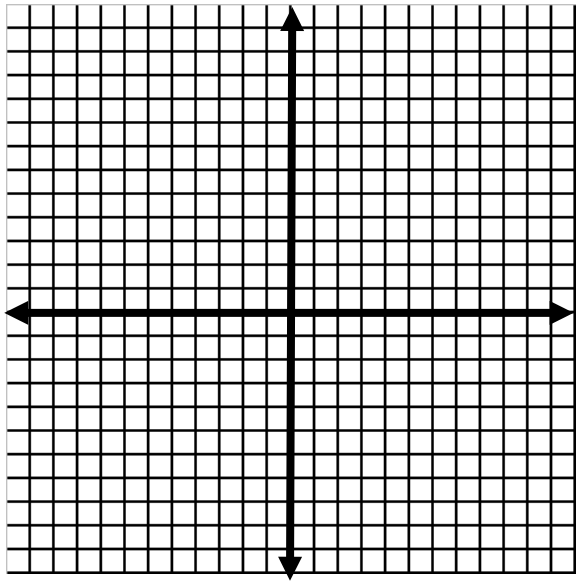
Description:

2. f(x) = $\sqrt[3]{x - 5}$



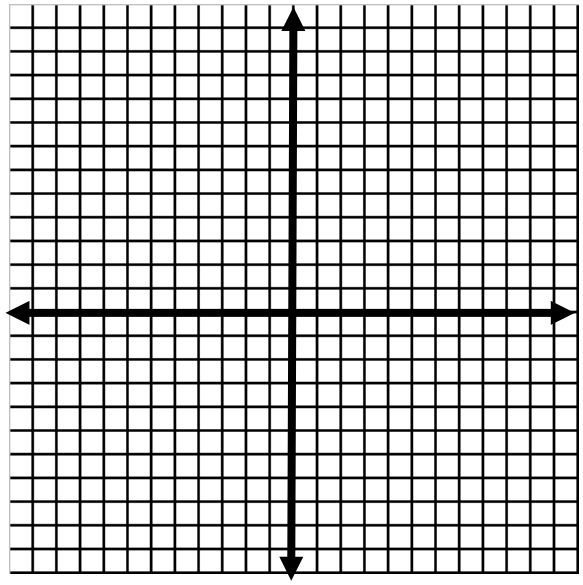
Description:

3. f(x) = $2\sqrt[3]{x}$



Description:

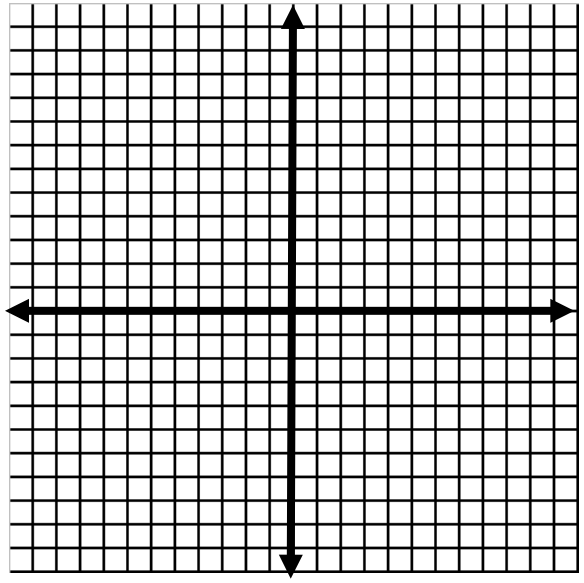
4. f(x) = $\sqrt[3]{x} + 3$



Description:

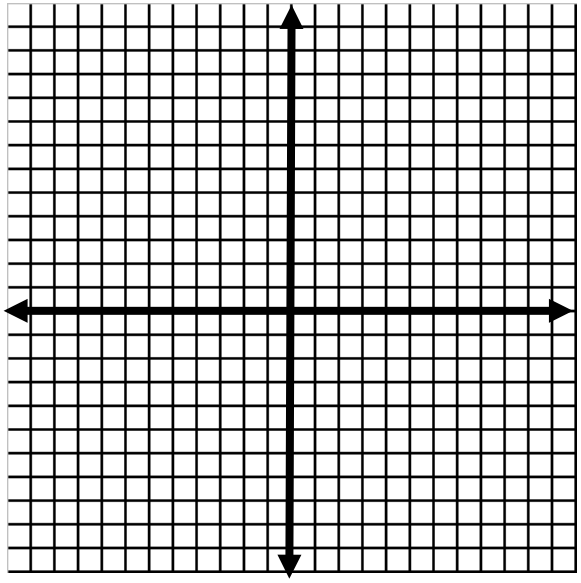
The Inverse Family

1. f(x) = $\frac{1}{x}$



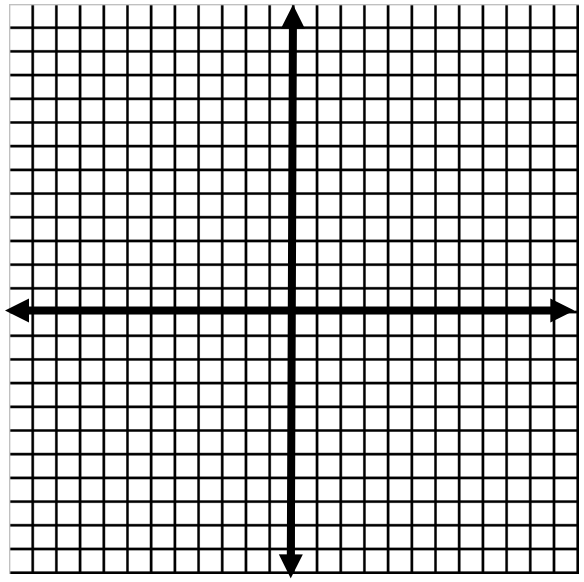
Description:

2. f(x) = $\frac{1}{x-2}$



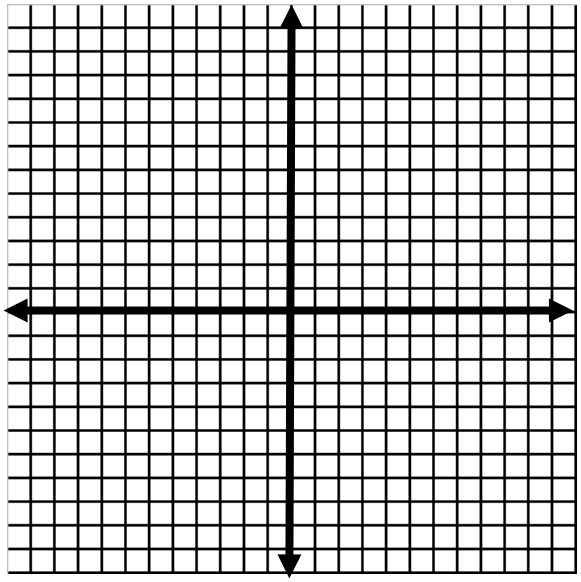
Description:

3. f(x) = $\frac{1}{x} + 3$



Description:

4. f(x) = $\frac{-1}{x}$



Description:

Function Family Shifts Summary

Now that we've taken a look at a few "function families" and their graphs let's try to generalize how "parent function" are shifted/transformed based on the addition of a constant.

For each of the following let's assume f(x) is any "parent function" and c is some constant where C70.

Shift/Transformation Rules

To graph y= f(x) + C, the "parent graph", f(x), is shifted _____

To graph y= f(x) –C, the "parent graph", f(x), is shifted _____

To graph y= f(x+C), the "parent graph", f(x), is shifted _____

To graph y= f(x-C), the "parent graph", f(x), is shifted _____

To graph y= -f(x), the "parent graph", f(x), is shifted _____

To graph y= f(-x), the "parent graph", f(x), is shifted _____

-We can combine any series of shifts that we choose.

Practice

-Even without knowing the graph you can describe the shift of the “parent function”.

-For each of the following describe the shifts performed on the “parent function”.

Parent Function	Shifted Function	Describe Shifts
F(x)= $\lvert x \rvert$	Y=$\lvert x+1 \rvert$	
F(x)= $\lvert x \rvert$	Y=$\lvert 3x \rvert$	
F(x)= $\lvert x \rvert$	Y= $\lvert x \rvert - 2$	
F(x)= $\sqrt{x}$	Y= $4\sqrt{x}$	
F(x)= $\sqrt{x}$	Y= $\sqrt{-x}$	
F(x)= $\sqrt{x}$	Y= $\sqrt{x-2}$	
F(x) =$\sqrt[3]{x}$	Y = $\sqrt[3]{x+4}$	
F(x) =$\sqrt[3]{x}$	Y = $\sqrt[3]{x} - 1$	
F(x)= $\frac{1}{x}$	Y= $\frac{1}{x+5}$	
F(x)= $\frac{1}{x}$	Y= $\frac{1}{x} - 4$	
F(x)= $\frac{1}{x}$	Y=$\frac{1}{-2x}$	

Name:_____Date:_____ Period:_____

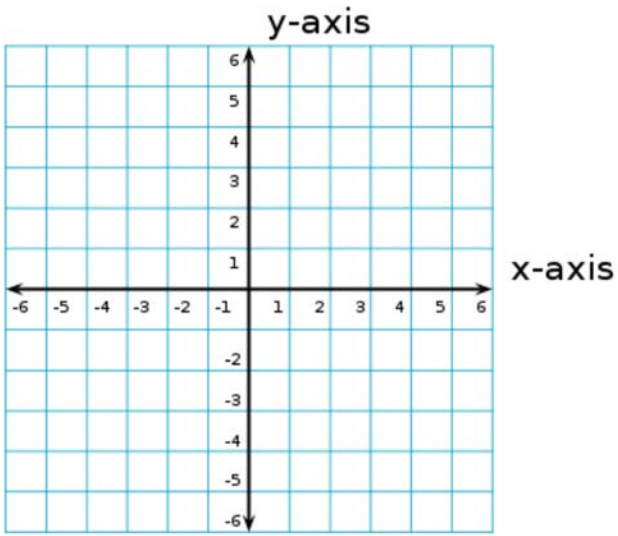
Algebra 1 CCSS
Inverse Functions

Example: Find the inverse of each function and determine whether or not the inverse is a function. $f(x) = x + 3$

STEP 1: Rewrite the function with y =
(if necessary)
STEP 2: Switch x and y in the equation
STEP 3: Solve for y
STEP 4: Replace y with the notation for an
inverse function (if necessary)

Find the inverse for $f(x) = x + 3$

Graph the function $f(x) = x + 3$ and also graph the inverse which is…_____



What do you notice about the graphs?_____

_____

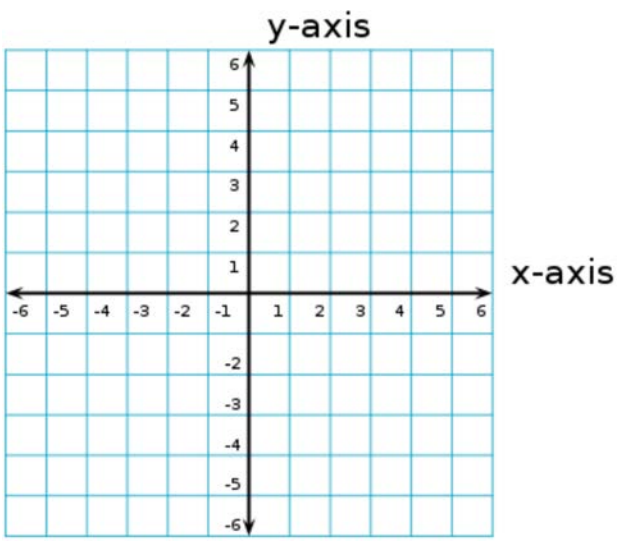
_____

Write the ordered pairs for the function $f(x) = x + 3$.

Write the ordered pairs for the inverse functions _____

Question 1: Find the inverse for $f(x) = -2x^3 + 1$

Graph the function $f(x) = -2x^3 + 1$ and also graph the inverse which is…_____



What do you notice about the graphs?_____

_____

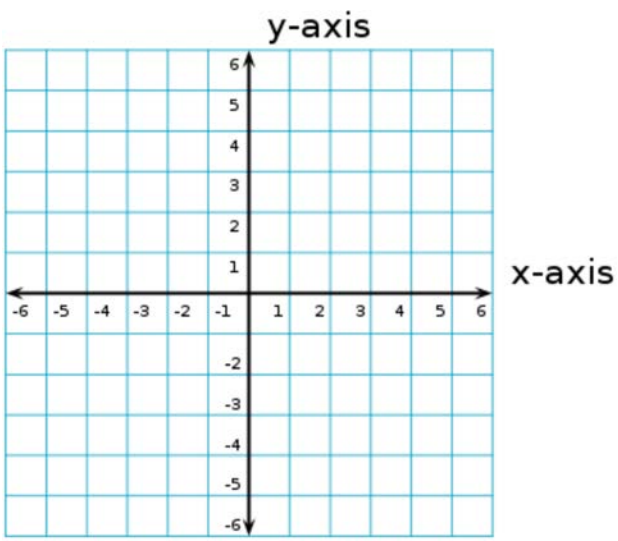
_____

Write the ordered pairs for the function $f(x) = -2x^3 + 1$.

Write the ordered pairs for the inverse functions _____

Question 2: Find the inverse for $f(x) = \frac{1}{x-1}$

Graph the function $f(x) = \frac{1}{x-1}$ and also graph the inverse which is…_____



What do you notice about the graphs?_____

_____

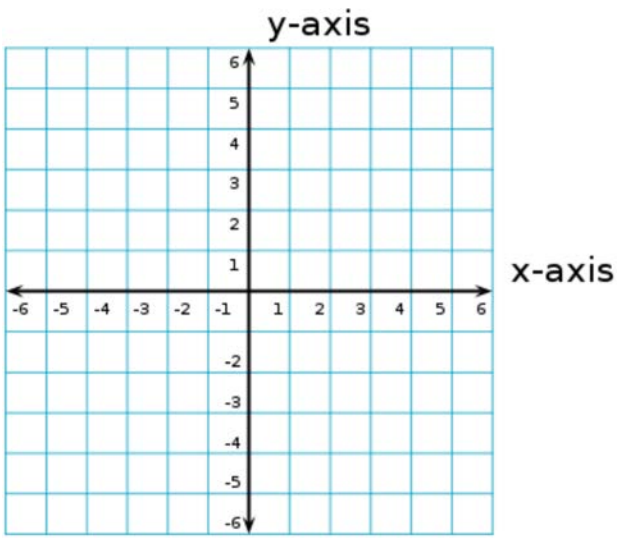
_____

Write the ordered pairs for the function $f(x) = \frac{1}{x-1}$

Write the ordered pairs for the inverse functions _____

Question 3: Find the inverse for $f(x) = x^2 - 2$

Graph the function $f(x) = x^2 - 2$ and also graph the inverse which is…_____



What do you notice about the graphs?_____

_____

_____

Write the ordered pairs for the function $f(x) = x^2 - 2$

Write the ordered pairs for the inverse functions _____

Unit 5 – Quadratic Functions and Modeling

Domain: Functions (F)
Essential Question: How do quadratic functions help model and solve problems?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Interpret, analyze and compare linear and quadratic models using varied representations.** **F-LE(***)** Construct and compare linear, quadratic, and exponential models and solve problems.	**How…** **F.LE.3:** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.	**Students will:** **Function Growth Comparison (Graphing Calculator)** F.LE. 3 Practice PT exponential vs linear. F.LE. 3 Practice PT quadratic.docx	**F.LE.3:** Example: Contrast the growth of the $f(x)=x^3$ and $f(x)=3^x$.

Vocabulary: axis of symmetry, end behavior, extreme values, intercepts, intervals, maximum/minimum values, polynomials, quadratic expression, quadratic functions, relative maximums and minimums, symmetries, zeros

Name:_____Date:_____ Period:_____

Algebra 1 CP
Practice Performance Task Unit 5

Car Trouble:

The tax collector in Manchester is responsible for evaluating the value of vehicles in town to determine how much in taxes the owner should pay. Mrs. Dias purchased a new car and received her tax bill. The bill reduced the value of her car at a flat rate each year. However, Mrs. Dias knows that cars depreciate in value more rapidly immediately after they are purchased and then level off, typically at a constant rate each year. She wants to appeal the value of the car to bring down her tax bill.

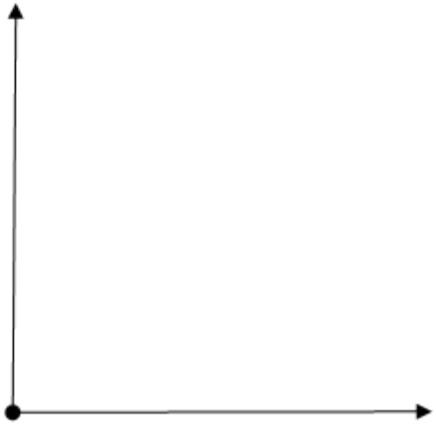
The car was purchased for $32,000.

1) Create a table of values comparing the two different scenarios of the value of the car:

Year	Tax Collector	Mrs. Dias
Staring Value	$32,000	$32,000
End of year 1	$30,750	$30,080
End of year 2	$29,500	$28,275
End of year 3	$28,250	$26,578
End of year 4	$27,000	$24,983
End of year 5	$25,750	$23,484

2) Using the graphing calculator, graph each table of values for the domain 1 to 10. Sketch the graph you see on the calculator below – be sure to label your axes:

X Min	
X Max	
X Scale	
Y Min	
Y Max	
Y Scale	



3) Describe the pattern of the function the tax collector is using:

_____

_____

_____

4) Describe the pattern of the function Mrs. Dias is using:

_____

_____

_____

5) After 10 years, what would each scenario determine the value to car to be.

Tax Collector: _____

Mrs. Dias': _____

6) Which scenario is more beneficial for the tax **payer** and why?

_____

_____

_____

_____

_____

Name:_____Date:_____ Period:_____

Algebra 1 CP
Practice Performance Task Unit 5

Rocket Science:

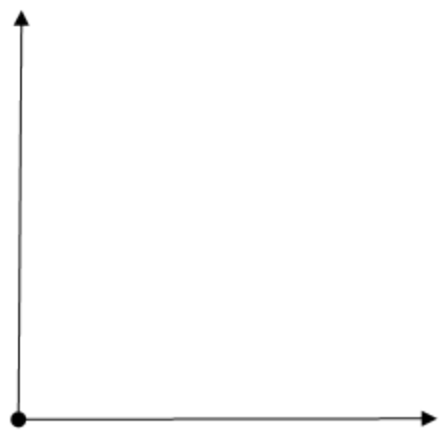
In physics class students were required to designed rockets. Two students have decided to test their rockets. Dan shoots his rocket from a platform that is 10 feet off the ground, the rocket is shot off at a velocity of 45 feet per second. Johanna shoots a rocket off the ground at a velocity of 55 feet per second.

1) Create a function that represents the path of each student's rocket. Remember to use the formula for the path of a traveling object: $h(t) = -16t^2 + v_0 t + s_0$

   Dan's rocket: _____

   Johanna's rocket: _____

2) Using the graphing calculator, graph each function for the domain 1 to 5. Sketch the graph you see on the calculator below – be sure to label your axes:



X Min	
X Max	
X Scale	
Y Min	
Y Max	
Y Scale	

3) Describe the path of Dan's rocket. Be sure to include the maximum height his rocket achieves and how long the rocket is in the air.

4) Describe the path of Johanna's rocket. Be sure to include the maximum height her rocket achieves and how long the rocket is in the air.

5) Can you find and use both roots for the equations of the rockets?

6) If the students are located in the same place at the launch point, can they launch both rockets off at the same time? Why or why not?

MHS Algebra 1 Mathematics Curriculum

Unit 5 Addendum

Performance Standard	Instructional Strategies
N.RN.3:	Since every difference is a sum and every quotient is a product, this includes differences and quotients as well. Explaining why the four operations on rational numbers produce rational numbers can be a review of students understanding of fractions and negative numbers. Explaining why the sum of a rational and an irrational number is irrational, or why the product is irrational, includes reasoning about the inverse relationship between addition and subtraction (or between multiplication and addition). Example: Explain why the number 2π must be irrational, given that π is irrational. Answer: if 2π were rational, then half of 2π would also be rational, so π would have to be rational as well.
F.IF.4:	Students may be given graphs to interpret or produce graphs given an expression or table for the function, by hand or using technology. Examples: A rocket is launched from 180 feet above the ground at time $t = 0$. The function that models this situation is given by $h = -16t^2 + 96t + 180$, where t is measured in seconds and h is height above the ground measured in feet. • What is a reasonable domain restriction for t in this context? • Determine the height of the rocket two seconds after it was launched. • Determine the maximum height obtained by the rocket. • Determine the time when the rocket is 100 feet above the ground. • Determine the time at which the rocket hits the ground. • How would you refine your answer to the first question based on your response to the second and fifth questions? Compare the graphs of $y = 3x^2$ and $y = 3x^3$. Let $R(x) = \frac{2}{\sqrt{x-2}}$. Find the domain of $R(x)$. Also find the range, zeros, and asymptotes of $R(x)$. Let $f(x) = 5x^3 - x^2 - 5x + 1$. Graph the function and identify end behavior and any intervals of constancy, increase, and decrease. It started raining lightly at 5am, then the rainfall became heavier at 7am. By 10am the storm was over, with a total rainfall of 3 inches. It didn't rain for the rest of the day. Sketch a possible graph for the number of inches of rain as a function of time, from midnight to midday.

MHS Algebra 1 Mathematics Curriculum

Unit 5 Addendum

<table>
<tr><th>Performance Standard</th><th>Instructional Strategies</th></tr>
<tr><td>

F.IF.6:

</td><td>

The average rate of change of a function $y = f(x)$ over an interval [a,b] is $\frac{4y}{4x} = \frac{f(b)-f(a)}{b-a}$. In addition to finding average rates of change from functions given symbolically, graphically, or in a table, Students may collect data from experiments or simulations (ex. falling ball, velocity of a car, etc.) and find average rates of change for the function modeling the situation.

Examples:

- Use the following table to find the average rate of change of *g* over the intervals [-2, -1] and [0,2]:

<table>
<tr><th>x</th><th>g(x)</th></tr>
<tr><td>-2</td><td>2</td></tr>
<tr><td>-1</td><td>-1</td></tr>
<tr><td>0</td><td>-4</td></tr>
<tr><td>2</td><td>-10</td></tr>
</table>

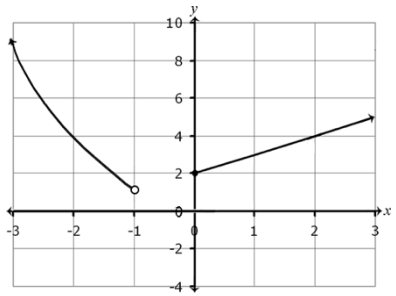
- The table below shows the elapsed time when two different cars pass a 10, 20, 30, 40 and 50 meter mark on a test track.
- For car 1, what is the average velocity (change in distance divided by change in time) between the 0 and 10 meter mark? Between the 0 and 50 meter mark? Between the 20 and 30 meter mark? Analyze the data to describe the motion of car 1.
- How does the velocity of car 1 compare to that of car 2?

<table>
<tr><th></th><th>Car 1</th><th>Car 2</th></tr>
<tr><th>d</th><th>t</th><th>t</th></tr>
<tr><td>10</td><td>4.472</td><td>1.742</td></tr>
<tr><td>20</td><td>6.325</td><td>2.899</td></tr>
<tr><td>30</td><td>7.746</td><td>3.831</td></tr>
<tr><td>40</td><td>8.944</td><td>4.633</td></tr>
<tr><td>50</td><td>10</td><td>5.348</td></tr>
</table>

</td></tr>
</table>

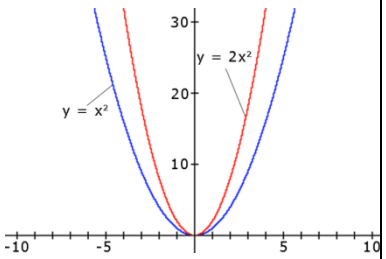
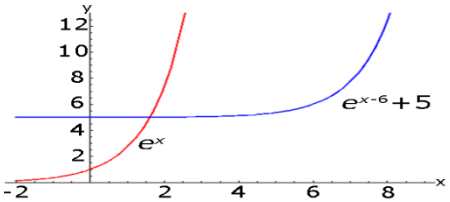
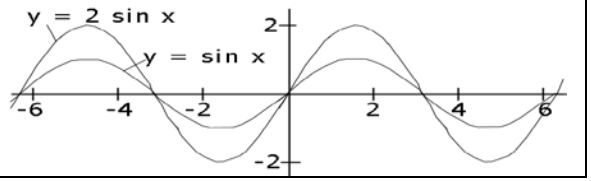
MHS Algebra 1 Mathematics Curriculum

Unit 5 Addendum

Performance Standard	Instructional Strategies
F.IF.7:	Key characteristics include but are not limited to maxima, minima, intercepts, symmetry, end behavior, and asymptotes. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to graph functions. Examples: • Describe key characteristics of the graph of $f(x) = \lvert x - 3 \rvert + 5$. • Sketch the graph and identify the key characteristics of the function described below. $F(x) = \begin{cases} x + 2 \text{ for } x \geq 0 \\ -x^2 \text{ for } x < -1 \end{cases}$  • Graph the function $f(x) = 2^x$ by creating a table of values. Identify the key characteristics of the graph. • Graph $f(x) = 2 \tan x - 1$. Describe its domain, range, intercepts, and asymptotes. Draw the graph of $f(x) = \sin x$ and $f(x) = \cos x$. What are the similarities and differences between the two graphs?
F.BF.1:	Students will analyze a given problem to determine the function expressed by identifying patterns in the function's rate of change. They will specify intervals of increase, decrease, constancy, and, if possible, relate them to the function's description in words or graphically. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model functions. Examples: • You buy a $10,000 car with an annual interest rate of 6 percent compounded annually and make monthly payments of $250. Express the amount remaining to be paid off as a function of the number of months, using a recursion equation. • A cup of coffee is initially at a temperature of 93º F. The difference between its temperature and the room temperature of 68º F decreases by 9% each minute. Write a function describing the temperature of the coffee as a function of time. • The radius of a circular oil slick after t hours is given in feet by $r = 10t^2 - 0.5t$, for $0 \leq t \leq 10$. Find the area of the oil slick as a function of time.

MHS Algebra 1 Mathematics Curriculum

Unit 5 Addendum

Performance Standard	Instructional Strategies
F.BF.3:	Students will apply transformations to functions and recognize functions as even and odd. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to graph functions. Examples: • Is $f(x) = x^3 - 3x^2 + 2x + 1$ even, odd, or neither? Explain your answer orally or in written format. • Compare the shape and position of the graphs of $f(x) = x^2$ and $g(x) = 2x^2$, and explain the differences in terms of the algebraic expressions for the functions  • Describe effect of varying the parameters *a*, *h*, and *k* have on the shape and position of the graph of $f(x) = a(x-h)^2 + k$. • Compare the shape and position of the graphs of $f(x) = e^x$ to $g(x) = e^{x-6} + 5$, and explain the differences, orally or in written format, in terms of the algebraic expressions for the functions  • Describe the effect of varying the parameters *a*, *h*, and *k* on the shape and position of the graph $f(x) = ab^{(x+h)} + k$., orally or in written format. What effect do values between 0 and 1 have? What effect do negative values have? • Compare the shape and position of the graphs of $y = \sin x$ to $y = 2 \sin x$. 

Geometry Mathematics Curriculum

Manchester High School
Common Core State Standards
Mathematics Curriculum

Description of Standards

The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in fourth credit courses or advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+). All standards without a (+) symbol should be in the common mathematics curriculum for all college and career ready students. Standards with a (+) symbol may also appear in courses intended for all students.

The high school standards are listed in conceptual categories including Number and Quantity, Algebra, Functions, Modeling, Geometry, and Statistics and Probability, and Contemporary Mathematics.
Conceptual categories portray a coherent view of high school mathematics; a student's work with functions, for example, crosses a number of traditional course boundaries, potentially up through and including calculus. Modeling is best interpreted not as a collection of isolated topics but in relation to other standards.

Number and Quantity • The Real Number System (N-RN) • Quantities (N-Q) • The Complex Number System (N-CN) • Vector and Matrix Quantities (N-VM) **Algebra** • Seeing Structure in Expressions (A-SSE) • Arithmetic with Polynomials and Rational Expressions (A-APR) • Creating Equations (A-CED) • Reasoning with Equations and Inequalities (A-REI) **Functions** • Interpreting Functions (F-IF) • Building Functions (F-BF) • Linear, Quadratic, and Exponential Models (F-LE) • Trigonometric Functions (F-TF)	**Geometry** • Congruence (G-CO) • Similarity, Right Triangles, and Trigonometry (G-SRT) • Circles (G-C) • Expressing Geometric Properties with Equations (G-GPE) • Geometric Measurement and Dimension (G-GMD) • Modeling with Geometry (G-MG) **Modeling** **Statistics and Probability** • Interpreting Categorical and Quantitative Data (S-ID) • Making Inferences and Justifying Conclusions (S-IC) • Conditional Probability and the Rules of Probability (S-CP) • Using Probability to Make Decisions (S-MD) **Contemporary Mathematics** • Discrete Mathematics (CM-DM)

NEASC CURRICULUM RESPONSIBILITY CHECKLIST

	ART	BUSINESS & FINANCE	CAREER SERVICES	ENGLISH	FAMILY & CONSUMER SCIENCE	GUIDANCE	LIBRARY	HF AI TH	MATHEMATICS	PERFORMING ARTS	PHYSICAL EDUCATION	SCIENCE	SOCIAL STUDIES	TECHNOLOGY	WORLD LANGUAGES
Academic Expectations All students will:															
1. Reads and understand literary, informational, and persuasive texts in order to analyze, interpret, and evaluate them.				1	1			1	2			2	1		2
2. produce written materials which develop thoughts, share information, influence, persuade, and entertain.				1	2			1	2			2	1	2	1
3. comprehend verbal and nonverbal presentations at the literal, inferential and evaluative levels, and speak using appropriate conventions, forms of expression, and tools.	2			1	1		2	2	2		1		2	2	1
4. comprehend and demonstrate number sense, probability and statistics, geometry, and algebra.				2	2				1			2		2	
5. understand and apply basic concepts, principles, and theories of biology and physical science, and their interrelationships.				2	1			1			1	1		2	
6. identify and apply appropriate technologies to conduct research, communicate information and ideas, produce original works, organize data, and solve problems.	2	1		1	2		1		2			2	2	1	2
7. demonstrate abstract thinking, curiosity, imagination, and creativity in the arts or through academic problem solving.	1	2		1	1				2	1	1	2	2	2	

NEASC CURRICULUM RESPONSIBILITY CHECKLIST

	ART	BUSINESS & FINANCE	CAREER SERVICES	ENGLISH	FAMILY & CONSUMER SCIENCE	GUIDANCE	HF AI TH	LIBRARY	MATHEMATICS	PERFORMING ARTS	PHYSICAL EDUCATION	SCIENCE	SOCIAL STUDIES	TECHNOLOGY	WORLD LANGUAGES
Civic and Social Expectations All students will:															
1. treat others with respect regardless of race, ethnicity, class, or orientation.	1	1				2	1		2	2	1		1		
2. demonstrate ethical behavior through honesty and integrity.	1	1				2	1	2	2	2	1		1		
3. be responsible for their words, actions, and commitments.	1	1				2	1		2	2	1		1		
4. contribute positively to the community.	1	1				1	2			1	2		2		
5. contribute to the school by participating in co-curricular or service activities.	2	2				1	2			1	1		2		

MHS Math Department
Curriculum Syllabus
Common Core State Standards 2012
Geometry

MONTH	CURRICULUM TOPICS
September (Quarter 1) **Unit 1** **Congruence, Proof, and Constructions**	• **Experiment with transformations in the plane -** (G.CO.1, G.CO.2, G.CO.3, G.CO.4, G.CO.5) • **Understand congruence in terms of rigid motions -** (G.CO.6, G.CO.7, G.CO.8)
October	• **Prove geometric theorems -** (G.CO.9, G.CO.10, G.CO.11) • **Make geometric constructions -** (G.CO.12, G.CO.13)
November (Quarter 2) **Unit 2** **Similarity, Proof and Trigonometry**	• **Understand similarity in terms of similarity transformations -** (G.SRT.1, G.SRT.2, G.SRT.3) • **Prove theorems involving similarity -** (G.SRT.4, G.SRT.5)
December	• **Define trigonometric ratios and solve problems involving right triangles -** (G.SRT.6, G.SRT.7, G.SRT.8)
January	• **Apply geometric concepts in modeling situations -** (G.MG.1, G.MG.2, G.MG.3) • **Apply trigonometry to general triangles -** (G.SRT.9(+), G.SRT.10(+), G.SRT.11(+))
February (Quarter 3) **Unit 5** **Circles With and Without Coordinates**	• **Understand and apply theorems about circles -** (G.C.1, G.C.2, G.C.3, G.C.4(+)) • **Find arc lengths and areas of sectors of circles -** (G.C.5) • **Translate between the geometric description and the equation for a conic section -** (G.GPE.1) • **Use coordinates to triple simple geometric theorems algebraically -** (G.GPE.4) • **Apply geometric concepts in modeling situations -** (G.MG.1)

MHS Math Department
Curriculum Syllabus
Common Core State Standards 2012
Geometry

MONTH	CURRICULUM TOPICS
March **Unit 3** **Extending to Three Dimensions**	• **Explain volume formulas and use them to solve problems -** (G.GMD.1, G.GMD.3) • **Visualize the relation between two-dimensional and three-dimensional objects -** (G.GMD.4) • **Apply geometric concepts in modeling situations -** (G.MG.1)
April (Quarter 4) **Unit 4** **Connecting Algebra and Geometry Through Coordinates**	• **Use coordinates to prove simple geometric theorems algebraically -** (G.GPE.4, G.GPE.5, G.GPE.6, G.GPE.7)
May	• **Translate between the geometric description and the equation for a conic section -** (G.GPE.2)
June **Unit 6** **Applications of Probability**	• **Understand independence and conditional probability and use them to interpret data -** (S.CP.1, S.CP.2, S.CP.3, S.CP.4, S.CP.5) • **Use the rules of probability to compute probabilities of compound events in a uniform probability model -** (S.CP.6, S.CP.7, S.CP.8(+), S.CP.9(+)) • **Use probability to evaluate outcomes of decisions -** (S.MD.6(+), S.MD.7(+))

Unit 1 – Congruence, Proof, and Constructions

Domain: Congruence (CO)

Essential Question: How do properties of congruence help define and prove geometric relationships?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Analyze geometric theorems through transformation, construction, and proof.** **G-CO(**)** Experiment with transformations in the plane	**How…** **G.CO.1:** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. **G.CO.2:** Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). **G.CO.3:** Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.	**Students will:** **Construction Activity**: ▪ Construct various geometric figures based on properties and descriptions **Transformations Worksheet**: ▪ Construct shapes on a coordinate plane then transformation them based on based on given instructions. **Transformation Exit Check:** ▪ Describe the transformation of a figure based on a given diagram or process.	**G.CO.1:** Students may use geometry software and/or manipulatives to model and compare transformations. **G.CO.2:** Students may use geometry software and/or manipulatives to model and compare transformations. **G.CO.3:** Students may use geometry software and/or manipulatives to model transformations.

Vocabulary: angle, bisector, circle, diagonals, line segment, parallel line, perpendicular line, reflections, rigid motion, rotations, similarity, transformations, translations

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Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Analyze geometric theorems through transformation, construction, and proof.** **G-CO(**)** Experiment with transformations in the plane **G-CO(**)** Understand congruence in terms of rigid motions.	**How…** **G.CO.4:** Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. **G.CO.5:** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. **G.CO.6:** Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.	**Students will:** **Movement Group Work:** ▪ Evaluate the manner in which an object was moved on a coordinate plane and identify the correct model for the motion. Justify the model based on the properties of the shape. **Translation Exit Check:** ▪ Analyze a figure and determine the sequence of transformations needed to move the figure to a new location. **Discovery of Congruence Activity:** ▪ Determine congruence of shapes based on mapping and aligning one to another through rigid motion.	**G.CO.4:** Students may use geometry software and/or manipulatives to model transformations. Students may observe patterns and develop definitions of rotations, reflections, and translations. **G.CO.5:** Students may use geometry software and/or manipulatives to model transformations and demonstrate a sequence of transformations that will carry a given figure onto another. **G.CO.6:** A rigid motion is a transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are assumed to preserve distances and angle measures.

Vocabulary: angle, bisector, circle, diagonals, line segment, parallel line, perpendicular line, reflections, rigid motion, rotations, similarity, transformations, translations

Unit 1 – Congruence, Proof, and Constructions

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Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Analyze geometric theorems through transformation, construction, and proof.** **G-CO(**)** Understand congruence in terms of rigid motions. **G-CO(**)** Prove geometric theorems	**How…** **G.CO.7:** Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. **G.CO.8:** Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motion. **G.CO.9:** Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.	**Students will:** **We Do Worksheet:** Unit Lessons\We Do Task Geometry Unit 1 G.CO.7.doc **Lines and Angles Exit Check:** ▪ Verify congruent relationships of angles formed by lines using transformations.	**G.CO.7:** A rigid motion is a transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Two triangles are said to be congruent if one can be exactly superimposed on the other by a rigid motion. The congruence theorems specify the conditions under which this can occur. **G.CO.8:** Students may use geometric simulations (computer software or graphing calculator) to explore theorems about lines and angles. **G.CO.9:** Students may use geometric simulations (computer software or graphing calculator) to explore theorems about triangles.

Vocabulary: angle, bisector, circle, diagonals, line segment, parallel line, perpendicular line, reflections, rigid motion, rotations, similarity, transformations, translations

Unit 1 – Congruence, Proof, and Constructions

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6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Analyze geometric theorems through transformation, construction, and proof.** **G-CO(**)** Prove geometric theorems	**How…** **G.CO.10:** Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.	**Students will:** **Properties of Triangles Activity:** ▪ Construct triangles, measure the interior angles and construct mid-segments to evaluate the relationship between segments, sides and angles of a triangle.	**G.CO.10:** Students may use geometric simulations (computer software or graphing calculator) to explore theorems about triangles.
	G.CO.11: Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.	**Properties of Parallelograms Worksheet:** ▪ Evaluate the properties of a parallelogram based on understanding of congruent triangles.	**G.CO.11:** Students may use geometric simulations (computer software or graphing calculator) to explore theorems about parallelograms.

Vocabulary: angle, bisector, circle, diagonals, line segment, parallel line, perpendicular line, reflections, rigid motion, rotations, similarity, transformations, translations

Unit 1 – Congruence, Proof, and Constructions

Domain: Congruence (CO)

Essential Question: How do properties of congruence help define and prove geometric relationships?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Analyze geometric theorems through transformation, construction, and proof.** **G-CO(*)** Make geometric constructions	**How…** **G.CO.12:** Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line. **G.CO.13:** Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.	**Students will:** **Match Me Activity:** ▪ Construct a copy of a series of shapes using multiple methods when given figures or descriptions of figures. **What's Inside? Worksheet:** ▪ Inscribe regular polygons inside a circle through construction.	**G.CO.12:** Students may use geometric software to make geometric constructions. Examples: • Construct a triangle given the lengths of two sides and the measure of the angle between the two sides. • Construct the circumcenter of a given triangle. **G.CO.13:** Students may use geometric software to make geometric constructions.

Vocabulary: angle, bisector, circle, diagonals, line segment, parallel line, perpendicular line, reflections, rigid motion, rotations, similarity, transformations, translations

Unit 2 – Similarity, Proof and Trigonometry

Domain: Similarity, Right Triangles, and Trigonometry (SRT)
Essential Question: How do properties of similarity help define and prove right triangle relationships?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Develop and apply triangle relationships using geometric properties and laws.** **G-SRT(**)** Understand similarity in terms of similarity transformations	**How…** **G.SRT.1:** Verify experimentally the properties of dilations given by a center and a scale factor: a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor. **G.SRT.2:** Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. **G.SRT.3:** Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.	**Students will:** **Dilation Experiment:** ▪ Perform a dilation given a scale factor and a shape. **What's the Scale? Activity:** ▪ Verify similarity through dilation and confirmation of corresponding congruent angles and proportional sides given two shapes that appear similar.	**G.SRT.1:** A dilation is a transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor. Students may use geometric simulation software to model transformations. Students may observe patterns and verify experimentally the properties of dilations. **G.SRT.2:** A similarity transformation is a rigid motion followed by dilation. Students may use geometric simulation software to model transformations and demonstrate a sequence of transformations to show congruence or similarity of figures.

Vocabulary: ASA, auxiliary line, congruence, corresponding sides, dilation, Laws of Sines and Cosines, Pythagorean Theorem, right triangles, similarity, SAS, SSS, trigonometric ratios, typographic grid systems

Unit 2 – Similarity, Proof and Trigonometry

Domain: Similarity, Right Triangles, and Trigonometry (SRT)
Essential Question: How do properties of similarity help define and prove right triangle relationships?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Develop and apply triangle relationships using geometric properties and laws.** **G-SRT(**)** Understand similarity in terms of similarity transformations	**How…** **G.SRT.4:** Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity. **G.SRT.5:** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.	**Students will:** **Prove It! Activity:** ▪ Evaluate properties of triangles based on transformations and congruent relationships. **Similarity Exit Check:** ▪ Analyze the similarity between two triangles to determine the postulate demonstrated.	**G.SRT.4:** Students may use geometric simulation software to model transformations and demonstrate a sequence of transformations to show congruence or similarity of figures. **G.SRT.5:** Similarity postulates include SSS, SAS, and AA. Congruence postulates include SSS, SAS, ASA, AAS, and H-L. Students may use geometric simulation software to model transformations and demonstrate a sequence of transformations to show congruence or similarity of figures.

Vocabulary: ASA, auxiliary line, congruence, corresponding sides, dilation, Laws of Sines and Cosines, Pythagorean Theorem, right triangles, similarity, SAS, SSS, trigonometric ratios, typographic grid systems.

Unit 2 – Similarity, Proof and Trigonometry

Domain: Similarity, Right Triangles, and Trigonometry (SRT)
Essential Question: How do properties of similarity help define and prove right triangle relationships?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Develop and apply triangle relationships using geometric properties and laws.** **G-SRT(**)** Define trigonometric ratios and solve problems involving right triangles	**How…** **G.SRT.6:** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. **G.SRT.7:** Explain and use the relationship between the sine and cosine of complementary angles. **G.SRT.8:** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.	**Students will:** **What's My Sine? Worksheet:** ▪ Compare sine and cosine values in acute angles right triangles to determine the relationship between the sides and angles. **Acute Angles Exit Check:** ▪ Compare acute angles and their trigonometric ratios in right triangles. **Measure that! Activity:** ▪ Apply trigonometric ratios to indirect measurement of large scale objects.	**G.SRT.6:** See Unit Addendum. **G.SRT.7:** Geometric the relationship between sine and simulation software, applets, and graphing calculators can be used to explore cosine. **G.SRT.8:** See Unit Addendum.

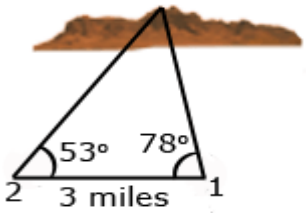
Vocabulary: ASA, auxiliary line, congruence, corresponding sides, dilation, Laws of Sines and Cosines, Pythagorean Theorem, right triangles, similarity, SAS, SSS, trigonometric ratios, typographic grid systems.

Unit 2 – Similarity, Proof and Trigonometry

Domain: <u>Similarity, Right Triangles, and Trigonometry</u> (SRT)
Essential Question: How do properties of similarity help define and prove right triangle relationships?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Develop and apply triangle relationships using geometric properties and laws.** **G-SRT(+)** Apply trigonometry to general triangles.	**How…** **G.SRT.9:** Derive the formula $A=\frac{1}{2}ab\sin C$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. **G.SRT.10:** Prove the Laws of Sines and Cosines and use them to solve problems. **G.SRT.11:** Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces)	**Students will:** **Exploration of Laws Activity:** ▪ Develop the law of Sines and the Law of Cosines through construction and measurement of non-right triangles. **Laws Applications Worksheet:** ▪ Evaluate unknown measurements in right and non-right triangles that model situations using the Law of Sines and the law of Cosines.	**G.SRT.11:** Example: Tara wants to fix the location of a mountain by taking measurements from two positions 3 miles apart. From the first position, the angle between the mountain and the second position is 78°. From the second position, the angle between the mountain and the first position is 53°. How can Tara determine the distance of the mountain from each position, and what is the distance from each position? 

Vocabulary: ASA, auxiliary line, congruence, corresponding sides, dilation, Laws of Sines and Cosines, Pythagorean Theorem, right triangles, similarity, SAS, SSS, trigonometric ratios, typographic grid systems.

Unit 2 – Similarity, Proof and Trigonometry

Domain: Modeling with Geometry (MG)

Essential Question: How do properties of similarity help define and prove right triangle relationships?

Mathematical Practices:

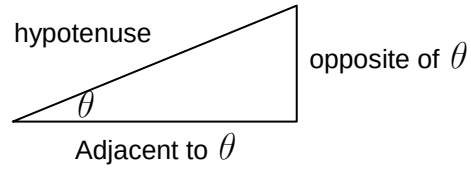
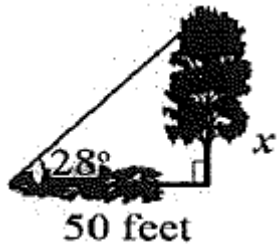
1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Develop and apply triangle relationships using geometric properties and laws.** **G-MG(***)** Apply geometric concepts in modeling situations	**How…** **G.MG.1:** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). **G.MG.2:** Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). **G.MG.3:** Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).	**Students will:** **Modeling Stations:** Describe the properties of real world objects in terms of geometric shapes; evaluate volume and area of real world scenarios; design an object to meet a set of geometric standards. **Modeling Exit Check:** Describe a manner in which a specific scenario can be modeled.	**G.MG.1:** Students may use simulation software and modeling software to explore which model best describes a set of data or situation. **G.MG.2:** Students may use simulation software and modeling software to explore which model best describes a set of data or situation. **G.MG.3:** Students may use simulation software and modeling software to explore which model best describes a set of data or situation.

Vocabulary: ASA, auxiliary line, congruence, corresponding sides, dilation, Laws of Sines and Cosines, Pythagorean Theorem, right triangles, similarity, SAS, SSS, trigonometric ratios, typographic grid systems.

MHS Geometry Mathematics Curriculum

Unit 2 Addendum

Performance Standard	Instructional Strategies
G.SRT.6:	Students may use applets to explore the range of values of the trigonometric ratios as θ ranges from 0 to 90 degrees.  $\text{sine of } \theta = \sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$ $\text{cosecant of } \theta = \csc\theta = \frac{\text{hypotenuse}}{\text{opposite}}$ $\text{cosine of } \theta = \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ $\text{secant of } \theta = \sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$ $\text{tangent of } \theta = \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$ $\text{cotangent of } \theta = \cot\theta = \frac{\text{adjacent}}{\text{opposite}}$
G.SRT.8:	Students may use graphing calculators or programs, tables, spreadsheets, or computer algebra systems to solve right triangle problems. Example: • Find the height of a tree to the nearest tenth if the angle of elevation of the sun is 28° and the shadow of the tree is 50 ft. 

Unit 3 – Extending to Three Dimensions

Domain: Geometric Measurement and Dimension (GMD)

Essential Question: How do two-dimensional cross sections help describe three-dimensional objects?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Evaluate and analyze relationships between two and three dimensional figures.** **G-GMD(*)** Explain volume formulas and use them to solve problems **G-GMD(***)** Visualize relationships between two-dimensional and three dimensional objects	**How…** **G.GMD.1:** Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.* **G.GMD.3:** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. **G.GMD.4:** Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	**Students will:** **Volume Discovery Activity:** Compare and contrast volume of given shapes to establish a relationship between the shapes. **Volume Scavenger Hunt:** Evaluate the volume of real world objects based on measurement then make conjectures about the changes to measurements if the volume is altered. **Cross Sections Worksheet:** Identify and evaluate the shape of the cross section of three-dimensional objects and the rotation of two-dimensional objects.	**G.GMD.1:** Cavalieri's principle is if two solids have the same height and the same cross-sectional area at every level, then they have the same volume. **G.GMD.3:** Missing measures can include but are not limited to slant height, altitude, height, diagonal of a prism, edge length, and radius. **G.GMD.4:** Students may use geometric simulation software to model figures and create cross sectional views. Example: ● Identify the shape of the vertical, horizontal, and other cross sections of a cylinder.

Vocabulary: Cavalieri's principle, circumference, cone, cylinder, limit, pyramid, three-dimensions, sphere

Unit 3 – Extending to Three Dimensions

Domain: Modeling with Geometry (MG)

Essential Question: How do properties of similarity help define and prove right triangle relationships?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Evaluate and analyze relationships between two and three dimensional figures.** **G-MG(***)** Apply geometric concepts in modeling situations	**How…** **G.MG.1:** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).	**Students will:** **Get Creative Project:** Design a model of real world objects using geometric shapes.	**G.MG.1:** Students may use simulation software and modeling software to explore which model best describes a set of data or situation.

Vocabulary: Cavalieri's principle, circumference, cone, cylinder, limit, pyramid, three-dimensions, sphere

Unit 4 – Connecting Algebra and Geometry Through Coordinates

Domain: <u>Expressing Geometric Properties with Equations</u> (GPE)
Essential Question: How does the coordinate system help to verify geometric relationships?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Interpret geometric relationships using algebraic reasoning.** **G-GPE(***)** Use coordinates to prove simple geometric theorems algebraically	**How…** **G.GPE.4:** Use coordinates to prove simple geometric theorems algebraically. *Prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).* **G.GPE.5:** Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	**Students will:** **Are You Sure? Worksheet:** ▪ Verify properties of figures using coordinate geometry and algebra. **What's My Line? Activity:** ▪ Construct parallel and perpendicular lines and verify their relationship using slope and the equations of those lines. **Is This Right? Exit Check:** ▪ Verify properties of right triangles using coordinate geometry and algebra.	**G.GPE.4:** Students may use geometric simulation software to model figures and prove simple geometric theorems. Example: Use slope and distance formula to verify the polygon formed by connecting the points (-3, -2), (5, 3), (9, 9), (1, 4) is a parallelogram. **G.GPE.5:** Lines can be horizontal, vertical, or neither. Students may use a variety of different methods to construct a parallel or perpendicular line to a given line and calculate the slopes to compare the relationships.

Vocabulary: angle, bisector, circle, conic section, diagonals, directed line segment, directrix, focus, line segment, parabola, parallel line, perpendicular line, reflections, rigid motion, rotations, similarity, transformations, translations

Unit 4 – Connecting Algebra and Geometry Through Coordinates

Domain: Expressing Geometric Properties with Equations (GPE)

Essential Question: How does the coordinate system help to verify geometric relationships?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Interpret geometric relationships using algebraic reasoning.** **G-GPE(***)** Use coordinates to prove simple geometric theorems algebraically	**How…** **G.GPE.6:** Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	**Students will:** **Proportional Reasoning Exit Check:** ▪ Construct a proportionally divided line segment from given information.	**G.GPE.6:** Students may use geometric simulation software to model figures or line segments. **Example:** Given A(3, 2) and B(6, 11), find the point that divides the line segment AB two-thirds of the way from A to B. The point two-thirds of the way from A to B has *x*-coordinate two-thirds of the way from 3 to 6 and *y* coordinate two-thirds of the way from 2 to 11. So, (5, 8) is the point that is two-thirds from point A to point B
	G.GPE.7: Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.	**Coordinate Geometry Worksheet:** ▪ Evaluate area and perimeter of shapes on a coordinate plane using formulas and algebra.	**G.GPE.7:** Students may use geometric simulation software to model figures.

Vocabulary: angle, bisector, circle, conic section, diagonals, directed line segment, directrix, focus, line segment, parabola, parallel line, perpendicular line, reflections, rigid motion, rotations, similarity, transformations, translations

Unit 4 – Connecting Algebra and Geometry Through Coordinates

Domain: <u>Expressing Geometric Properties with Equations</u> (GPE)
Essential Question: How does the coordinate system help to verify geometric relationships?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Interpret geometric relationships using algebraic reasoning.** **G-GPE(***)** Translate between the geometric description and the equation for a conic section	**How…** **G.GPE.2:** Derive the equation of a parabola given a focus and directrix.	**Students will:** **Parabola Practice Worksheet:** ▪ Create the equations of a parabola based on given information and diagram.	**G.GPE.2:** Students may use geometric simulation software to explore parabolas. Examples: ● Write and graph an equation for a parabola with focus (2, 3) and directrix $y = 1$.

Vocabulary: angle, bisector, circle, conic section, diagonals, directed line segment, directrix, focus, line segment, parabola, parallel line, perpendicular line, reflections, rigid motion, rotations, similarity, transformations, translations

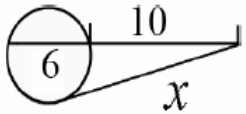
Unit 5 – Circles with and without Coordinates

Domain: Circles (C)

Essential Question: How do geometric relationships help to represent the properties of circles and conics?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Evaluate and model situations involving circles using geometric theorems and coordinate algebra.** **G-C(*)** Understand and apply theorems about circles	**How…** **G.C.1:** Prove that all circles are similar. **G.C.2:** Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*	**Students will:** **Circles Are The Same Exit Check:** ▪ Compare and contrast the properties of similar circles. **Circle's Do Have Angles Worksheet:** ▪ Classify and describe relationships between angles and lines in circles.	**G.C.1:** Students may use geometric simulation software to model transformations and demonstrate a sequence of transformations to show congruence or similarity of figures. **G.C.2:** Examples: Given the circle below with radius of 10 and chord length of 12, find the distance from the chord to the center of the circle. Find the unknown length in the picture below. 

Vocabulary: arc, area of sector, central angle, circumscribed angle, diameter, inscribed angle, inscribed chord, inscribed radius, radian measure

Unit 5 – Circles with and without Coordinates

Domain: Circles (C)

Essential Question: How do geometric relationships help to represent the properties of circles and conics?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Evaluate and model situations involving circles using geometric theorems and coordinate algebra.** **G-C(*)** Understand and apply theorems about circles **G-C(**)** Find arc lengths and areas of sectors of circles	**How…** **G.C.3:** Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. **G.C.4:** Construct a tangent line from a point outside a given circle to the circle. **G.C.5:** Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.	**Students will:** **Inscribe This Activity:** ▪ Construct inscribed polygons and analyze the angle relationships that exist. **Tangent Exit Check:** ▪ Construct a tangent line from a point to a given circle and analyze the segment and angle relationships that exist. **How Much Pizza? Worksheet:** ▪ Evaluate the relationship between the central angle, radius and the area of a circle to develop the formula for the area of a sector.	**G.C.3:** Students may use geometric simulation software to make geometric constructions. **G.C.4:** Students may use geometric simulation software to make geometric constructions. **G.C.5:** Students can use geometric simulation software to explore angle and radian measures and derive the formula for the area of a sector.

Vocabulary: arc, area of sector, central angle, circumscribed angle, diameter, inscribed angle, inscribed chord, inscribed radius, radian measure

Unit 5 – Circles with and without Coordinates

Domain: Circles (C)

Essential Question: How do geometric relationships help to represent the properties of circles and conics?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Evaluate and model situations involving circles using geometric theorems and coordinate algebra.** **G-GPE(***)** Translate between the geometric description and the equation for a conic section	**How…** **G.GPE.1:** Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	**Students will:** **Circle Practice Worksheet:** ▪ Derive the equation of a circle from given information and diagrams and construct a circle from a given equation.	**G.GPE.1:** Students may use geometric simulation software to explore the connection between circles and the Pythagorean Theorem. Examples: ● Write an equation for a circle with a radius of 2 units and center at (1, 3). ● Write an equation for a circle given that the endpoints of the diameter are (-2, 7) and (4, -8). Find the center and radius of the circle $4x^2 + 4y^2 - 4x + 2y - 1 = 0$.

Vocabulary: arc, area of sector, central angle, circumscribed angle, diameter, inscribed angle, inscribed chord, inscribed radius, radian measure

Unit 5 – Circles with and without Coordinates

Domain: Modeling with Geometric (MG)

Essential Question: How do geometric relationships help to represent the properties of circles and conics?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Evaluate and model situations involving circles using geometric theorems and coordinate algebra.** **G-GPE(***)** Use coordinates to prove simple geometric theorems algebraically **G-MG(***)** Apply geometric concepts in modeling situations	**How…** **G.GPE.4:** Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).* **G.MG.1:** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).	**Students will:** **How Can You Be Sure? Activity:** ▪ Verify shapes in a coordinate plane using slope and distance formula. **Get Creative Project:** ▪ Design a model of real world objects using geometric shapes and their properties.	**G.GPE.4:** Students may use geometric simulation software to model figures and prove simple geometric theorems. Example: Use slope and distance formula to verify the polygon formed by connecting the points (-3, -2), (5, 3), (9, 9), (1, 4) is a parallelogram. **G.MG.1:** Students may use simulation software and modeling software to explore which model best describes a set of data or situation.

Vocabulary: arc, area of sector, central angle, circumscribed angle, diameter, inscribed angle, inscribed chord, inscribed radius, radian measure

Unit 6 – Applications of Probability

Domain: Statistics

Essential Question: How does probability help to develop informed decisions?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Evaluate probability models using algebraic and geometric reasoning.** **S-CP(*)** Understand independence and conditional probability and use them to interpret data.	**How…** **S.CP.1:** Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events. **S.CP.2:** Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.	**Students will:** **What's Common? Worksheet:** ▪ Compare shapes to determine what is shared and not shared to establish the sample space, union and intersection of the shapes. **Probability Rules Exit Check:** ▪ Evaluate events and determine if they are independent.	**S.CP.1:** See Unit Addendum.

Vocabulary: combinations, complements, conditional probability, complements, events, independent events, intersections, permutations, sample space, subsets, unions

Unit 6 – Applications of Probability

Domain: Statistics

Essential Question: How does probability help to develop informed decisions?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Evaluate probability models using algebraic and geometric reasoning.** **S-CP(*)** Understand independence and conditional probability and use them to interpret data.	**How…** **S.CP.3:** Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*. **S.CP.4:** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Compare the results.*	**Students will:** **Probability Models Stations:** ▪ Evaluate conditional probability scenarios and model them appropriately given the construct of the scenario. **Conditional Probability Exit Check:** ▪ Assess the probability of a specified event given a series of conditions.	**S.CP.4:** Students may use spreadsheets, graphing calculators, and simulations to create frequency tables and conduct analyses to determine if events are independent or determine approximate conditional probabilities.

Vocabulary: combinations, complements, conditional probability, complements, events, independent events, intersections, permutations, sample space, subsets, unions

Unit 6 – Applications of Probability

Domain: Statistics

Essential Question: How does probability help to develop informed decisions?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Evaluate probability models using algebraic and geometric reasoning.** **S-CP(*)** Understand independence and conditional probability and use them to interpret data.	**How…** **S.CP.5:** Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*	**Students will:** **Conditional Probability Exit Check:** ▪ Evaluate a conditional probability scenario to determine the outcomes and establish independence of the events.	**S.CP.5:** Examples: • What is the probability of drawing a heart from a standard deck of cards on a second draw, given that a heart was drawn on the first draw and not replaced? Are these events independent or dependent? • At Johnson Middle School, the probability that a student takes computer science and French is 0.062. The probability that a student takes computer science is 0.43. What is the probability that a student takes French given that the student is taking computer science?

Vocabulary: combinations, complements, conditional probability, complements, events, independent events, intersections, permutations, sample space, subsets, unions

Unit 6 – Applications of Probability

Domain: Statistics

Essential Question: How does probability help to develop informed decisions?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Evaluate probability models using algebraic and geometric reasoning.** **S-CP** Use the rules of probability to compute probabilities of compound events in a uniform probability model.	**How…** **S.CP.6:** Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model. **S.CP.7:** Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model. **S.CP.8:** Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B\|A) = P(B)P(A\|B)$, and interpret the answer in terms of the model. **S.CP.9:** Use permutations and combinations to compute probabilities of compound events and solve problems.	**Students will:** **Interpreting Probability Activity:** ▪ Interpret the outcome of probability scenarios in terms of the model given using various representations. **How Many Ways Worksheet:** ▪ Evaluate the number of possible outcomes and combinations to solve real world problems.	**S.CP.6:** Students could use graphing calculators, simulations, or applets to model probability experiments and interpret the outcomes. **S.CP.7:** See Unit Addendum. **S.CP.8:** Students could use graphing calculators, simulations, or applets to model probability experiments and interpret the outcomes. **S.CP.9:** See Unit Addendum.

Vocabulary: combinations, complements, conditional probability, complements, events, independent events, intersections, permutations, sample space, subsets, unions

Unit 6 – Applications of Probability

Domain: Statistics

Essential Question: How does probability help to develop informed decisions?

Mathematical Practices:

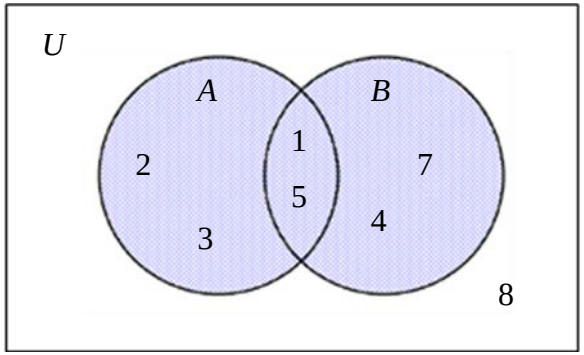
1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Evaluate probability models using algebraic and geometric reasoning.** **S-MD** Use probability to evaluate outcomes of decisions.	**How…** **S.MD.6:** Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). **S.MD.7:** Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).	**Students will:** **Model Real World Events Activity:** ▪ Evaluate probability models of real world problems and analyze results to make decisions. **Probability Analysis Worksheet:** ▪ Analyze a real world scenario and determine if the strategies utilized result in a fair decision.	**S.MD.6:** Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model and interpret parameters in linear, quadratic or exponential functions. **S.MD.7:** Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model and interpret parameters in linear, quadratic or exponential functions.

Vocabulary: combinations, complements, conditional probability, complements, events, independent events, intersections, permutations, sample space, subsets, unions

MHS Geometry Mathematics Curriculum

Unit 6 Addendum

Performance Standard	Instructional Strategies
S.CP.1:	<u>Intersection</u>: The **intersection** of two sets *A* and *B* is the set of elements that are common to both set *A* **and** set *B*. It is denoted by $A \cap B$ and is read '*A* intersection *B*'. • $A \cap B$ in the diagram is {1, 5} • this means: BOTH/AND  <u>Union</u>: The **union** of two sets *A* and *B* is the set of elements, which are in *A* **or** in *B* **or** in both. It is denoted by $A \cup B$ and is read '*A* union *B*'. • $A \cup B$ in the diagram is {1, 2, 3, 4, 5, 7} • this means: EITHER/OR/ANY • *could* be both <u>Complement</u>: The **complement of the set $A \cup B$** is the set of elements that are members of the universal set U but are not in $A \cup B$. It is denoted by $(A \cup B)'$ $(A \cup B)'$ in the diagram is {8}
S.CP.9:	Students may use calculators or computers to determine sample spaces and probabilities. Example: You and two friends go to the grocery store and each buys a soda. If there are five different kinds of soda, and each friend is equally likely to buy each variety, what is the probability that no one buys the same kind?

Algebra 2 Mathematics Curriculum

Manchester High School
Common Core State Standards
Mathematics Curriculum

Description of Standards

The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in fourth credit courses or advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+). All standards without a (+) symbol should be in the common mathematics curriculum for all college and career ready students. Standards with a (+) symbol may also appear in courses intended for all students.

The high school standards are listed in conceptual categories including Number and Quantity, Algebra, Functions, Modeling, Geometry, and Statistics and Probability, and Contemporary Mathematics.
Conceptual categories portray a coherent view of high school mathematics; a student's work with functions, for example, crosses a number of traditional course boundaries, potentially up through and including calculus. Modeling is best interpreted not as a collection of isolated topics but in relation to other standards.

Number and Quantity • The Real Number System (N-RN) • Quantities (N-Q) • The Complex Number System (N-CN) • Vector and Matrix Quantities (N-VM) **Algebra** • Seeing Structure in Expressions (A-SSE) • Arithmetic with Polynomials and Rational Expressions (A-APR) • Creating Equations (A-CED) • Reasoning with Equations and Inequalities (A-REI) **Functions** • Interpreting Functions (F-IF) • Building Functions (F-BF) • Linear, Quadratic, and Exponential Models (F-LE) • Trigonometric Functions (F-TF)	**Geometry** • Congruence (G-CO) • Similarity, Right Triangles, and Trigonometry (G-SRT) • Circles (G-C) • Expressing Geometric Properties with Equations (G-GPE) • Geometric Measurement and Dimension (G-GMD) • Modeling with Geometry (G-MG) **Modeling** **Statistics and Probability** • Interpreting Categorical and Quantitative Data (S-ID) • Making Inferences and Justifying Conclusions (S-IC) • Conditional Probability and the Rules of Probability (S-CP) • Using Probability to Make Decisions (S-MD) **Contemporary Mathematics** • Discrete Mathematics (CM-DM)

NEASC CURRICULUM RESPONSIBILITY CHECKLIST

	ART	BUSINESS & FINANCE	CAREER SERVICES	ENGLISH	FAMILY & CONSUMER SCIENCE	GUIDANCE	LIBRARY	HF AI TH	MATHEMATICS	PERFORMING ARTS	PHYSICAL EDUCATION	SCIENCE	SOCIAL STUDIES	TECHNOLOGY	WORLD LANGUAGES
Academic Expectations All students will:															
1. Reads and understand literary, informational, and persuasive texts in order to analyze, interpret, and evaluate them.				1	1			1	2			2	1		2
2. produce written materials which develop thoughts, share information, influence, persuade, and entertain.				1	2			1	2			2	1	2	1
3. comprehend verbal and nonverbal presentations at the literal, inferential and evaluative levels, and speak using appropriate conventions, forms of expression, and tools.	2			1	1		2	2	2		1		2	2	1
4. comprehend and demonstrate number sense, probability and statistics, geometry, and algebra.				2	2				1			2		2	
5. understand and apply basic concepts, principles, and theories of biology and physical science, and their interrelationships.				2	1			1			1	1		2	
6. identify and apply appropriate technologies to conduct research, communicate information and ideas, produce original works, organize data, and solve problems.	2	1		1	2		1		2			2	2	1	2
7. demonstrate abstract thinking, curiosity, imagination, and creativity in the arts or through academic problem solving.	1	2		1	1				2	1	1	2	2	2	

NEASC CURRICULUM RESPONSIBILITY CHECKLIST

	ART	BUSINESS & FINANCE	CAREER SERVICES	ENGLISH	FAMILY & CONSUMER SCIENCE	GUIDANCE	HF AI TH	LIBRARY	MATHEMATICS	PERFORMING ARTS	PHYSICAL EDUCATION	SCIENCE	SOCIAL STUDIES	TECHNOLOGY	WORLD LANGUAGES
Civic and Social Expectations All students will:															
1. treat others with respect regardless of race, ethnicity, class, or orientation.	1	1				2	1		2	2	1		1		
2. demonstrate ethical behavior through honesty and integrity.	1	1				2	1	2	2	2	1		1		
3. be responsible for their words, actions, and commitments.	1	1				2	1		2	2	1		1		
4. contribute positively to the community.	1	1				1	2			1	2		2		
5. contribute to the school by participating in co-curricular or service activities.	2	2				1	2			1	1		2		

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MONTH	CURRICULUM TOPICS
September - Quarter 1 **Unit 1** **Polynomial, Rational, and Radical Relationships**	***Operations on polynomials [Text: P6]*** • Perform arithmetic operations on polynomials (+,-,*,Binomial Theorem) (A.APR.1, A.APR.5) ***Factoring, ZPP [Text: P7]*** • Write expressions in equivalent forms to solve problems (Factoring) (A.SSE.4) • Understand the relationship between zeros and factors of polynomials (A.APR.2, A.APR.3) • Interpret the structure of expressions (A.SSE.1, A.SSE.2) • Rewrite rational expressions (multiplication, division) (A.APR.6, A.APR.7(+)) ***Solving [Text: Ch. 1 and 2.3][Solving quadratics, roots, absolute value, inequalities. Solving graphically]*** • Understand solving equations as a process of reasoning and explain the reasoning (A.REI.2)
October	• Represent and solve equations and inequalities graphically (A.REI.11) • Create equations that describe numbers or relationships (Literal Equations) (A.CED.1, A.CED.2, A.CED.3, A.CED.4) ***Complex Numbers[Text: 1.4]*** • Perform arithmetic operations with complex numbers (N.CN.1, N.CN.2) • **Use complex number in polynomial identities and equations** (N.CN.7, N.CN.8, N.CN.9) • Solve quadratic equations with real coefficients that have complex solutions.(N.CN.7)
November (Quarter 2) **Unit 3** **Modeling with Functions**	***Functions [Text: 3]*** • Build new functions from existing functions (Composition, Inverses, Families, Domain, Range, Transformations on Linear, Quadratic, Power, Root, Absolute Value, Reciprocal, Greatest Integer, Piecewise-Defined) (F.BF.3, F.BF.4.a)

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December	• Interpret functions that arise in applications in terms of a context (Average rate of change, maxima, minima, increasing, decreasing, Getting info from the graph of a function) (F.IF.4, F.IF.5, F.IF.6) *Quadratic Functions [Text 4.1]* Find and interpret vertex, intercepts and graph a quadratic function • Build a function that models a relationship between two quantities (F.BF.1)
January (Quarter 3)	• Analyze functions using different representations (F.IF.7.b, F.IF.7.e, F.IF.8, F.IF.9) • Interpret functions that arise in applications in terms of a context (F.IF.4, F.IF.5, F.IF.6)
February and March	***Exponential and Log Functions [Text ch. 5]*** Graph, Solve exponential and log equations, Connect exponent rules to laws of logs, build, analyze, interpret,,model • Build a function that models a relationship between two quantities (F.BF.1) • Analyze functions using different representations (F.IF.7.b, F.IF.7.e, F.IF.8, F.IF.9) • Interpret functions that arise in applications in terms of a context(F.IF.4, F.IF.5, F.IF.6)
April	***Systems of Equations and Inequalities [Text: ch. 6]*** *Create and Compare Functions [Text Resources for Integrating Modeling Problems Throughout Unit: ch.3 pp.280; 4.1, ch. 4 pp.364, 5.5,pp.431]* • Construct and compare linear, quadratic, and exponential models and solve problems (F.LE.4) • Build a function that models a relationship between two quantities (F.BF.1)
May and June	***Unit 1 continued: Polynomial Functions*** • Perform arithmetic operations on polynomials (closed systems) (A.APR.1) • Understand the relationship between zeros and factors of polynomials (A.APR.2, A.APR.3) • Analyze functions using different representations (F.IF.7.b, F.IF.7.c , F.IF.7.e, F.IF.8, F.IF.9) • Use polynomial identities to solve problems (A.APR.4, A.APR.5(+)) (+)Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

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To insert for 2013-14:

Rational Expressions and Functions

- **Rewrite rational expressions** (A.APR.6, A.APR.7(+)) (polynomial division, closed systems)

- **Use complex number in polynomial identities and equations** (N.CN.7, N.CN.8, N.CN.9)
- **Use polynomial identities to solve problems**
- **Analyze functions using different representations** (F.IF.7.c)

(A.APR.4, A.APR.5(+))

Unit 2
Trigonometric Functions

- **Extend the domain of trigonometric functions using the unit circle** (F.TF.1, F.TF.2)
- **Model periodic phenomena with trigonometric functions** (F.TF.5)

- **Prove and apply trigonometric identities** (F.TF.8)

Unit 4 Inferences and Conclusions from Data

- **Summarize, represent, and interpret data on a single count of measurement variable** (S.ID.4)
- **Understand and evaluate random processes underlying statistical experiments** (S.IC.1, S.IC.2)
- **Make inferences and justify conclusions from sample surveys, experiments, and observational studies** (S.IC.3, S.IC.4, S.IC.5, S.IC.6)
- **Use probability to evaluate outcomes of decisions** (S.MD.6(+), S.MD.7(+))

http://dese.mo.gov/divimprove/curriculum/math_model_curriculum/patternslinear/lesson_four.pdf
Illuminations: Egg Launch Contest
http://dese.mo.gov/divimprove/curriculum/math_model_curriculum/patternslinear/lesson_three.pdf
Compound Interest Derivations
Polynomials - Long Division (flip the classroom)

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Unit 1 Pacing Guide:

Text Resources: P6,P7,Ch 1, 2.3

- Operations on Polynomials: Add, Subtract, Multiply, Binomial Theorem
- Factoring: Common Factor, Trinomials, Difference of Squares, Difference of Cubes, Sum of Cubes, Grouping
- Solving Equations: Quadratic Equations (Factor, Root Property, Complete the Square, Quadratic Formula) , Radical Equations, Absolute Value
- Solving Inequalities
- Solving Equations and Inequalities Graphically
- Complex Numbers: Definition, Powers of i, Operations on Complex Numbers, Closed Systems

Unit 3 Pacing Guide:

Functions

Text Resources: Ch. 3

- Composition
- Transformations on Linear, Quadratic , Power, Root, Absolute Value, Reciprocal, Greatest Integer, Piecewise-Defined
- Average Rate of Change
- Getting Info from a Graph
- Inverses
- Modeling with Functions

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Quadratic Functions and Models

Text Resources: 4.1

- Graphing
- Vertex
- Vertex Form
- Modeling and Interpreting

Exponential and Log Functions

Text Resources: Ch. 5

- Exponential Functions and Graphs
- Logarithmic Functions and Graphs

- Laws of Logarithms
- Solving Exponential and Log Equations
- Modeling with Exponential and Log Functions, Growth and Decay

Fitting Exponential and Power Curves to Data

Text Resources: ch. 5 supplement

t

Systems of Equations and Inequalities

Text Resources: Ch. 6

- Strategies for Solving: Graphing, Substitution, Elimination,
- Create and solve systems in 3 variables
- Systems of Inequalities
- Linear Programming

Unit 1 – Polynomial, Rational, and Radical Relationships

Domain: Number and Quantity (N)

Essential Question: How do polynomial relationships compare to numerical relationships?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Illustrate arithmetic operations of polynomials through the comparison of operations on rational numbers.** **N-CN** Perform arithmetic operations with complex numbers. **N-CN** Use complex numbers in polynomial identities and equations.	**How…** **N.CN.1:** Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real. **N.CN.2:** Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. **N.CN.7:** Solve quadratic equations with real coefficients that have complex solutions. **N.CN.8:** (+) Extend polynomial identities to the complex numbers. *For example, rewrite* $x2 + 4$ *as* $(x + 2i)(x – 2i)$. **N.CN.9:** (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.	**Students will…** **How Complex Is It? Activity:** ▪ Perform various operations with imaginary and complex numbers using operations from the real number system **Quadratic Revisit Investigation:** ▪ Factor and solve quadratic equations that may have complex solutions and analyze the solutions based on the real or imaginary roots of the equation. polynomial functions summary.docx Representing Polynomials	**N.CN.1:** See Unit Addendum **N.CN.7:** See Unit Addendum **N.CN.9:** See Unit Addendum

Vocabulary: complex solutions, cube root, decreasing behavior, end behavior, extraneous solutions, increasing behavior, intervals, piece-wise function, polynomial function, quadratic equation, rational equation, real coefficients, relative maximum, relative minimum, square root, suitable factorization, symmetries, zeros of polynomial

In summary: What do we know so far about polynomial functions and their roots?

Technical Statement *In your own words*

I. There is a difference between zeros, x-intercepts, roots, factors.

Zeros/Roots	
X-intercepts	
Factors	

II. Theorem: Fundamental Theorem of Algebra

A polynomial function of degree *n* has *n* complex zeros. (real and nonreal) Some of these zeros may be repeated.	

III. Theorem: Conjugate Zeros Theorem

Suppose that f(x) is a polynomial function with real coefficients. If *a+bi* is a zero of *f(x)*, then its complex conjugate *a-bi* is also a zero of *f(x)*. ($b \neq 0$)	

III. Theorem: Factor Theorem

A polynomial function has a factor *(x-k)* if and only if *f(k)=0*	

IV. Theorem: Local Extrema and Zeros of Polynomial Functions

A polynomial function of degree *n* has at most *(n-1)* local extrema and at most *n* zeros.	

MHS Algebra 2 Mathematics Curriculum

Unit 1 – Polynomial, Rational, and Radical Relationships

Domain: <u>Algebra</u> (A)

Essential Question: How do polynomial relationships compare to numerical relationships?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Illustrate arithmetic operations of polynomials through the comparison of operations on rational numbers.** **A-SSE** Interpret the structure of expressions **A-SSE** Write expressions in equivalent forms to solve problems	**How…** **A.SSE.1:** Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of P and a factor not depending on P.* **A.SSE.2:** Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$ **A.SSE.4:** Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *Mortgage problems*	**Students will…** **Expression Check-In Quiz:** ▪ Interpret parts of an expression as they are defined within the expression ▪ Manipulate expressions to isolate different variables within the expression **Equations and Identities Always, Sometimes, Never Activity:** equations and identities card set.pdf **Savings Plan Group Activity:** ▪ Model and solve a financial situation through pattern recognition and prediction.	**A.SSE.1:** Students should understand the vocabulary for the parts that make up the whole expression and be able to identify those parts and interpret their meaning in terms of a context. **A.SSE.2:** Students should extract the greatest common factor (whether a constant, a variable, or a combination of each). If the remaining expression is quadratic, students should factor the expression further. Example: Factor $x^3 - 2x^2 - 35x$ **A.SSE.4:** Example: In February, the Bezanson family starts saving for a trip to Australia in September. The Bezanson's expect their vacation to cost $5375. They start with $525. Each month they plan to deposit 20% more than the previous month. Will they have enough money for their trip?

Vocabulary: complex solutions, cube root, decreasing behavior, end behavior, extraneous solutions, increasing behavior, intervals, piece-wise function, polynomial function, quadratic equation, rational equation, real coefficients, relative maximum, relative minimum, square root, suitable factorization, symmetries, zeros of polynomial

CONCEPT DEVELOPMENT

Mathematics Assessment Project

CLASSROOM CHALLENGES

A Formative Assessment Lesson

Sorting Equations and Identities

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley

Beta Version

For more details, visit: http://map.mathshell.org
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Sorting Equations and Identities

MATHEMATICAL GOALS

This lesson unit is intended to help you assess how well students are able to:

- Recognize the differences between equations and identities.
- Substitute numbers into algebraic statements in order to test their validity in special cases.
- Resist common errors when manipulating expressions such as $2(x - 3) = 2x - 3$; $(x + 3)^2 = x^2 + 3^2$.
- Carry out correct algebraic manipulations.

It also aims to encourage discussion on some common misconceptions about algebra.

COMMON CORE STATE STANDARDS

This lesson relates to the following *Standards for Mathematical Content* in the *Common Core State Standards for Mathematics*:

A-SSE: Interpret the structure of expressions.
Write expressions in equivalent forms to solve problems.
A-REI: Solve equations and inequalities in one variable.

This lesson also relates to the following *Standards for Mathematical Practice* in the *Common Core State Standards for Mathematics*:

3. Construct viable arguments and critique the reasoning of others.
7. Look for and make use of structure.

INTRODUCTION

The lesson unit is structured in the following way:

- Before the lesson, students work individually on an assessment task that is designed to reveal their current understandings and difficulties. You then review their work, and create questions for students to answer in order to improve their solutions.
- After a whole-class introduction, students work in small groups on a collaborative discussion task.
- Students return to their original task and try to improve their own responses.

MATERIALS REQUIRED

- Each student will need two copies of the assessment task *Equations and Identities,* a mini-whiteboard, a pen, and an eraser.
- Each small group of students will need *Card Set*: *Always, Sometimes, or Never True?* (cut into cards before the lesson), a marker pen, a glue stick, and a large sheet of paper for making a poster.
- There is a projector resource to support the whole-class introduction.

TIME NEEDED

10 minutes before the lesson for the assessment task, a 1-hour lesson, and 10 minutes in a follow-up lesson (or for homework). Timings given are approximate and will depend on the needs of the class.

BEFORE THE LESSON

Assessment task: *Equations and Identities* (15 minutes)

Set this task, in class or for homework, a few days before the formative assessment lesson. This will give you an opportunity to assess the work, to find out the kinds of difficulties students have with it. You will then be able to target your help more effectively in the follow-up lesson.

Give each student a copy of the assessment task *Equations and Identities*.

Read through the questions and try to answer them as carefully as you can.

It is important that, as far as possible, students are allowed to answer the questions without your assistance.

Students should not worry too much if they cannot understand or do everything, because in the next lesson they will work on a similar task, which should help them. Explain to students that by the end of the next lesson, they should be able to answer questions such as these confidently. This is their goal.

Sorting Equations and Identities | Student Materials | Beta Version

Equations and Identities

1. Write down an example of an equation that has:

   (a) One solution. .....

   (b) Two solutions. .....

   (c) An infinite number of solutions. .....

   (d) No solutions. .....

2. For each of the following statements, indicate whether it is "Always true," "Never true," or "Sometimes true." Circle the correct answer. If you choose "Sometimes true" then state on the line below when it is true. The first one is done for you as an example.

$x + 2 = 3$	Always true	Never true	Sometimes true
	It is true when $x = 1$.		
$x - 12 = x + 30$	Always true	Never true	Sometimes true
	It is true when		
$2(x + 6) = 2x + 12$	Always true	Never true	Sometimes true
	It is true when		
$3(x - 2) = 3x - 2$	Always true	Never true	Sometimes true
	It is true when		
$(x + 4)^2 = x^2 + 4^2$	Always true	Never true	Sometimes true
	It is true when		
$x^2 + 4 = 0$	Always true	Never true	Sometimes true
	It is true when		

3. Which of the equations in question 2 are also identities?

   .....

   .....

   In your own words, explain what is meant by an identity.

   .....

   .....

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Assessing students' responses

Collect students' responses to the task, and make some notes on what their work reveals about their current levels of understanding. The purpose of doing this is to forewarn you of the difficulties students will experience during the lesson itself, so that you may prepare carefully.

We suggest that you do not write scores on students' work. The research shows that this is counterproductive as it encourages students to compare scores, and distracts their attention from what they are to do to improve their mathematics.

Instead, help students to make further progress by asking questions that focus their attention on aspects of their work. Some suggestions for these are given on the next page. These have been drawn from common difficulties observed in trials of this unit.

We suggest that you write your own list of questions, based on your own students' work, using the ideas that follow. You may choose to write questions on each student's work. If you do not have time to do this, write a few questions that will help the majority of students. These can then be displayed on the board at the end of the lesson.

Common issues:	Suggested questions and prompts:
Student writes expressions rather than equations For example: The student writes $y + 3$ for an equation with an infinite number of solutions.	• What is the difference between an equation and an expression? • How can you change your expression to an equation?
Student fails to include a variable in their equation For example: The student has written $5 + 5 = 10$ as an example of an equation with one solution.	• Can you include an unknown number or a variable in the equation so that we can look at all possible values of that unknown?
Student fails to provide an example of an equation with an infinite number of solutions	• What would an equation with an infinite number of solutions look like?
Student provides a quadratic with non-integer solutions as an example of an equation with no solutions For example: The student gives $x^2 + 8x + 13 = 0$ as an answer to Q1d. The student has assumed that, because it won't factorize there are no solutions.	• Can a quadratic equation that will not factorize still have solutions/cross the x-axis? How can you check whether or not a quadratic equation has solutions?
Student assumes that $-(x^2)$ is the same as $(-x)^2$ For example: The student classifies $x^2 + 4 = 0$ as true when $x = -2$.	• What does $(-x)^2$ mean? What kind of number do we get when we multiply two negative numbers together? • Is x^2 positive or negative?
Student correctly answers all the questions The student needs an extension task.	• Use algebra to justify one of your answers to Question 2. • Draw a diagram to justify one of your answers to Question 2.

SUGGESTED LESSON OUTLINE

Whole-class introduction (10 minutes)

Use slide P-1 of the projector resource in this introduction.

Give each student a mini-whiteboard, a pen, and an eraser. Write the following equation on the board:

$$(x + 2)(y + 2) = xy + 4$$

Is this equation 'always true', 'never true' or 'sometimes true'? [Write 'always', 'never' or 'sometimes' on your whiteboard.]

Typically, most students will begin by saying that this is never true.

Can you show me values for x and y that make the equation false?

Can you show me values for x and y that make the equation true?

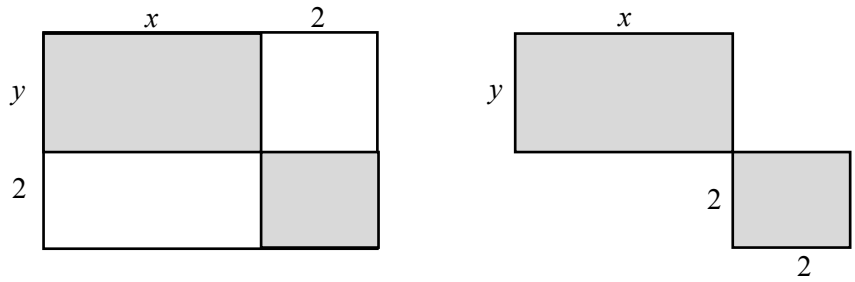
Hold a discussion about the responses, asking students to provide values for x and y to support their response.

Can the values of x and y be the same number? Can you figure out one?

This misconception needs to be explicitly addressed. Some students may assume that because x and y are different letters, they should take different values.

Students may spot that the equation is true when $x = y = 0$.

If students are struggling to find any values of x and y for which the equation is true, drawing an area diagram may be helpful (Slide P-1)



Total area = $(x + 2)(y + 2)$ Total area = $xy + 4$

For these two area diagrams to be equal, what are the values of x and y?

For the area diagrams to be the equal, $2y$ must equal 0 and $2x$ must equal 0. This is true when x and y are both equal to 0.

When students are comfortable that when $x = y = 0$ the equation is true, ask them to summarize their findings.

We have found values of x and y that make the equation false and values of x and y that make the equation true. Is the equation always, sometimes or never true? [equation is sometimes true.]

Next, ask the students:

Are x = 0 and y = 0, the only values that make the equation true? How could we find out?

Using an algebraic approach here might be helpful, as we are unable to describe a negative area. The following method may be appropriate:

$$(x + 2)(y + 2) = xy + 2x + 2y + 4.$$

We want to know when this is the same as $xy + 4$, which must be when $2x + 2y = 0$, i.e. when $x + y = 0$ or when $x = -y$. We can therefore conclude that the equation is true when $x = -y$.

Now write this equation on the board:

$$(x + 2)(x - 2) = x^2 + 4$$

How about this equation? Is it 'always true', 'never true' or 'sometimes true'?

Students will probably find values for x for which the equation is false. After a discussion of a couple of these examples, encourage students to justify their conclusions:

Give me a value of x that will make the equation false/true? And another? [There are no values of x that will make the equation true.]

Do you think the equation is never true? Convince me. [Students should simplify the left side of the equation to $x^2 - 4$. The equation is never true, because $4 = -4$.]

After a few minutes, ask one or two students to explain their answers. Encourage other students to challenge their reasoning.

In this activity, the students use the term **identity**.

*If an equation is always true, we say it is an **identity**.*

Teachers may be accustomed to varying uses of the term 'identity'. While this is not the main focus of this activity, for the purpose of the lesson, the term 'identity' is used to describe equations that are always true.

Collaborative activity: *Always, Sometimes, or Never True?* (30 minutes)

Ask students to work in groups of two or three.

Give each group *Card Set*: *Always, Sometimes, or Never True?*, a large sheet of paper, a marker pen, and a glue stick.

Ask students to divide their large sheet into three columns and head respective columns with the words: *Always True*, *Sometimes True*, *Never True*.

You may want to use slide P-2 of the projector resource to display the following instructions.

You are now going to consider whether the equations on your desk are Always, Sometimes, or Never True.

In your groups, take turns to place a card in a column and justify your answer to your partner.

If you think the equation is sometimes true, you will need to find values of x for which it is true and values of x for which it is not true.

If you think the equation is always true or never true, you will need to explain how we can be sure that this is the case. Remember, showing it is true, or never true, for just a few values is not sufficient.

Another member of the group should then either explain their reasoning again in his or her own words, or challenge the reasons you gave.

It is important that everyone in the group understands the categorization of each card.

When everyone in the group agrees, glue the card onto your poster. Write the reason for your choice of category next to the card.

It does not matter if you do not manage to place all of the cards. It is more important that everyone in the group understands the categorization of each card.

The purpose of this structured work is to encourage each student to engage with their partner's explanations, and to take responsibility for their partner's understanding.

While students work in small groups you have two tasks: to make a note of student approaches to the task, and to support student reasoning.

Make a note of student approaches to the task

Listen and watch students carefully. In particular, listen to see whether students are addressing the difficulties they experienced in the assessment. You can use this information to focus the whole-class discussion towards the end of the lesson.

Support student reasoning

Use the questions in the *Common issues* table to help address misconceptions.

Encourage students to explain their reasoning carefully.

You have shown the statement is true for this specific value of x. Now convince me it is always true for every number!

Can you use algebra to justify your decision for this card?

Can you draw a diagram to explain your categorization for this card?

(Card 8) Can you sketch a graph to show why $x^2 = 2x$ has only two solutions?

(Card 9) Draw an area diagram to show that $(x + 3)^2$ means something different from $x^2 + 3^2$.

(Card 11) Can you draw an area diagram to show why $(3x)^2$ is always equal to $9x^2$?

If some students try to solve the equations by algebraic manipulation, they may notice that while sometimes this gives them possible solutions, sometimes they just get $0 = 0$. These are, of course, the identities. Equations that have no solutions give absurdities such as $1 = 2$.

If students finish the task quickly, ask them to create new examples.

Can you make up an identity? And another one?

Can you make up an equation that has two solutions?

Can you make up an equation that has no solutions and shows a common algebraic mistake? [E.g. $3(x + 4) = 3x + 4$.]

Whole-class discussion (20 minutes)

Organize a whole-class discussion about different methods of justification used for two or three equations.

Ask each group to choose an equation from their poster that meets some given criteria. For example:

Show me an equation that has no solutions.

Show me an equation that has just one solution. Write this solution on your mini-whiteboard.

Show me an equation that has two solutions. What are they?

Show me an equation that has an infinite number of solutions.

Show me an identity.

You may find that numerous different equations are displayed in response to a given criterion. If more than one group shows the same equation, ask each of these groups of students to give a justification of their thinking. Then ask other students to contribute ideas of alternative approaches, and their views on which reasoning method was easier to follow. It is important that students consider a variety of methods, and begin to develop a repertoire of approaches.

Why did you put this equation in this column? How else can you explain that decision?

Can anyone improve this explanation?

Which explanation do you prefer? Why?

Draw out issues you have noticed as students worked on the activity. Make specific reference to the misconceptions you noticed during the collaborative activity.

Improving individual solutions to the assessment task (10 minutes)

Return their original assessment *Equations and Identities* to the students, together with a second blank copy of the task.

Look at your original responses and think about what you have learned this lesson.

Using what you have learned, try to improve your work.

If you have not added questions to individual pieces of work, write your list of questions on the board. Students should select from this list only the questions they think are appropriate to their own work.

If you find you are running out of time, then you could set this task in the next lesson or for homework.

SOLUTIONS

Always true (Identities)	Sometimes true	Never true
$2(x+3)=2x+6$	$x-6=6-x$ True when $x = 6$.	$2(x-3)=2x-3$
$x^2-1=(x+1)(x-1)$	$x+6=y+6$ True when $x = y$.	$x^2+6=0$ (unless you include complex numbers.)
$(x-6)^2=(6-x)^2$	$\frac{x}{6}=\frac{6}{x}$ True when x = +6 or −6.	
$(3x)^2=9x^2$	$6+2x=8x$ True when x = 1.	
	$x^2=2x$ True when x = 0 or 2.	
	$(x+3)^2=x^2+3^2$ True when x = 0.	
	$(x+1)(x+4)=x^2+14$ True when x = 2.	
	$\frac{x+6}{2}=x+3$ True when x = 0.	

Equations and Identities

1. Write down an example of an equation that has:

   (a) One solution. .....

   (b) Two solutions. .....

   (c) An infinite number of solutions. .....

   (d) No solutions. .....

2. For each of the following statements, indicate whether it is 'Always true', 'Never true' or 'Sometimes true'. Circle the correct answer. If you choose 'Sometimes true' then state on the line below when it is true. The first one is done for you as an example.

$x + 2 = 3$	Always true	Never true	Sometimes true
	It is true when $\boldsymbol{x = 1}$.		
$x - 12 = x + 30$	Always true	Never true	Sometimes true
	It is true when		
$2(x + 6) = 2x + 12$	Always true	Never true	Sometimes true
	It is true when		
$3(x - 2) = 3x - 2$	Always true	Never true	Sometimes true
	It is true when		
$(x + 4)^2 = x^2 + 4^2$	Always true	Never true	Sometimes true
	It is true when		
$x^2 + 4 = 0$	Always true	Never true	Sometimes true
	It is true when		

3. Which of the equations in question 2 are also identities?

   .....

   .....

   In your own words, explain what is meant by an identity.

   .....

   .....

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Card Set: Always, Sometimes, or Never True?

1 $x - 6 = 6 - x$	2 $x + 6 = y + 6$
3 $\frac{x}{6} = \frac{6}{x}$	4 $6 + 2x = 8x$
5 $2(x - 3) = 2x - 3$	6 $2(x + 3) = 2x + 6$
7 $\frac{x + 6}{2} = x + 3$	8 $x^2 = 2x$
9 $(x + 3)^2 = x^2 + 3^2$	10 $(x - 6)^2 = (6 - x)^2$
11 $(3x)^2 = 9x^2$	12 $x^2 - 1 = (x + 1)(x - 1)$
13 $x^2 + 6 = 0$	14 $(x + 1)(x + 4) = x^2 + 14$

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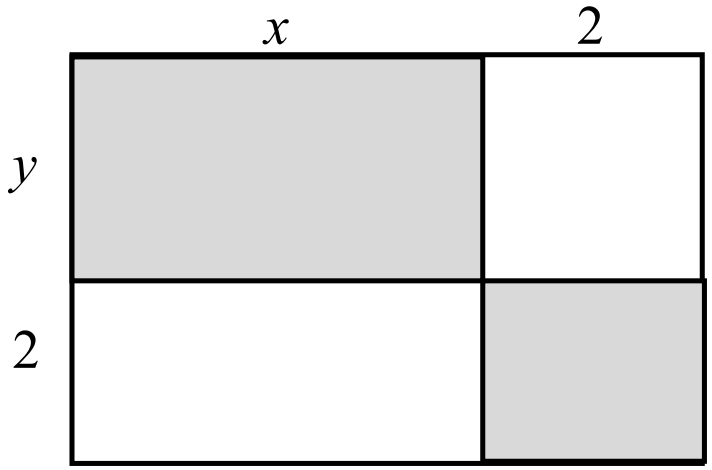
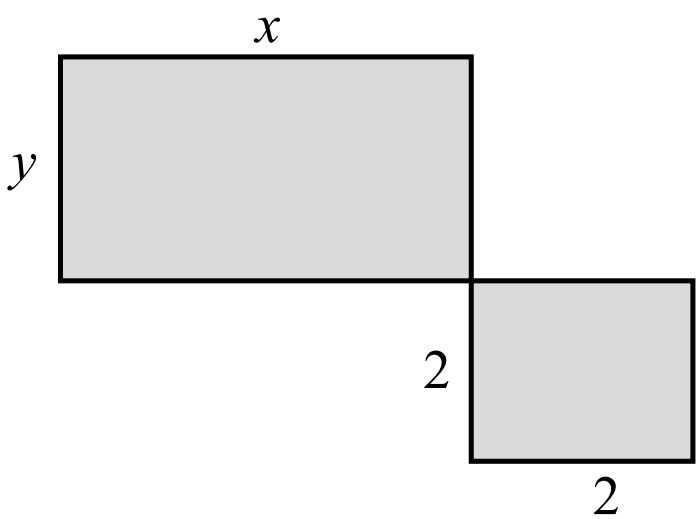
Always, Sometimes, or Never True?

$$(x + 2)(y + 2) = xy + 4$$





Always, Sometimes, or Never True?

- In your groups, take turns to place a card in a column and justify your answer to your partner.
- If you think the equation is ‘sometimes true’, find values of x for which it is true and values of x for which it is not true.
- If you think the equation is ‘always true’ or ‘never true’, explain how we can be sure that this is the case.
- Another member of the group should then either explain that reasoning again in his or her own words, or challenge the reasons you gave.
- When everyone in the group agrees, glue the card onto the poster. Write the reason for your choice next to the card.

Mathematics Assessment Project

CLASSROOM CHALLENGES

This lesson was designed and developed by the
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It was refined on the basis of reports from teams of observers led by
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based on their observation of trials in US classrooms
along with comments from teachers and other users.

This project was conceived and directed for
MARS: Mathematics Assessment Resource Service
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and based at the University of California, Berkeley

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Unit 1 – Polynomial, Rational, and Radical Relationships

Domain: Algebra (A)

Essential Question: How do polynomial relationships compare to numerical relationships?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Illustrate arithmetic operations of polynomials through the comparison of operations on rational numbers.** **A-APR** Perform arithmetic operations on polynomials **A-APR** Understand the relationship between zeros and factors of polynomials	**How…** **A.APR.1:** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. **A.APR.2:** Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$. **A.APR.3:** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	**Students will…** **Polynomial Concept Check***:* Polynomial Functions Concept Check.docx ***Polynomial TI Exploration: Zeroes and Factors*** Exploration intercepts zeroes roo	**A.APR.2:** See Unit Addendum **A.APR.3:** See Unit Addendum

Vocabulary: complex solutions, cube root, decreasing behavior, end behavior, extraneous solutions, increasing behavior, intervals, piece-wise function, polynomial function, quadratic equation, rational equation, real coefficients, relative maximum, relative minimum, square root, suitable factorization, symmetries, zeros of polynomial

Concept Check: **Intro to Polynomial Functions (No calculator)**

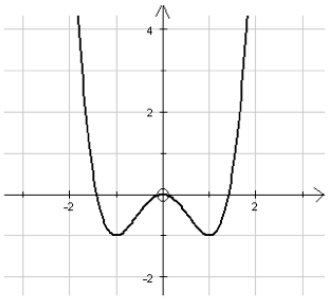
1. Which of the following represent polynomial functions? Explain your reasoning.

a. $y = \dfrac{2}{x+3}$

b. $y = x^3 + 6x^2 - 4$

c. $y = 3x^{-2}$

d.



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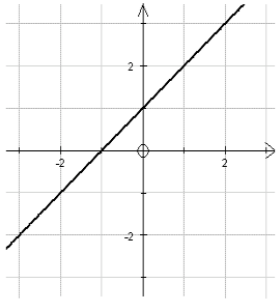

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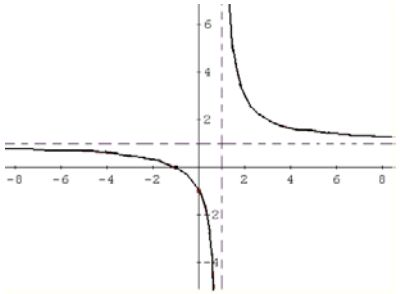


a._____

b._____

c._____

d._____

e._____

f._____

2. For each polynomial below describe the end behavior.

a. $y = -x^3 + 6x^2 - x + 8$

as $x \to \infty$ as $x \to -\infty$

b. $y = 2x^4 - 8x - 15$

as $x \to \infty$ as $x \to -\infty$

3. If P(x) is a function with odd degree, which of the following statements must be true? Explain your reasoning.

a. It has an odd number of maxima or minima.

b. The end behaviors as $x \to \infty$ and as $x \to -\infty$ are the same.

c. Its leading coefficient is an odd number.

4. Which of the following statements couldn't possibly be true about the polynomial function P? Explain your reasoning.

   a. P has degree 4, 3 local maxima and 1 local minimum

   b. P has degree 3, 1 local maxima or minima

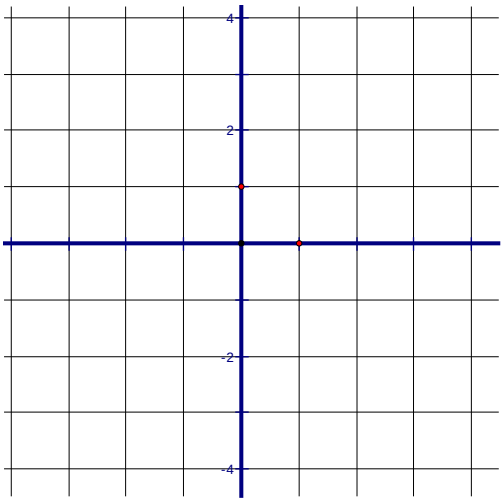
   c. P has degree 2, 2 local maxima or minima

Name_____Exploration: Intercepts, Zeros, Roots, Factors & Solutions

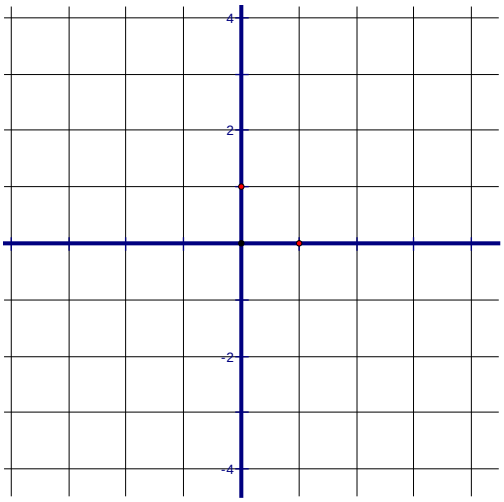
Part A

Sketch the following functions below with the aid of your grapher. Be sure to label intercepts (zeros) and relative extrema (maximums and minimums) found by using the CALCULATE functions on the TRACE key.

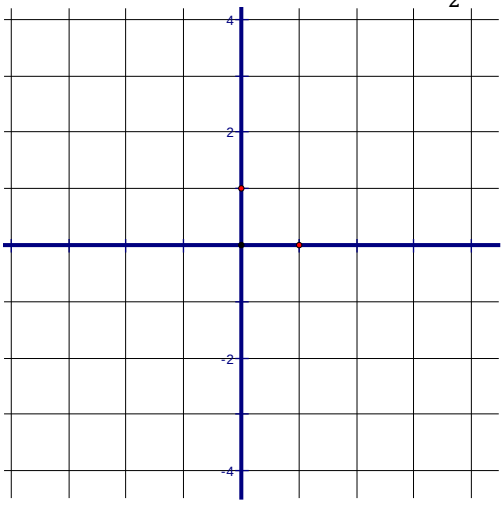
1. $f(x) = -x^3 + 3x^2 + x - 3$
$= -(x-1)(x+1)(x-3)$



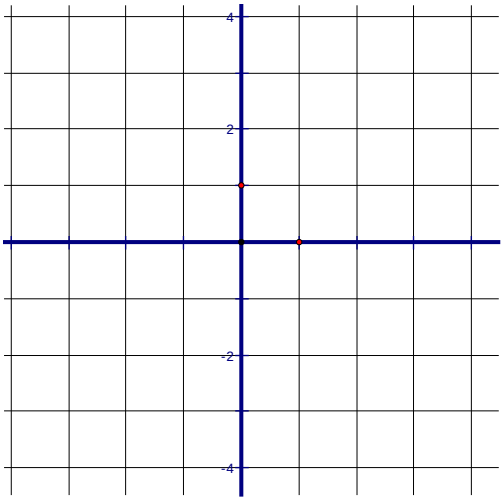
2. $g(x) = x^3 + 6x^2 + 11x + 6$
$= (x+3)(x+2)(x+1)$



3. $h(x) = 2x^3 - x^2 - 13x - 6$
$= (x+2)(x-3)(x+\frac{1}{2})$



4. $k(x) = -3x^3 + 2x^2 + 11x - 10$
$= -(x+2)(x-1)(3x-5)$



How are the graphs and the factored form of the equations related?

How is the number of local extrema related to the degree of the function?

. Examine the graph of $f(x) = x^3 - 3x^2 - x + 3$

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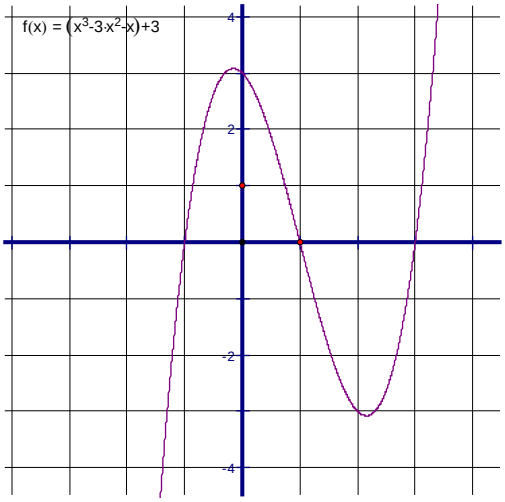


x- intercepts:

zeros:

roots:

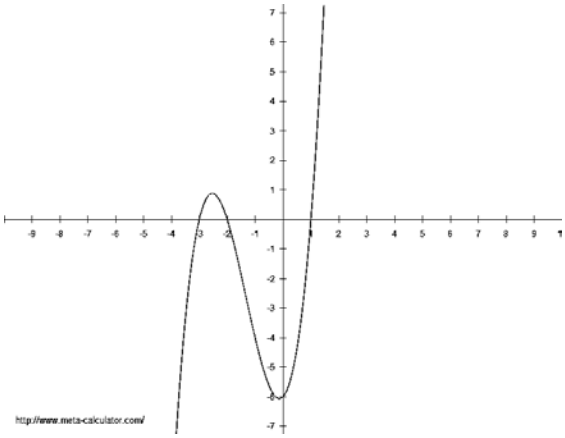
factors:

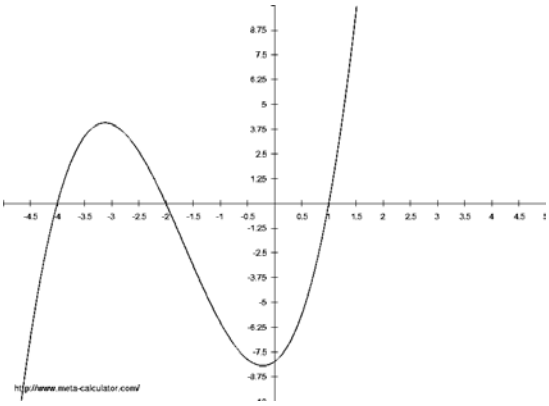
How can knowing the x-intercepts, zeros or roots lead to information about the factors of the polynomial equation?

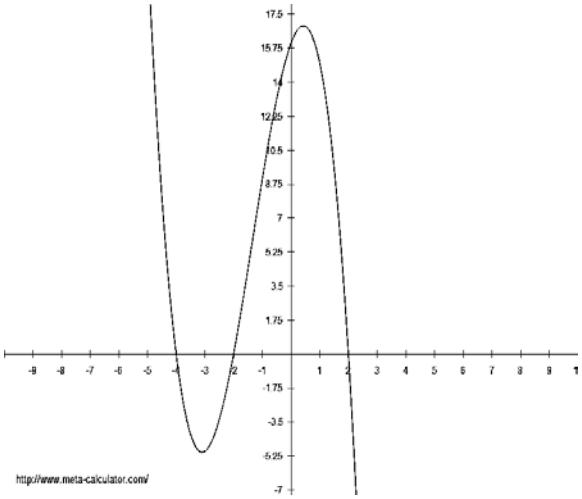
Create and use a graphing calculator to sketch a cubic function with zeroes at x = -2, 3, and 1

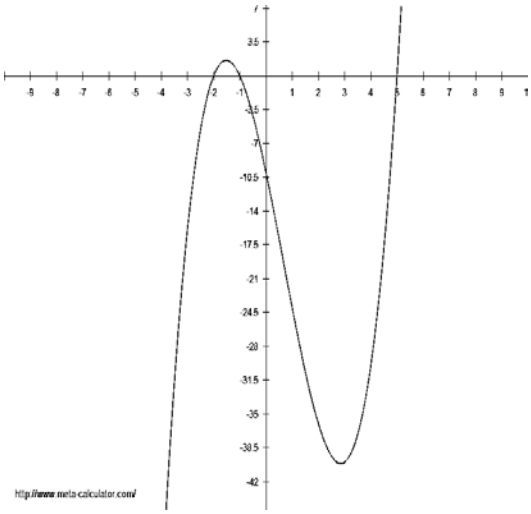
Create and use a graphing calculator to sketch a cubic function with zeroes at x = 0,1,-1 and 5

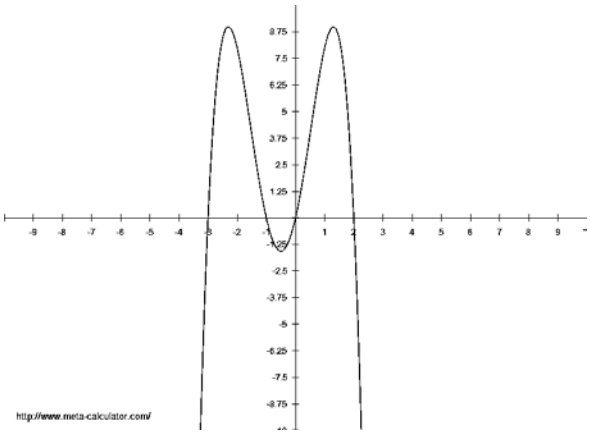
Write a function to represent each graph below. You may keep your function in factored form.

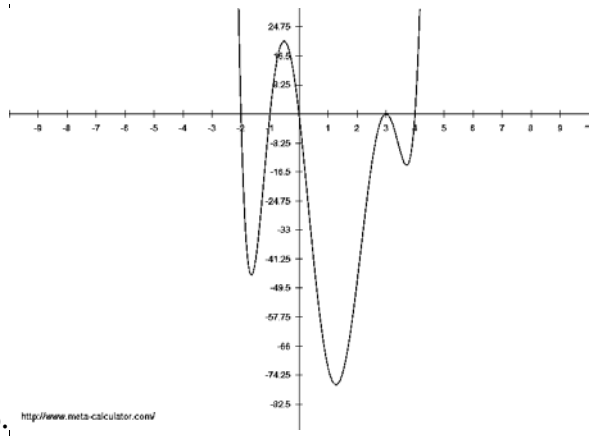
1. 

2. 

3. 

4. 

5. 

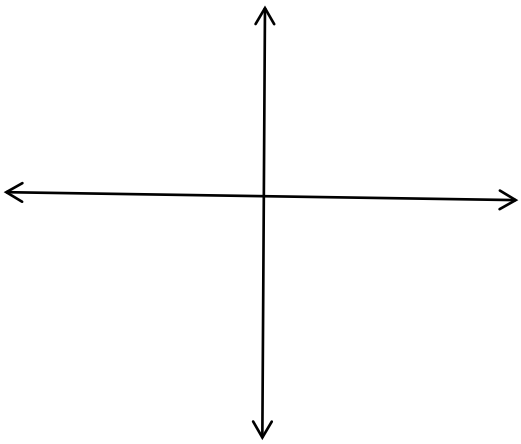
6. 

Something to think about…….

Sketch the graph of $f(x) = (x - 3)\{x + 4\}$

.

:



x- intercepts:

zeros:

Sketch the graph of $f(x) = (x - 3)(x - 3)\{x + 4\}$

.

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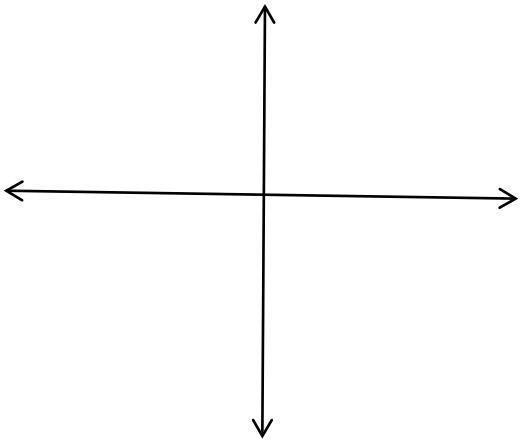


x- intercepts:

zeros:

Discuss the differences between the two graphs

How could you use a graph to decide if a graph has multiplicity of roots?

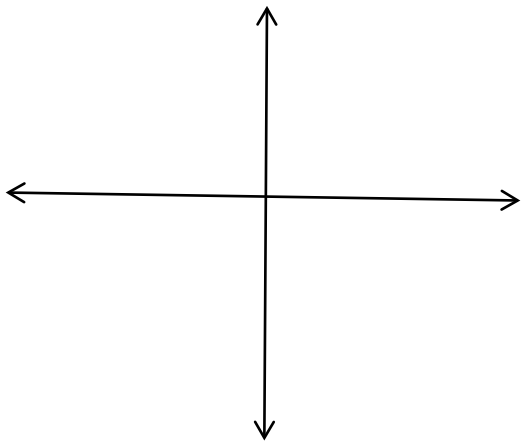
Name_____ Part II Exploration: Intercepts, Zeros, Roots, Factors & Solutions

Part A

1. Given $f(x) = x^2 - 4x + 5$

a. Use the quadratic formula to solve f(x) = 0

b. Use your graphing calculator to sketch:



x- intercepts:

zeros:

roots:

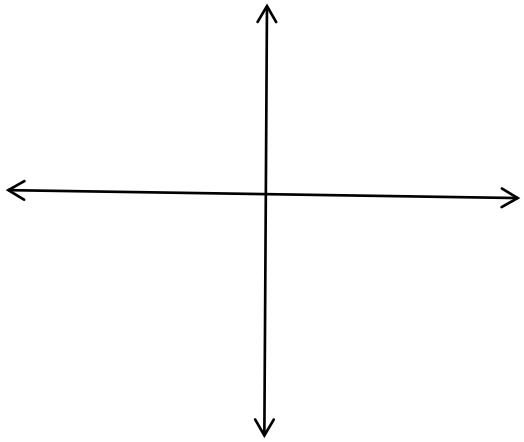
factors (See note below):

Note: We cannot connect the factors to the intercepts of the graph. We must use the solutions from part a to create complex factors

How can knowing the x-intercepts, zeros or roots lead to information about the factors of the polynomial equation?

2. a. Create a polynomial function with roots of $x = -i, i$, and -3

b. Use a graphing calculator to sketch



x- intercepts:

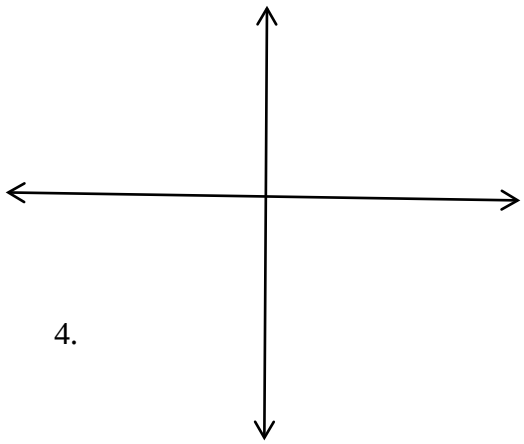
zeros:

roots:

factors:

3. a. Create a polynomial function with roots of $x = -i, i$, and -3

c. Use a graphing calculator to sketch



x- intercepts:

zeros:

roots:

factors:

4.

Unit 1 – Polynomial, Rational, and Radical Relationships

Domain: Algebra (A)

Essential Question: How do polynomial relationships compare to numerical relationships?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Illustrate arithmetic operations of polynomials through the comparison of operations on rational numbers.** **A-APR** Use polynomial identities to solve problems	**How…** **A.APR.4:** Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2+y^2)^2=(x^2-y^2)^2+(2xy)^2$ *can be used to generate Pythagorean triples.* **A.APR.5:** (+) Know and apply the Binomial Theorem for the expansion of $(x+y)^n$ in powers of *x* and *y* for a positive integer *n*, where *x* and *y* are any numbers, with coefficients determined for example by Pascal's Triangle.	**Students will…** **Pattern Exploration:** ▪ Discover patterns through manipulating polynomials and generalize relationships to explain mathematical identities. ▪ Extend polynomial generalizations to include the Binomial Theorem	**A.APR.4:** See Unit Addendum **A.APR.5:** See Unit Addendum
A-APR Rewrite rational expressions	**A.APR.6:** Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, a computer algebra system.	**Polynomial Division Check-In:** ▪ Express the division of polynomials in a variety of forms including a comparison of graphical and algebraic representations	**A.APR.6:** See Unit Addendum

Vocabulary: complex solutions, cube root, decreasing behavior, end behavior, extraneous solutions, increasing behavior, intervals, piece-wise function, polynomial function, quadratic equation, rational equation, real coefficients, relative maximum, relative minimum, square root, suitable factorization, symmetries, zeros of polynomial

Unit 1 – Polynomial, Rational, and Radical Relationships

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8. **Look for and express regularity in repeated reasoning.**

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Illustrate arithmetic operations of polynomials through the comparison of operations on rational numbers.** **A-APR** Rewrite rational expressions	**How…** **A.APR.7:** (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.	**Students will…** **Division Extension Practice:** ▪ Manipulate rational expressions by applying factoring and polynomial division properties.	**A.APR.7:** See Unit Addendum
A-REI Understand solving equations as a process of reasoning and explain the reasoning	**A.REI.2:** Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	**Equation Worksheet:** ▪ Manipulate rational and radical equations to determine real and complex solutions. ▪ Analyze a solution process and evaluate the reasoning with justification.	**A.REI.2:** See Unit Addendum

Vocabulary: complex solutions, cube root, decreasing behavior, end behavior, extraneous solutions, increasing behavior, intervals, piece-wise function, polynomial function, quadratic equation, rational equation, real coefficients, relative maximum, relative minimum, square root, suitable factorization, symmetries, zeros of polynomial

MHS Algebra 2 Mathematics Curriculum

Unit 1 – Polynomial, Rational, and Radical Relationships

Domain: <u>Functions</u> (F)

Essential Question: How do polynomial relationships compare to numerical relationships?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Illustrate arithmetic operations of polynomials through the comparison of operations on rational numbers.** **A-REI** Represent and solve equations and inequalities graphically **F-IF** Analyze functions using different representations	**How…** **A.REI.11:** Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find approximations. Include cases where functions are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. **F.IF.7:** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.	**Students will…** **Function Investigation Calculator Activity:** ▪ Compare the intersection points of two functions graphically, algebraically and numerically. ▪ Analyze given information about the intersections of functions and represent the information in other forms. **Polynomial Intro and End Behavior Exploration** intro to polynomial functions.docx ▪ **Polynomial Graph Card Matching Activity** matching activity polynomials (factored **Polynomial Theorems Summary** polynomial functions summary.docx	**A.REI.11:** See Unit Addendum **F.IF.7:** See Unit Addendum

Vocabulary: complex solutions, cube root, decreasing behavior, end behavior, extraneous solutions, increasing behavior, intervals, piece-wise function, polynomial function, quadratic equation, rational equation, real coefficients, relative maximum, relative minimum, square root, suitable factorization, symmetries, zeros of polynomial

Course: Algebra 2

Unit: Polynomial, Rational, and Radical Relationships

F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

Lesson: Intro to Polynomial Functions

A. Definition, vocabulary, discussion of end behavior

B. Student investigation. Students create equations and graphs and conduct comparisons of maxima, minima, and end behavior based on the degree of the polynomial.

Polynomial Functions

A polynomial function of degree n is a function of the form

$P(x) = a_n x^n + a_{n-1} x^{n-1} + ... + a_1 x + a_0$ where $a_n, a_{n-1}, \ldots, a_{1,}\ and\ \ a_0$ are real numbers, n is a **nonnegative integer** and $a_n \neq 0$.

Vocab: Coefficients:

Constant term:

Leading coefficient:

Leading term:

Degree:

Graphs of polynomials are always smooth curves, they have no corners, breaks or cusps.

Example: Given $f(x) = -3x^4 + 2x^3 - 4x^2 + 2x - 1$

*Leading coefficient:*_____

*Leading term:*_____

*Coefficients:*_____

*Degree:*_____

*Constant Term:*_____

End behavior of a Function

End behavior of a polynomial is a description of what happens as x becomes large in the positive or negative direction.

For $f(x) = x^2$ *as* $x \to \infty$ $y \to$ _____

$x \to -\infty$ $y \to$ _____

Graph:

For $f(x) = x^3$ *as* $x \to \infty$ $y \to$ _____

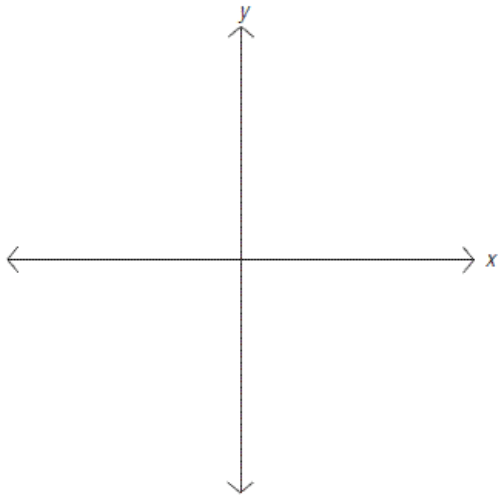
$x \to -\infty$ $y \to$ _____

Graph:

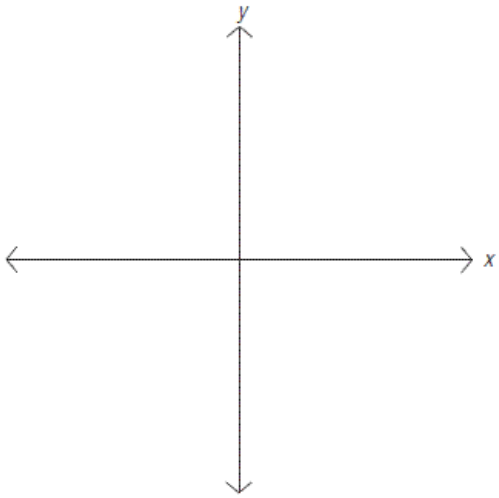
Exploring Graphs of Polynomials

A. *Create four polynomial functions with odd degree. Vary the* ***degree*** *and the* ***sign*** *of the* ***lead coefficient****. Sketch the graphs and label the functions.*

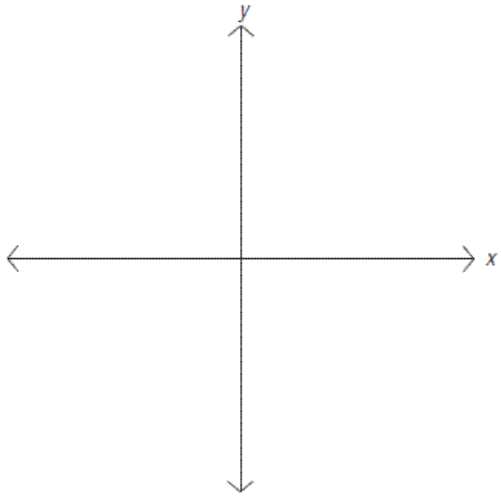
*f(x)=*_____



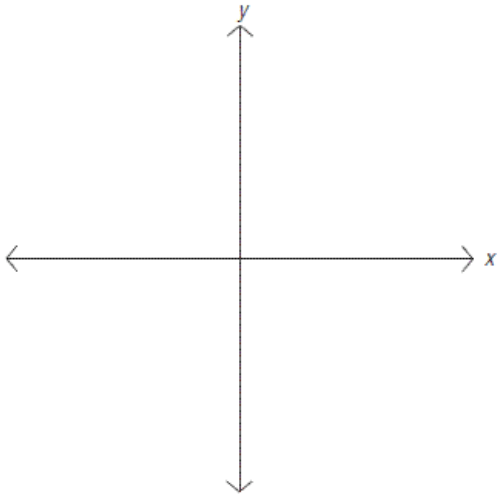
*f(x)=*_____



*f(x)=*_____

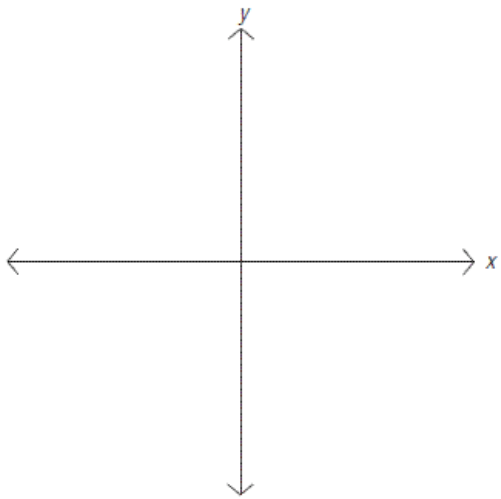


*f(x)=*_____

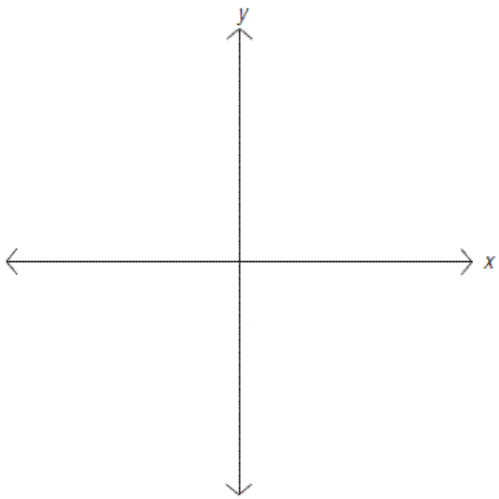


B. Create four polynomial functions with even degree. Vary the degree and the sign of the lead coefficient. Sketch the graphs and label the functions.

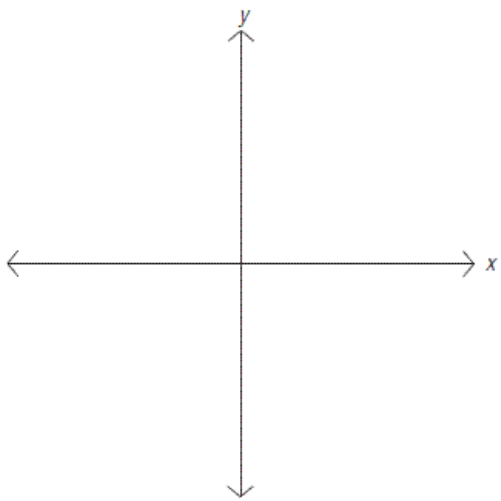
*f(x)=*_____



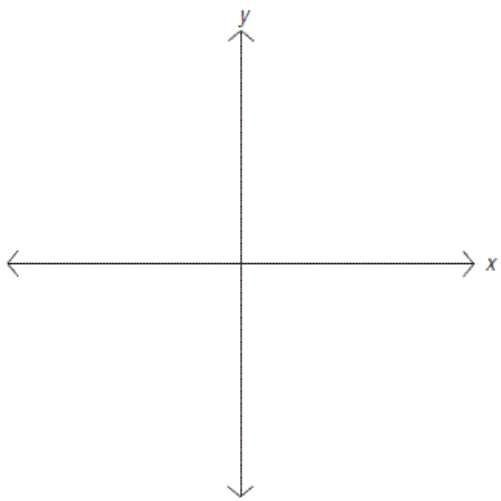
*f(x)=*_____



*f(x)=*_____



*f(x)=*_____



Observations:

1. *What do you notice about the degree of the polynomial and the amount of minima and maxima?*

2. *What do you notice about the shapes of “odd” polynomials vs. “even” polynomials?*

3. *How does the lead coefficient impact the end behavior of the graphs?*

4. *Create 3 equations that do not represent polynomial functions.*

5. *Create 3 equations that do represent polynomial functions.*

6. *Draw a graph that does represent a polynomial function.*

7. *Draw a graph that does not represent a polynomial function.*

Polynomial Matching Warm up Activity

j. $y = x^2(x-3)(x+3)$

i. $y = -x(x+2)(x-4)$

h. $y = x(x-2)(x+4)$

g. $y = -x^2(x-3)(x+3)$

f. $y = (x+2)(x-4)(x+6)$

e. $y = -(x+2)(x-4)(x+6)$

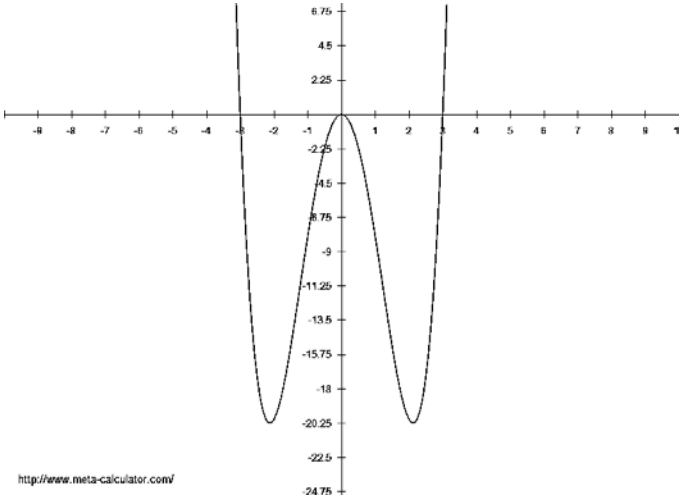
d. $y = -(x+5)^2(x+8)$

c. $y = (x+2)(x-3)(x+4)(x-5)(x+6)$

b. $y = (x-3)(x+3)(x-7)(x+7)$

a. $y = -(x+3)(x-3)(x-7)(x+7)$

1.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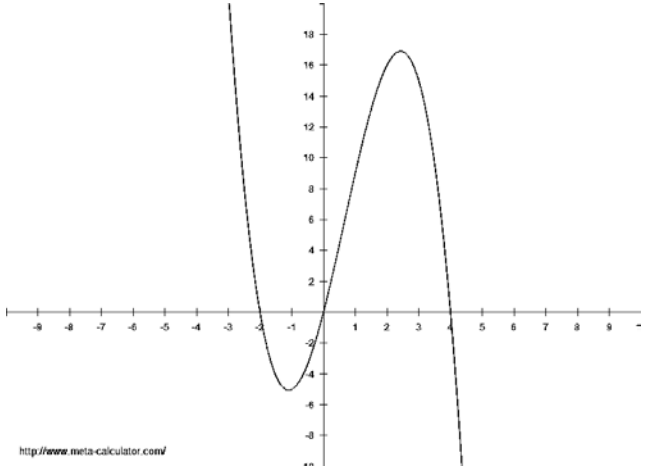
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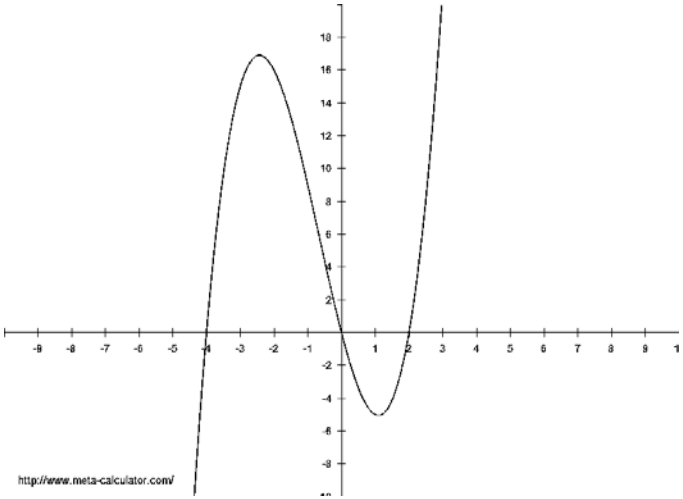
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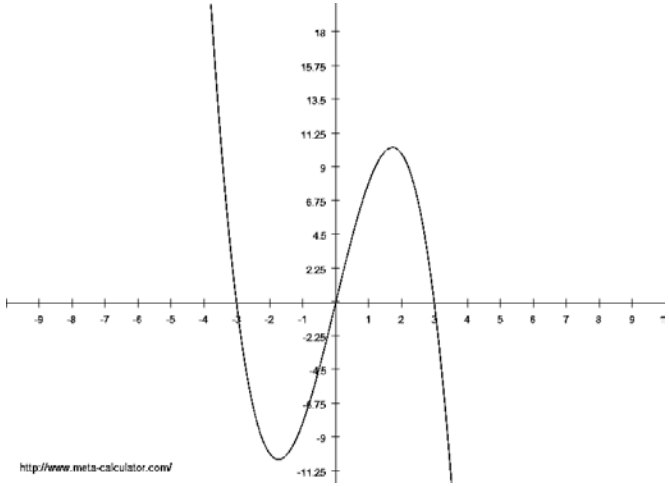
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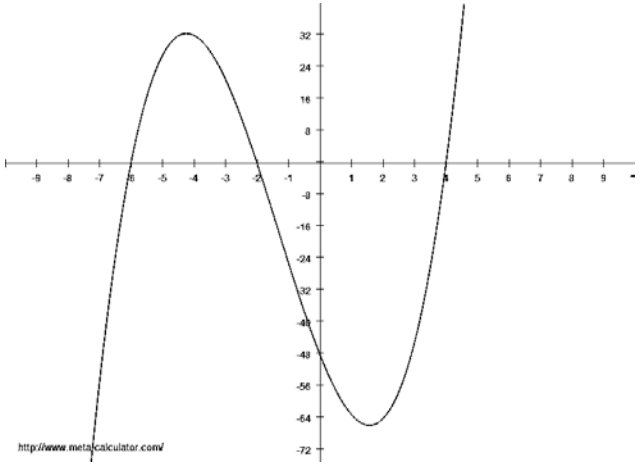
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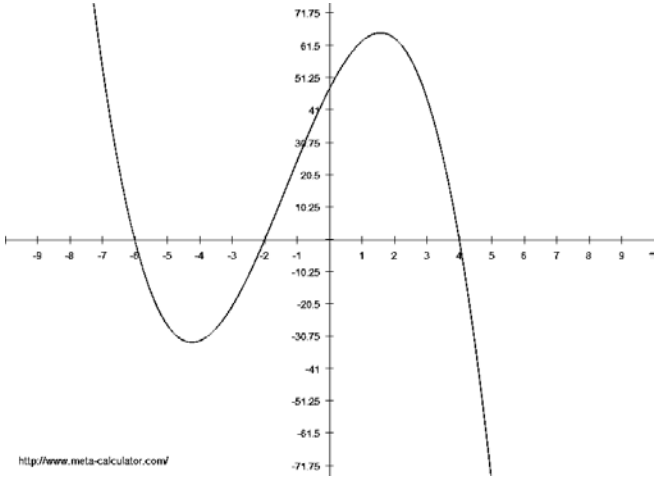
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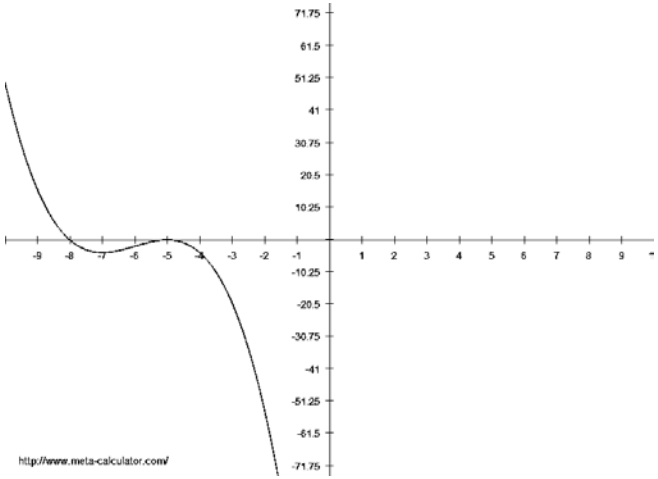
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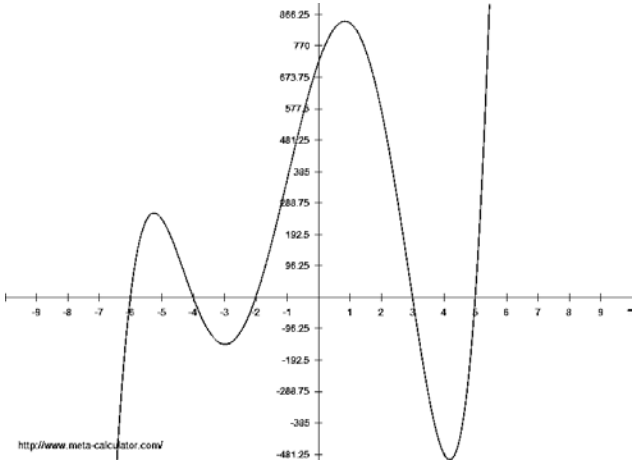
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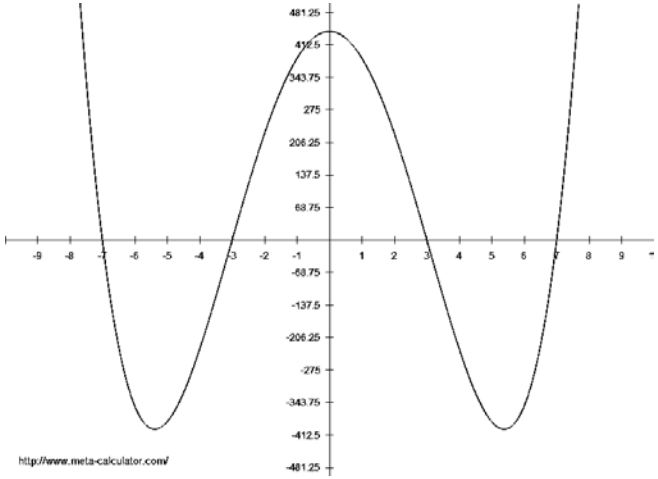
9. 
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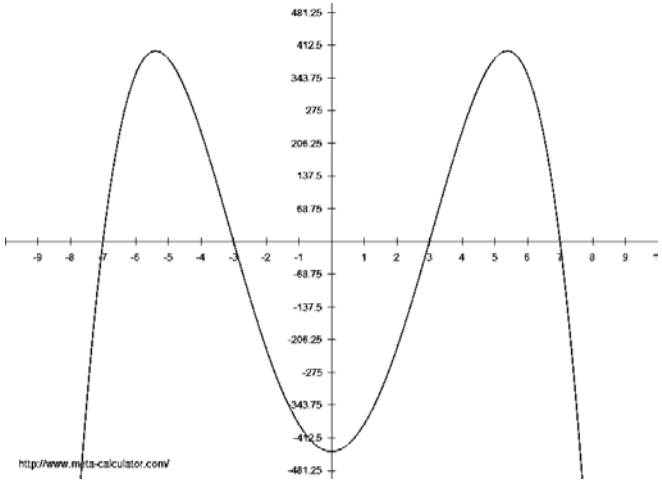
10. 
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-412.5
-481.25
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http://www.meta-calculator.com/

In summary: What do we know so far about polynomial functions and their roots?

Technical Statement *In your own words*

I. There is a difference between zeros, x-intercepts, roots, factors.

Zeros/Roots	
X-intercepts	
Factors	

II. Theorem: Fundamental Theorem of Algebra

A polynomial function of degree *n* has *n* complex zeros. (real and nonreal) Some of these zeros may be repeated.	

III. Theorem: Conjugate Zeros Theorem

Suppose that f(x) is a polynomial function with real coefficients. If *a+bi* is a zero of *f(x)*, then its complex conjugate *a-bi* is also a zero of *f(x)*. ($b \neq 0$)	

III. Theorem: Factor Theorem

A polynomial function has a factor *(x-k)* if and only if *f(k)=0*	

IV. Theorem: Local Extrema and Zeros of Polynomial Functions

A polynomial function of degree *n* has at most *(n-1)* local extrema and at most *n* zeros.	

MHS Algebra 2 Mathematics Curriculum

Unit 1 Addendum

Performance Standard	Instructional Strategies
N.CN.2:	Example: ● Simplify the following expression. Justify each step using the commutative, associative and distributive properties. $(3 - 2i)(-7 + 4i)$ Solutions may vary; one solution follows: $(3 - 2i)(-7 + 4i)$ $3(-7 + 4i) - 2i(-7 + 4i)$ Distributive Property $-21 + 12i + 14i - 8i^2$ Distributive Property $-21 + (12i + 14i) - 8i^2$ Associative Property $-21 + i(12 + 14) - 8i^2$ Distributive Property $-21 + 26i - 8i^2$ Computation $-21 + 26i - 8(-1)$ $i^2 = -1$ $-21 + 26i + 8$ Computation $-21 + 8 + 26i$ Commutative Property $-13 + 26i$ Computation
N.CN.7:	Examples: ● Within which number system can $x^2 = -2$ be solved? Explain how you know. ● Solve $x^2 + 2x + 2 = 0$ over the complex numbers. Find all solutions of $2x^2 + 5 = 2x$ and express them in the form $a + bi$.
N.CN.9:	Examples: • How many zeros does $-2x^2 + 3x - 8$ have? Find all the zeros and explain, orally or in written format, your answer in terms of the Fundamental Theorem of Algebra. • How many complex zeros does the following polynomial have? How do you know? $P(x) = (x^2 - 3)(x^2 + 2)(x - 3)(2x - 1)$

MHS Algebra 2 Mathematics Curriculum

Unit 1 Addendum

Performance Standard	Instructional Strategies
A.APR.2:	The Remainder theorem says that if a polynomial $p(x)$ is divided by $x - a$, then the remainder is the constant $p(a)$. That is, $p(x)=q(x)(x-a)+p(a)$. So if p(a) = 0 then p(x) = q(x)(x-a). Let $p(x)=x^5 -3x^4 +8x^2 -9x+30$. Evaluate p(-2). What does your answer tell you about the factors of p(x)? [Answer: p(-2) = 0 so x+2 is a factor.]
A.APR.3:	Graphing calculators or programs can be used to generate graphs of polynomial functions. Example: Factor the expression $x^3 + 4x^2 - 59x - 126$ and explain how your answer can be used to solve the equation $x^3 + 4x^2 - 59x - 126 = 0$. Explain why the solutions to this equation are the same as the x-intercepts of the graph of the function $f(x) = x^3 + 4x^2 - 59x = 126$.
A.APR.4:	Examples: Use the distributive law to explain why $x^2 - y^2 = (x - y)(x + y)$ for any two numbers x and y. Derive the identity $(x - y)^2 = x^2 - 2xy + y^2$ from $(x + y)^2 = x^2 + 2xy + y^2$ by replacing y by $-y$. Use an identity to explain the pattern $2^2 - 1^2 = 3$ $3^2 - 2^2 = 5$ $4^2 - 3^2 = 7$ $5^2 - 4^2 = 9$ [Answer: $(n + 1)^2 - n^2 = 2n + 1$ for any whole number n.]

MHS Algebra 2 Mathematics Curriculum

Unit 1 Addendum

Performance Standard	Instructional Strategies
A.APR.5:	Examples: • Use Pascal's Triangle to expand the expression $(2x-1)^4$. • Find the middle term in the expansion of $(x^2+2)^{18}$. • $\begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 1 & & 1 & & & \\ & & 1 & & 2 & & 1 & & \\ & 1 & & 3 & & 3 & & 1 & \\ 1 & & 4 & & 6 & & 4 & & 1 \\ \uparrow & & \uparrow & & \uparrow & & \uparrow & & \uparrow \\ {}_4C_0 & & {}_4C_1 & & {}_4C_2 & & {}_4C_3 & & {}_4C_4 \end{array}$ $\longleftarrow (x+1)^3 = x^3+3x^2+3x+1$
A.APR.6:	The polynomial $q(x)$ is called the quotient and the polynomial $r(x)$ is called the remainder. Expressing a rational expression in this form allows one to see different properties of the graph, such as horizontal asymptotes. Examples: • Find the quotient and remainder for the rational expression $\frac{x^3-3x^2+x-6}{x^2+2}$ and use them to write the expression in a different form. • Express f(x) = $\frac{2x+1}{x-1}$ in a form that reveals the horizontal asymptote of its graph. [Answer: f(x) = $\frac{2x+1}{x-1} = \frac{2(x-1)+2}{x-1} = 2 + \frac{2}{x-1}$, so the horizontal asymptote is y = 2.]
A.APR.7:	Examples: • Use the formula for the sum of two fractions to explain why the sum of two rational expressions is another rational expression. • Express $\frac{1}{x^2+1} - \frac{1}{x^2-1}$ in the form a(x)/b(x), where a(x) and b(x) are polynomials.

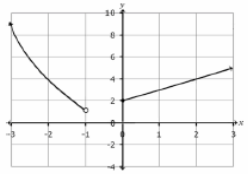
MHS Algebra 2 Mathematics Curriculum

Unit 1 Addendum

Performance Standard	Instructional Strategies
A.REI.2:	Examples: • $\sqrt{x+2} = 5$ • $\frac{7}{8}\sqrt{2x-5} = 21$ • $\frac{x+2}{x+3} = 2\text{Solv}$ • $\sqrt{3x-7} = -4$
A.REI.11:	Students need to understand that numerical solution methods (data in a table used to approximate an algebraic function) and graphical solution methods may produce approximate solutions, and algebraic solution methods produce precise solutions that can be represented graphically or numerically. Students may use graphing calculators or programs to generate tables of values, graph, or solve a variety of functions Example: ● Given the following equations determine the *x* value that results in an equal output for both functions. $f(x) = 3x - 2$ $g(x) = (x + 3)^2 - 1$

MHS Algebra 2 Mathematics Curriculum

Unit 1 Addendum

Performance Standard	Instructional Strategies
A.F.IF.7:	Key characteristics include but are not limited to maxima, minima, intercepts, symmetry, end behavior, and asymptotes. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to graph functions. Examples: • Describe key characteristics of the graph of $f(x) = \lvert x - 3 \rvert + 5$. • Sketch the graph and identify the key characteristics of the function described below. $F(x) = \begin{cases} x + 2 \text{ for } x \geq 0 \\ -x^2 \text{for } x < -1 \end{cases}$  • Graph the function $f(x) = 2^x$ by creating a table of values. Identify the key characteristics of the graph. • Graph $f(x) = 2 \tan x - 1$. Describe its domain, range, intercepts, and asymptotes. • Draw the graph of $f(x) = \sin x$ and $f(x) = \cos x$. What are the similarities and differences between the two graphs?

Unit 2 – Trigonometric Functions

Domain: Functions (F)

Essential Question: How do trigonometric functions help to model periodic phenomena?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Model and solve periodic situations using trigonometric theorems and identities.** **F-TF** Extend the domain of trigonometric functions using the unit circle **F-TF** Model periodic phenomena with trigonometric functions	**How…** **F.TF.1:** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. **F.TF.2:** Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. **F.TF.5:** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.	**Trigonometry Exploration** ▪ Evaluate models using the unit circle and trigonometry and determine the accuracy of the model based on the periodic properties of the function. ▪ Extend the study of functions to include trigonometric functions and their properties. **Trig Exit Check:** Evaluate and analyze a real world situation of a periodic phenomenon through the use of a trigonometric model.	**F.TF.2:** Students may use applets and animations to explore the unit circle and trigonometric functions. Students may explain (orally or in written format) their understanding.

Vocabulary: amplitude, cosine, extension of trigonometric functions, midline, periodic, phenomena, periodicity, radian measure, sine, tangent, trigonometric functions, unit circle

Unit 2 – Trigonometric Functions

Domain: <u>Functions</u> (F)

Essential Question: How do trigonometric functions help to model periodic phenomena?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Model and solve periodic situations using trigonometric theorems and identities.** **F-TF** Prove and apply trigonometric identities	**How…** **F.TF.8:** Prove the Pythagorean identity sin2(θ) + cos2(θ) = 1 and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle.	**Identity Worksheet:** Compare trigonometric functions and their properties to analyze and prove trigonometric identities and apply them to situations.	**F.TF.8:** See Unit Addendum

Vocabulary: amplitude, cosine, extension of trigonometric functions, midline, periodic, phenomena, periodicity, radian measure, sine, tangent, trigonometric functions, unit circle

MHS Algebra 2 Mathematics Curriculum

Unit 2 Addendum

Performance Standard	Instructional Strategies
F.TF.8:	Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model trigonometric functions and periodic phenomena. Example: ● The temperature of a chemical reaction oscillates between a low of 20°C and a high of 120°C. The temperature is at its lowest point when $t = 0$ and completes one cycle over a six hour period. a) Sketch the temperature, T, against the elapsed time, t, over a 12 hour period. b) Find the period, amplitude, and the midline of the graph you drew in part a. c) Write a function to represent the relationship between time and temperature. d) What will the temperature of the reaction be 14 hours after it began? e) At what point during a 24 hour day will the reaction have a temperature of 60° C?

Unit 3 – Modeling with Functions

Domain: Algebra (A)

Essential Question: How do functions help to model, analyze, and predict situations?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Synthesize properties of functions to compare model situations using various non-linear functions.** **A-CED** Create equations that describe numbers or relationships *For A.CED.1, use all available types of functions to create such equations including root functions, but constrain to simple cases. While functions used in A.CED.2, 3, and 4 will often be linear, exponential, or quadratic the types of problems should draw from more complex situations than those addressed in Algebra I.*	**How…** **A.CED.1:** Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* **A.CED.2:** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. **A.CED.3:** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*	**Students will…** **One Variable Models Worksheet:** ▪ Analyze given situations and create an equation or inequality in one variable to model and solve the situation. **Two Variable Activity** ▪ Analyze given situations and create an equation or inequality in two variables to model and solve the situation. **Systems Exit Check** ▪ Analyze given situations and create a system of equations or inequalities in two variables to model and solve the situation.	**A.CED.1:** See Unit Addendum **A.CED.3:** See Unit Addendum

Vocabulary: average rate of change, base, equivalent forms of expressions, exponent, exponential function, exponential model, logarithm, logarithmic function, step function, system of equations, system of inequalities, translation of function

Unit 3 – Modeling with Functions

Domain: Algebra (A)

Essential Question: How do functions help to model, analyze and predict situations?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Synthesize properties of functions to compare model situations using various non-linear functions.** **A-CED** Create equations that describe numbers or relationships *Consider extending this unit to include the relationship between properties of logarithms and properties of exponents,* **F-IF** Interpret functions that arise in applications in terms of the context	**How…** **A.CED.4:** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's Law V=IR to highlight resistance R.* **F.IF.4:** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*	**Students will…** **Functions and Everyday Situations Activity** ▪ Analyze formulas and equations to highlight specific variables to model a situation. functions and everyday situations.p ▪ Quadratics Functions Performance Task: Egg Launch Quadratics Performance Task Eg	**A.CED.4:** See Unit Addendum **F.IF.4:** See Unit Addendum

Vocabulary: average rate of change, base, equivalent forms of expressions, exponent, exponential function, exponential model, logarithm, logarithmic function, step function, system of equations, system of inequalities, translation of function

CONCEPT DEVELOPMENT

Mathematics Assessment Project

CLASSROOM CHALLENGES

A Formative Assessment Lesson

Functions and Everyday Situations

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley

Beta Version

For more details, visit: http://map.mathshell.org
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Functions and Everyday Situations

MATHEMATICAL GOALS

This lesson unit is intended to help you assess how well students are able to:

- Articulate verbally the relationships between variables arising in everyday contexts.
- Translate between everyday situations and sketch graphs of relationships between variables.
- Interpret algebraic functions in terms of the contexts in which they arise.
- Reflect on the domains of everyday functions and in particular whether they should be discrete or continuous.

COMMON CORE STATE STANDARDS

This lesson relates to the following *Standards for Mathematical Content* in the *Common Core State Standards for Mathematics*:

F-IF Interpret functions that arise in applications in terms of a context.
F-IF Analyze functions using different representations.
F-LE Construct and compare linear, quadratic, and exponential models and solve problems.

This lesson also relates to the following *Standards for Mathematical Practice* in the *Common Core State Standards for Mathematics*:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics.
5. Use appropriate tools strategically. (Graphing calculator.)

INTRODUCTION

The lesson unit is structured in the following way:

- Before the lesson, students work individually on an assessment task that is designed to reveal their current understanding and difficulties. You then review their solutions and create questions for students to consider that may help them improve their work.
- During the lesson, small groups work on a collaborative task, matching situations, sketch graphs, and algebraic functions. They also refine the graphs and interpret the formulas to answer questions.
- In a whole-class discussion, students discuss what has been learned and the strategies used.
- In a follow-up lesson, students return to their original task to consider their responses and the questions posed, and use what they have learned to complete a similar task; *Another Four Situations*.

MATERIALS REQUIRED

- Each student will need a copy of the assessment tasks *Four Situations* and *Another Four Situations*.
- Each small group of students will need a mini-whiteboard, a pen, and an eraser, the cut-up card sets: *Everyday Situations 1, 2, and 3; Graphs 1, and 2;* and *Algebraic Functions,* and a graphing calculator to check answers.

There is a projector resource to support whole-class discussions. You may also want to copy the card sets onto transparencies to be used on an overhead projector.

TIME NEEDED

20 minutes before the lesson, a 75-minute lesson, and 20 minutes in a follow-up lesson (or for homework). These timings are approximate. Exact timings will depend on the needs of the class.

BEFORE THE LESSON

Assessment task: *Four Situations* (20 minutes)

Have the students complete this task, in class or for homework, a few days before the formative assessment lesson. This will give you an opportunity to assess the work, and to find out the kinds of difficulties students have with it. You should then be able to target your help more effectively in the follow-up lesson.

Give each student a copy of the task *Four Situations*.

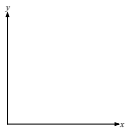
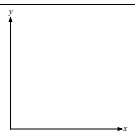
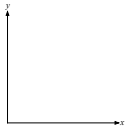
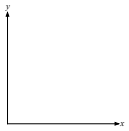
> *Read through the questions and try to answer them as carefully as you can.*
>
> *You are **not** to use a graphing calculator to do this task.*

It is important that, as far as possible, students are allowed to answer the questions without your assistance.

Students should not worry too much if they cannot understand or do everything because, in the next lesson they will engage in a similar task that should help them. Explain to students that by the end of the next lesson, they should expect to answer questions such as these confidently. This is their goal.

Four Situations

1. Sketch a graph to model each of the following situations.
Think about the shape of the graph and whether it should be a continuous line or not.

A: Candle Each hour a candle burns down the same amount. x = the number of hours that have elapsed. y = the height of the candle in inches.	
B: Letter When sending a letter, you pay quite a lot for letters weighing up to an ounce. You then pay a smaller, fixed amount for each additional ounce (or part of an ounce.) x = the weight of the letter in ounces. y = the cost of sending the letter in cents.	
C: Bus A group of people rent a bus for a day. The total cost of the bus is shared equally among the passengers. x = the number of passengers. y = the cost for each passenger in dollars.	
D: Car value My car loses about half of its value each year. x = the time that has elapsed in years. y = the value of my car in dollars.	

2. The formulas below are models for the situations.
Which situation goes with each formula?
Write the correct letter (A, B, C or D) under each one.

$y = \frac{300}{x}$	$y = 12 - 0.5x$	$y = 50 + 20x$	$y = 2000 \times (0.5)^x$
Situation ……	*Situation* ……	*Situation* ……	*Situation* ……

3. Answer the following questions using the formulas.
Under each answer show your reasoning.
   a. How long will the candle last before it burns completely away? ……
   b. How much will it cost to send a letter weighing 8 ounces? ……
   c. If 20 people go on the coach trip, how much will each have to pay? ……
   d. How much will my car be worth after 2 years? ……

Assessing students' responses

Collect students' responses to the task and note what their work reveals about their current levels of understanding, and their different approaches.

We suggest that you do not score students' work. The research shows that this will be counterproductive, as it will encourage students to compare their scores, and will distract their attention from what they can do to improve their mathematics.

Instead, help students to make further progress by summarizing their difficulties as a series of questions. Some suggestions for these are given in the *Common issues* table on the next page. These have been drawn from difficulties observed in trials of this unit.

We suggest you make a list of your own questions, based on your students' work. We recommend you either:

- Write one or two questions on each student's work, or
- Give each student a printed version of your list of questions, and highlight the appropriate questions for individual students.

If you do not have time to do this, you could select a few questions that will be of help to the majority of students, and write these on the board when you return the work to the students.

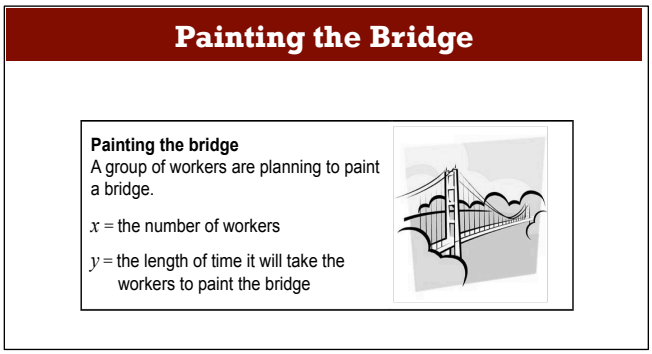
Common issues:	Suggested questions and prompts:
Students have a limited repertoire of graphs (Q1) For example: The student draws increasing graphs for A, C, D. Or: The student draws straight line graphs for C, D.	• As x increases does y increase or decrease? Why? • As x goes up in equal steps, does y go up in equal steps? Why?
Student draws continuous lines for all the graphs (Q1)	• Is x a discrete or continuous variable? Why? • Is y a discrete or continuous variable? Why?
Students' graphs cut the axes at inappropriate places (Q1) For example: The graph meets or crosses *y*-axis for C. Or: The graph meets or crosses *x*-axis for C, D.	• What happens when no passengers go on the coach trip? • How many passengers would you need for y =0? (cost for each passenger is nothing!).
Student draws a graph that consists of two straight lines of differing slopes	• What does the slope represent? Why does the steepness of your slope change?
Student matches the formulas incorrectly (Q2)	• Check your matching by trying a few values for x. • As x increases, does y increase or decrease? How can you tell? • As x increases in equal steps, does y change in equal steps?
Students are unable to interpret and use the formulas correctly (Q3)	• What does each statement tell you about the value of x? • What does each statement tell you about the value of y?

SUGGESTED LESSON OUTLINE

Whole-class introduction (15 minutes)

Give each student a mini-whiteboard, a pen, and an eraser. Maximize participation in the whole-class discussion by asking all students to show you a solution on their mini-whiteboards.

Use Slide P-1 to write or project the following situation:



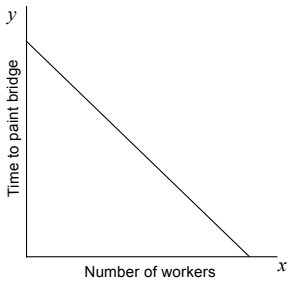
Ask the students to sketch a graph that describes this situation:

Look at this situation carefully. Can you sketch a graph to show how y will depend on x?

Show me your answer on your mini-whiteboard and be prepared to give an explanation for what you have drawn.

Students may well come up with a range of answers. If this is the case, draw a few on the board and discuss which are appropriate models for the situation. Help students to develop their ideas until they are able to fully understand the most appropriate graph.

Some students, for example may draw a linear graph:



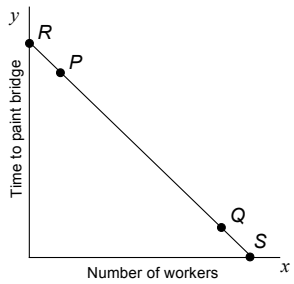
If this is the case, spend some time discussing the meaning of a linear graph. It is likely that students who have viewed the problem as a linear situation understand that the more workers that are involved, the less time it will take to paint the bridge. Explore the specific features of the graph and encourage students who have drawn non-linear graphs to participate in the discussion as well. It may be helpful to label some points on the graph:

What does point P tell you? [If only a few paint the bridge, it will take a long time.]

What about point Q? [A lot of workers will take a shorter time.]

What about the points R and S, where the line crosses the axes?

How can we change this graph so that it is a better model of the situation?



This kind of questioning should help students to see that the graph should not cross the axes. However, some students may simply suggest erasing the ends of the graph as a way of making it a better model of the situation. Help students to understand why a curved graph is a more appropriate model. It might be appropriate to consider what happens to the amount of time taken to paint the bridge as the number of workers varies:

Suppose we know the time it takes for a specific numbers of workers to paint the bridge.

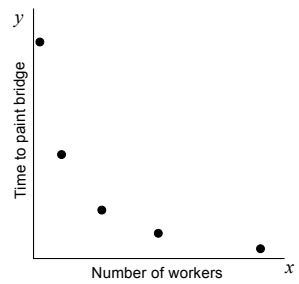
What would happen if we doubled the number of workers that paint the bridge? [You would halve the time it takes.]

What would this look like on the graph?

If we halved the number of workers, what would this look like?

Will these points be in a straight line?

What shape will these points make?



If some students in the class have successfully drawn a curved graph, encourage them to explain their reasoning to the rotest of the class. Emphasize that strictly speaking the graph should not be continuous, as it does not make sense to have fractions of people painting a bridge. The domain of the graph should be discrete.

Once students are happy with the shape of the graph, ask them to think about an appropriate function that describes it.

Can you suggest a possible algebraic function that will fit this graph? [$y = \frac{A}{x}$ *]*

If students are struggling to come up with a possible function, consider a specific situation. Drawing up a table of values may also help students to see the relationship between x and y.

If it takes 20 people 5 years to paint a bridge, how long would it take 10 people? [10 years.]

How long would it take 50 people? [2 years.]

What would the function be in this case? [$xy = 100$ or $y = 100/x$]

Explain to students that during this lesson they are going to be considering a number of similar situations and thinking about the graphs and algebraic functions that can be used to model them.

Collaborative activity: Matching situations to graphs (20 minutes)

Organize the class into groups of two or three students.

Give each group the cut-up card sets: *Everyday Situations 1, 2* and *3*, and *Graphs 1* and *2*. **Do not give out graphing calculators!**

Explain how students are to work collaboratively. Slide P-2 of the projector resource summarizes these instructions:

Take turns to match a situation card to one of the sketch graphs.

If you place a card, explain why that situation matches that graph.

Everyone in your group should agree on, and be able to explain, your choice.

If you think the graph could be improved in any way, then say how it should be changed. For example you may think that it should be discrete rather than continuous.

There are two blank graphs for you to complete!

Arrange pairs side by side (not on top of one another) so I can see them as I walk around.

You may want to check that everyone understands each situation before students begin on the task. Tell students to ignore the questions on the situation cards for the moment.

The purpose of this structured group work is to make students engage with each others' reasoning, and take responsibility for the understanding of the group. Encourage students not to rush, but spend time justifying fully the pairing of the cards.

Whilst students work on the collaborative activity you have two tasks, to notice students' approaches and difficulties and to support student reasoning:

Note different student approaches to the task

Notice how students make a start on the task, where they get stuck, and how they interpret the situations. Do they sort the cards in any way before they try to match them? If so, what is their strategy? How do students go about completing the blank cards? Notice whether students are addressing the difficulties they experienced in the assessment task. You may want to use the questions in the *Common issues* table to help to address any misconceptions.

If students are struggling to get started on the task suggest that they focus on Situations A, B, C and D and see if they can find graphs to match these four situations first.

If you find that students are having difficulty imagining a situation, then talk it through with them. Ask them to describe orally what happens to y as x changes.

Support student reasoning

Try not to make suggestions that move students towards a particular approach to the task. Instead, ask questions that help students clarify their thinking. It is important that students are encouraged to engage with their partner's explanations and take responsibility for each other's understanding.

> *Peter, you matched this graph with this situation. Gail, can you explain why Peter matched these two cards?*

Prompt students to think and reason more deeply.

> *What happens to the temperature of the boiling kettle if you leave it standing?*
>
> *Is the water in the kettle cooling at a constant rate?*
>
> *When the kettle is very hot, does it cool down more quickly or more slowly than when it is nearly at room temperature? Why?*
>
> *Which graph best shows this relationship?*
>
> *Try out some values for this situation.*

Encourage students to label their axes with the descriptions. Organizing their values into a table may help students to see a relationship between x and y.

Collaborative activity: Matching situations and graphs to formulas (20 minutes)

When students have had a chance to match up the situations and graphs, give each group the cut-up cards: *Algebraic Functions*, a large sheet of paper, and glue stick for making a poster.

> *Now match these cards to the pairs you already have on the table.*
>
> *You should be able to do this without using a graphing calculator.*
>
> *After you have matched the function, try to answer the question on the right hand side of the situations card.*

Students should be able to match the linear functions more easily, but may find the others much more challenging. Encourage them to use 'strategic substitutions' (**not** graphing calculators) to make their decisions:

Can you group these functions in a way that will help you match them?

Which of these functions are linear? Non-linear?

Which are increasing/ decreasing?

Where does this function cross the x-axis / y-axis?

Where does this function reach its maximum/ minimum value?

Show me a function that never crosses the x-axis. How do you know?

Show me a function that intersects the origin. How do you know?

Show me a function that has two x-intercepts. How do you know?

Support the students as in the first collaborative activity.

When students have matched an algebraic function card to the other two, allow them to check their answers using graphing calculators, then encourage them to try to answer the question on the situations card by looking at the **form** of the algebraic function. For example:

OK, you think that the function that shows how the height of the golf ball changes is:
$y = 30x - 5x^2$
How can we answer the question: "When does the ball hit the ground?" [Let y = 0.]

At this point, the cards may be stuck down and the questions answered on the poster.

Whole-class discussion (20 minutes)

Organize a discussion about what has been learned and the different strategies students have used to match the cards. You may, first, want to select cards that most groups matched correctly to encourage good explanations, before selecting one or two cards that groups found difficult to match. It is likely that some groups will not have matched all the cards. The aim of this discussion is to encourage contributions from everyone in the class, rather than merely checking for correct matches.

Give me a situation/graph/algebraic function that was easy to match. Why was this?

Once one group has justified their choice for a particular match, ask students from other groups to contribute ideas of alternative matches/approaches. The intention is that you focus on getting students to understand and share their **reasoning** and explore the different approaches used within the class.

Did anyone find this situation/graph/algebraic function difficult to match or use a different strategy to the one explained?

When discussing a match, encourage students to listen carefully to each other's explanations and comment on which explanation was easier to follow.

Did anyone manage to match a graph with the Folding Paper situation? Which graph did you choose?

Why did you decide this graph best matched this situation?

Can anyone explain this match in a different way?

Did anyone choose a different graph for the Folding Paper situation?

Which explanation is easiest to understand?

Explore the situation in as much depth as possible, rather than just discussing the answer to the question on the card. Consider features of the graph as well as exploring in detail the ways in which the algebraic function relates to the situation.

In what ways could we improve graph G3 as a model for the Folding Paper situation? [The graph should be a series of discrete points as the number of folds can only be whole numbers. It should also stop at x = 8 as you cannot physically fold paper more than 8 times.]

Which algebraic function best matches graph G3? [A12: $y = \frac{2^x}{1000}$*.]*

What does the 1000 stand for in this function? [The thickness of the paper in inches.]

How thick would the paper be if you could fold it ten times? [about 1 inch.]

How thick would the paper be if you could fold it 50 times? [17 million miles!]

There will not be time to discuss all of the cards in this way and it is not necessary to do so. A discussion of a few cards that maximizes student involvement, and encourages a thoughtful discussion and reflection of students' work, is recommended.

Follow-up lesson: *Another Four Situations* (20 minutes)

Return the original assessment task *Four Situations* to the students, together with a copy of the task *Another Four Situations*.

If you have not added questions to individual pieces of work, then write your list of questions on the board. Students should select from this list only those questions they think are appropriate to their own work.

Look at your original responses and think about what you have learned this lesson.

Carefully read through the questions I have written.

Spend a few minutes thinking about how you could improve your work.

You may want to make notes on your mini-whiteboard.

Using what you have learned, try to answer the questions on the new task Another Four Situations.

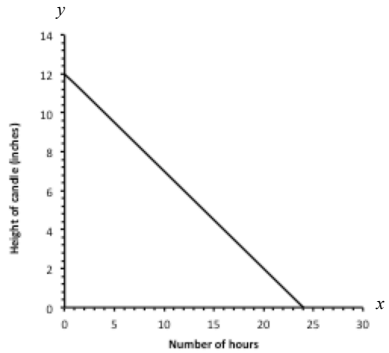
Some teachers give this for homework.

SOLUTIONS

Assessment Task: Four Situations.

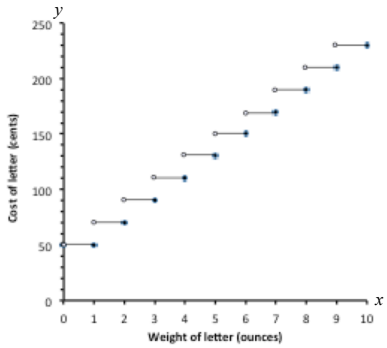
A: Candle

1. The candle sketch graph should be linear and decreasing as shown. Both time and height are continuous variables.
2. It corresponds to the function $y = 12 - 0.5x$
3. It will burn out completely after 24 hours.



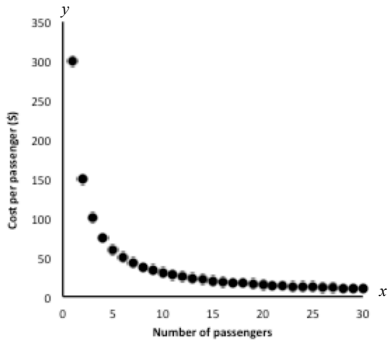
B: Letter

1. The graph should be linear and increasing and passing through a point (0,A) where A>0.
2. Some students may realize that the *x*-axis is continuous and the *y*-axis will be discrete. Also the costs will jump as shown here.
3. The cost of a letter weighing x ounces is given by the function: $y = 30 + 20x$
4. A letter weighing 8 ounces will cost 190 cents.



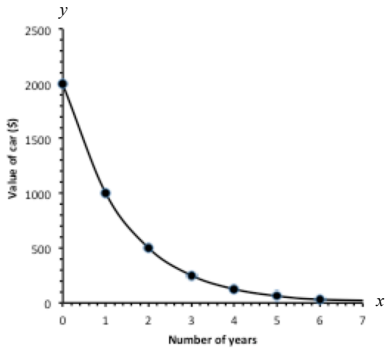
C: Bus

1. The graph should be non-linear (a hyperbola), decreasing. It should not meet either axis. Some students may realize that the x-axis is discrete and the graph will consist of points.
2. The function modeling this is: $y = \frac{300}{x}$
3. If 20 people go on the trip, it will cost each person 15 dollars.



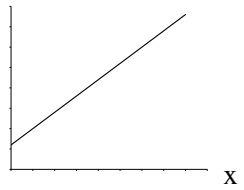
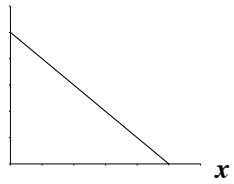
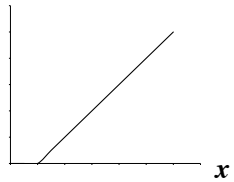
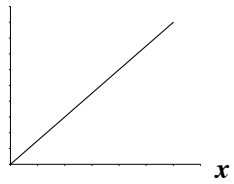
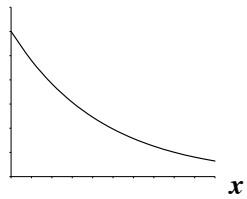
D: Car value

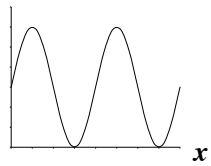
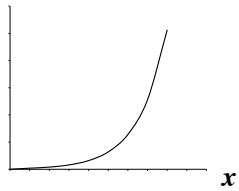
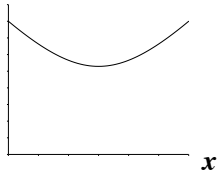
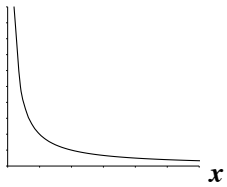
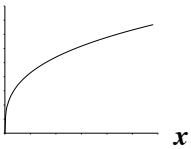
1. The graph should be smooth, continuous, decreasing exponential.
2. The function modeling this is: $y = 2000 \times (0.5)^x$
3. After 2 years the car will be worth $500.

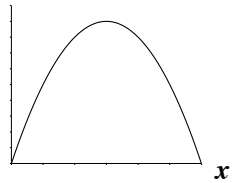
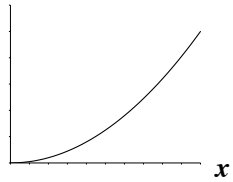


Collaborative activity: Making Posters

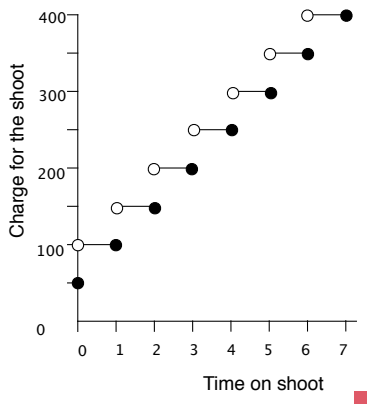
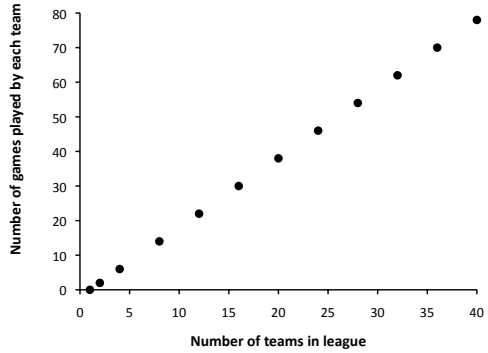
Students may change the shape of some of the curves or show the graph as discrete rather than continuous.

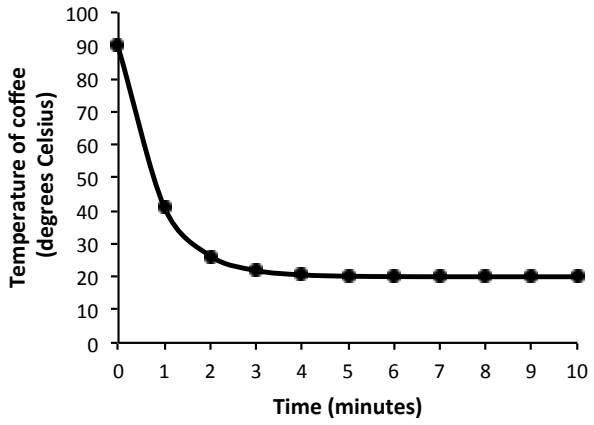
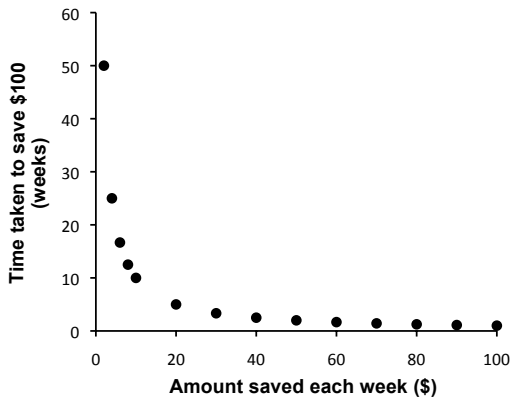
Situation	Graph	Formula
A. Plumber A plumber charges a fixed fee for coming to your house, then charges a fixed amount per hour on top of this. x = the time the job takes in hours. y = the total cost of the plumber's time in dollars.	**G2** *y*  Step function is better	**A3** $y = 40x + 60$ Plumber charges $180 for a 3-hour job.
B. Cycling A cyclist travels along a direct route from town A to town B. x = the distance of the cyclist from town A in miles. y = the distance of the cyclist from town B in miles.	**G1** *y* **0**  	**A4** $y = -x + 100$ Towns are 100 miles apart.
C. Movie subscription You get two movies free, but then you get charged at a fixed rate per movie. x = the number of movies seen. y = the total money spent in dollars.	**G4** *y*  Graph to be drawn on by student. Discrete points would be better.	**A1** $y = 5x - 10$ The fixed rate per movie is $5.
D. Internet café An internet café charges a fixed amount per minute to use the internet. x = the number of minutes spent on the internet. y = the cost of using the internet in dollars.	**G7** *y*  	**A2** $y = \frac{3x}{4}$ $8 will buy 6 minutes.
E. Cooling kettle A kettle of boiling water cools in a warm kitchen. x = the time that has elapsed in minutes. y = the temperature of the kettle in degrees Celsius.	**G9** *y*  	**A11** $y = 20 + 80 \times (0.27)^x$ As the water cools it approaches the room temperature of 20°C.

F. Ferris wheel A ferris wheel turns round once. x = the time that has elapsed in seconds. y = the height of a seat from the ground in meters.	**G11** *y*  *x*	**A10** $y = 30 + 30\sin(18x)$ The starting point is $x = 0$ and $y = 30$. After one complete turn $y = 30$.The Ferris wheel is at this height at x = 10 (half a turn) and x = 20 (complete turn). It turns once every 20 seconds.
G. Folding paper A piece of paper is folded in half. It is then folded in half again, and again…. x = the number of folds. y = the thickness of the paper in inches.	**G3** *y*  *x* The student may change the shape of the curve. Paper can be folded up to 8 times. Discrete points would be better.	**A12** $y = \frac{2^x}{1000}$ Folding it ten times results in thickness of about an inch. This is impossible! $x \le 8$ in practice (try it!).
H. Speed of golf shot A golfer hits a ball. x = the time that has elapsed in seconds. y = the speed of the ball in meters per second.	**G8** *y*  *x*	**A7** $y = 10\sqrt{(x-3)^2 + 7}$ Speed is a minimum after 3 seconds.
I. Test drive A car drives along a test track. x = the average speed of the car in meters per second. y = the time it takes to travel the length of the track in seconds.	**G1** *y*  *x*	**A5** $y = \frac{200}{x}$ Distance = speed × time = $y \times x$. The track is 200 meters long
J. Balloon A man blows up a balloon. x = volume of air he has blown in cubic inches. y = diameter of the balloon in inches.	**G6** *y*  *x* If puffs are allowed the graph will show steps. At some point the balloon will pop.	**A6** $y = \frac{5}{4}\sqrt[3]{x}$ $x = 1{,}000$. Diameter of balloon is about 1 foot. (12.5 inches)

K. Height of golf shot. A golfer hits a ball. x = the time that has elapsed in seconds. y = the height of the ball in meters.	**G5** y  x	**A9** $y = 30x - 5x^2$ The ball hits the ground when: $5x(6 - x) = 0;\quad x = 0$ or $x = 6$. Ball hits ground after 6 seconds.
L. Film projector A film is shown on a screen using a small projector. x = the distance from the projector to the screen in feet. y = the area of the picture in square feet.	**G12** y  x	**A8** $y = \frac{1}{4}x^2$ When projector is 10 feet away, area of picture is 25 square feet.

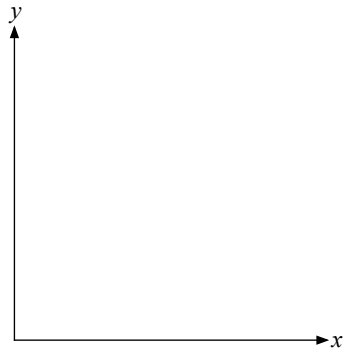
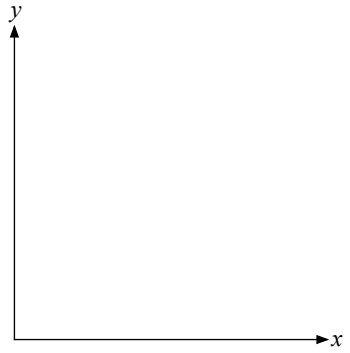
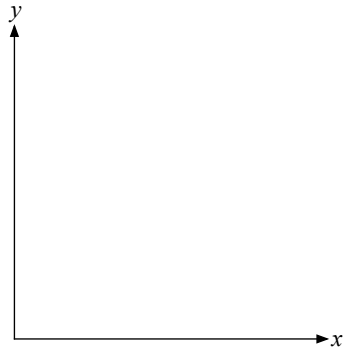
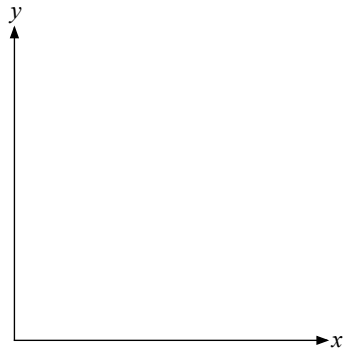
Assessment Task: *Another Four Situations*

A: Photographer 1. The graph should be linear and increasing and passing through a point (0,A) where A>0. Some students may realize that the *x*-axis is continuous and the *y*-axis will be discrete. The charges will jump as shown here. 2. It corresponds to the function $y = 50(1 + x)$ 3. The photographer will charge $400 for a 7 hour 'shoot'.	
B: Football 1. The graph should be linear and increasing and passing through a point (1,0). Some students may realize that both the *x*-axis and the *y*-axis are discrete. 2. The function modeling this is: $y = 2x - 2$ 3. When there are 20 teams in the league, each team will play 38 games.	

C: Cup of coffee 1. The graph should be smooth, continuous, decreasing exponential. 2. The function modeling this is: $y = 20 + 70 \times (0.3)^x$ 3. After 3 minutes, the temperature of the coffee will be 21.89°C.	
D: Saving up 1. The graph should be non-linear (a hyperbola), decreasing. It should not meet either axis. Some students may realize that the *y*-axis is discrete and the graph will consist of points. 2. The function modeling this is: $y = \frac{100}{x}$ 3. It will take Tanya 20 weeks to save for the coat.	

Four Situations

1. Sketch a graph to model each of the following situations.
   Think about the shape of the graph and whether it should be a continuous line or not.

A: Candle Each hour a candle burns down the same amount. x = the number of hours that have elapsed. y = the height of the candle in inches.	
B: Letter When sending a letter, you pay quite a lot for letters weighing up to an ounce. You then pay a smaller, fixed amount for each additional ounce (or part of an ounce.) x = the weight of the letter in ounces. y = the cost of sending the letter in cents.	
C: Bus A group of people rent a bus for a day. The total cost of the bus is shared equally among the passengers. x = the number of passengers. y = the cost for each passenger in dollars.	
D: Car value My car loses about half of its value each year. x = the time that has elapsed in years. y = the value of my car in dollars.	

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2. The formulas below are models for the situations.

Which situation goes with each formula?

Write the correct letter (A, B, C or D) under each one.

$y = \frac{300}{x}$	$y = 12 - 0.5x$	$y = 30 + 20x$	$y = 2000 \times (0.5)^x$
Situation ……	*Situation ……*	*Situation ……*	*Situation ……*

3. Answer the following questions using the formulas.
Under each answer show your reasoning.

a. How long will the candle last before it burns completely away?

---

b. How much will it cost to send a letter weighing 8 ounces?

---

c. If 20 people go on the coach trip, how much will each have to pay?

---

d. How much will my car be worth after 2 years?

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Everyday Situations 1

A. Plumber

A plumber charges a fixed fee for coming to your house, then charges a fixed amount per hour on top of this.

x = the time the job takes in hours.

y = the total cost of the plumber's time in dollars.

How much does the plumber charge for a 3-hour job?

B. Cycling

A cyclist travels along a direct route from town A to town B.

x = the distance of the cyclist from town A in miles.

y = the distance of the cyclist from town B in miles.

How far apart are the towns?

C. Movie subscription

You get two movies free, but then you get charged at a fixed rate per movie.

x = the number of movies seen.

y = the total money spent in dollars.

What is the fixed rate per movie?

D. Internet café

An internet café charges a fixed amount per minute to use the internet.

x = the number of minutes spent on the internet.

y = the cost of using the internet in dollars.

How many minutes will $8 buy?

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Everyday Situations 2

E. Cooling kettle

A kettle of boiling water cools in a warm kitchen.

x = the time that has elapsed in minutes.

y = the temperature of the kettle in degrees Celsius.

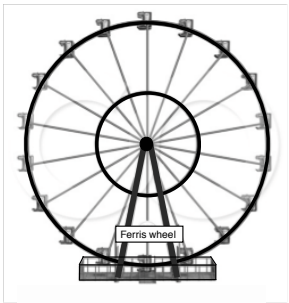
What is the temperature of the room?

F. Ferris wheel

A Ferris wheel turns round once.

x = the time that has elapsed in seconds.

y = the height of a seat from the ground in meters.



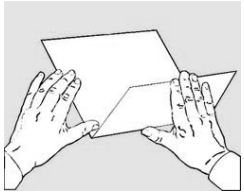
How long does it take the Ferris wheel to turn once?

G. Folding paper

A piece of paper is folded in half. It is then folded in half again, and again….

x = the number of folds.

y = the thickness of the paper in inches.



How thick would the paper be if you could fold it 10 times?

H. Speed of golf shot

A golfer hits a ball.

x = the time that has elapsed in seconds.

y = the speed of the ball in meters per second.

When is the ball travelling most slowly?

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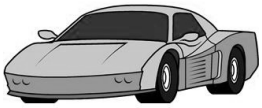
Everyday Situations 3

I. Test drive

A car drives along a test track.

x = the average speed of the car in meters per second.

y = the time it takes to travel the length of the track in seconds.



How long is the track?

J. Balloon

A man blows up a balloon.

x = the volume of air he has blown in cubic inches.

y = the diameter of the balloon in inches.

What is the diameter of the balloon when the man has blown in 1000 cubic inches?

K. Height of golf shot.

A golfer hits a ball.

x = the time that has elapsed in seconds.

y = the height of the ball in meters.

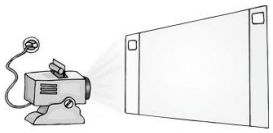
When does the ball hit the ground?

L. Film projector

A film is shown on a screen using a small projector.

x = the distance from the projector to the screen in feet.

y = the area of the picture in square feet.



How large is the picture when the screen is 10 feet away?

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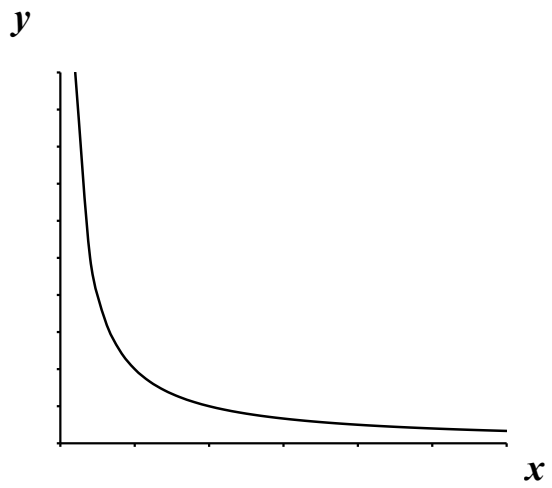
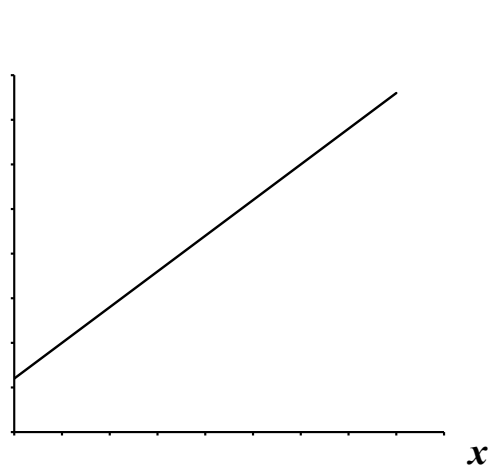
Graphs 1

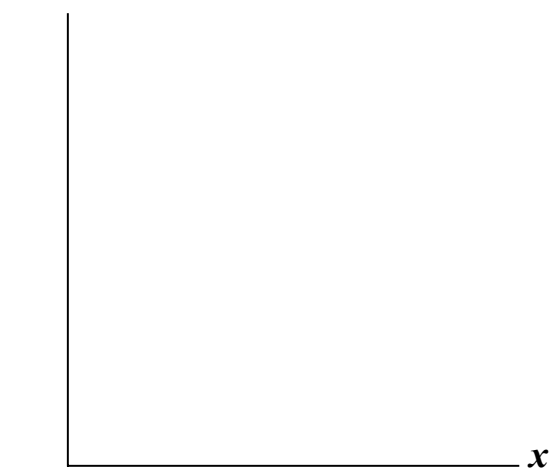
G1



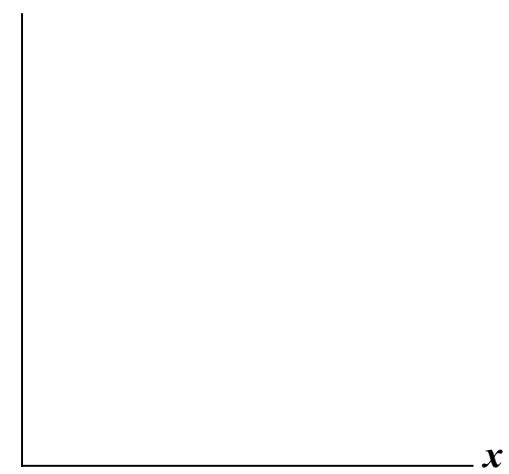
G2



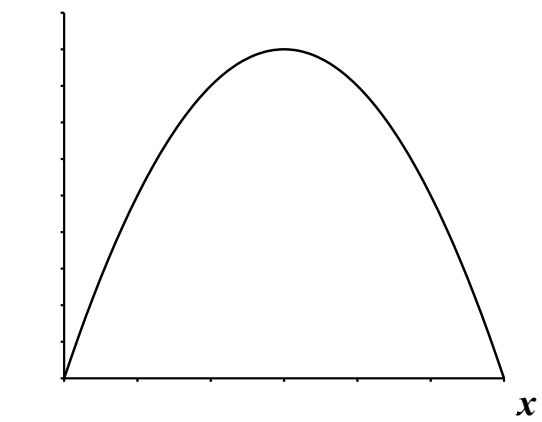
G3



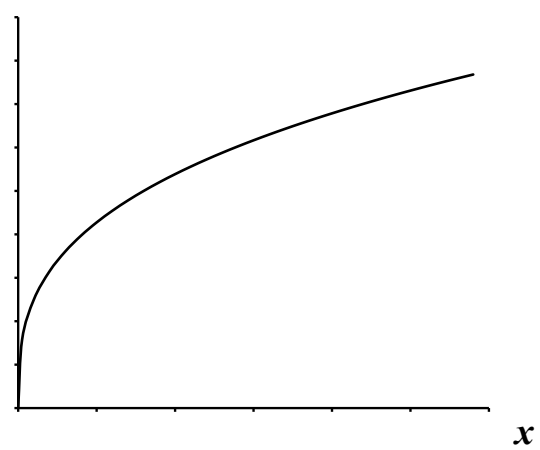
G4



G5



G6



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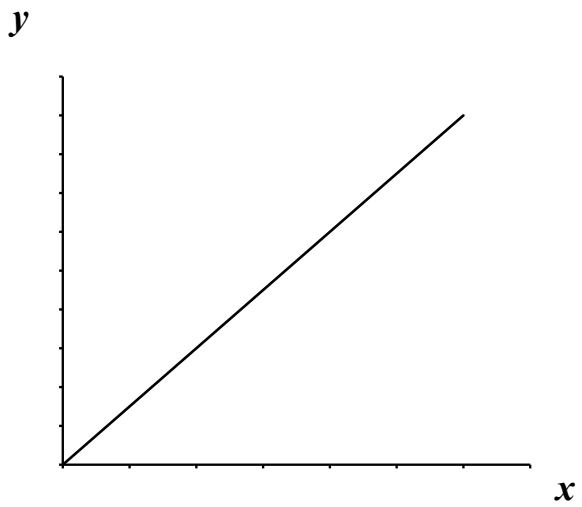
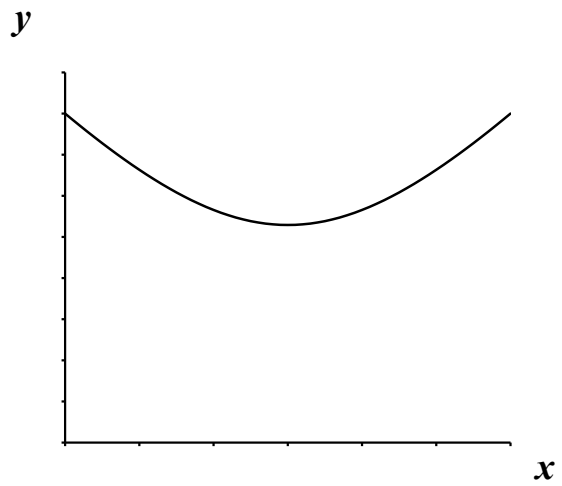
Graphs 2

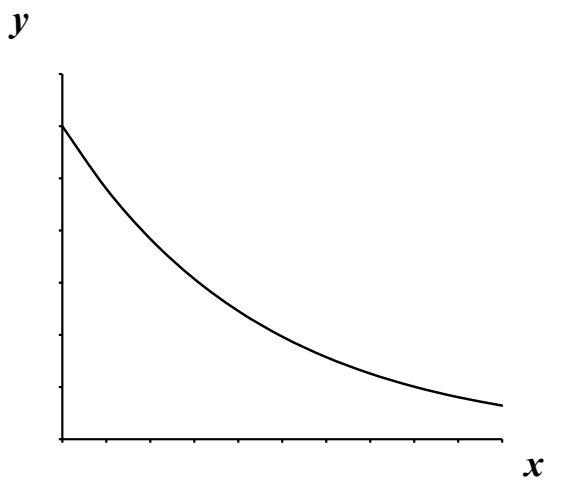
G7



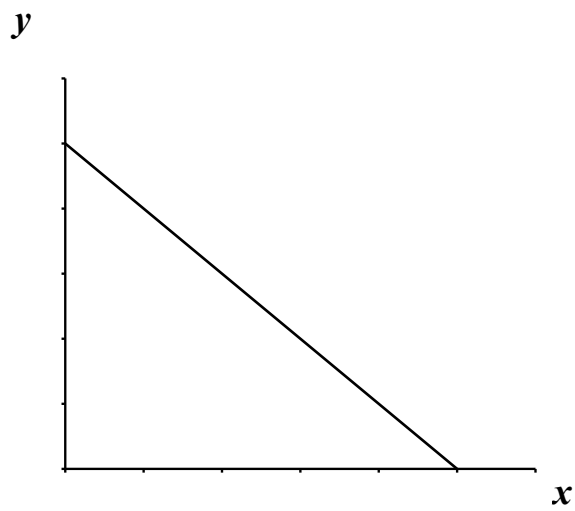
G8



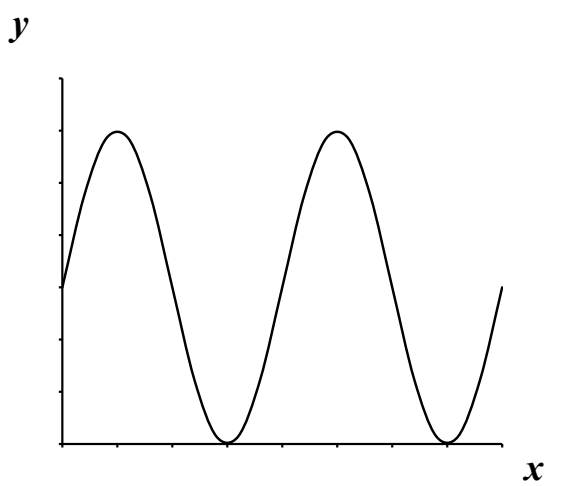
G9



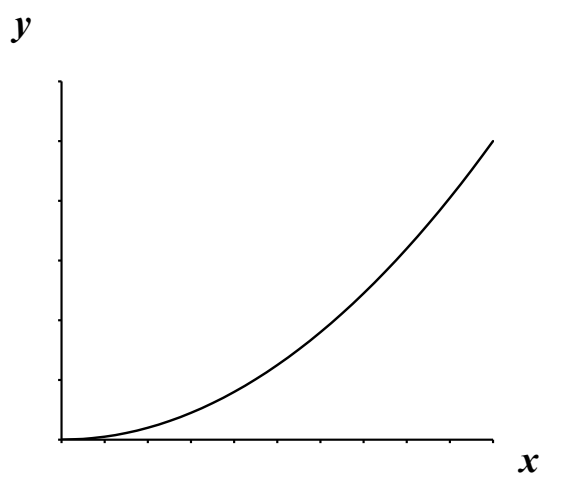
G10



G11



G12



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Algebraic Functions

A1 $$y = 5x - 10$$	A2 $$y = \frac{3x}{4}$$
A3 $$y = 40x + 60$$	A4 $$y = -x + 100$$
A5 $$y = \frac{200}{x}$$	A6 $$y = \frac{5}{4}\sqrt[3]{x}$$
A7 $$y = 10\sqrt{(x-3)^2 + 7}$$	A8 $$y = \frac{1}{4}x^2$$
A9 $$y = 30x - 5x^2$$	A10 $$y = 30 + 30\sin(18x)$$
A11 $$y = 20 + 80 \times (0.27)^x$$	A12 $$y = \frac{2^x}{1000}$$

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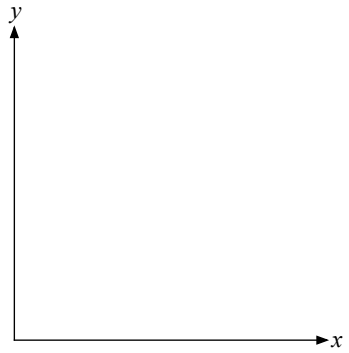
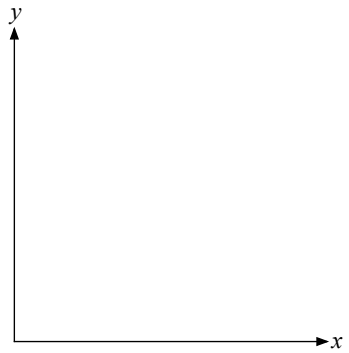
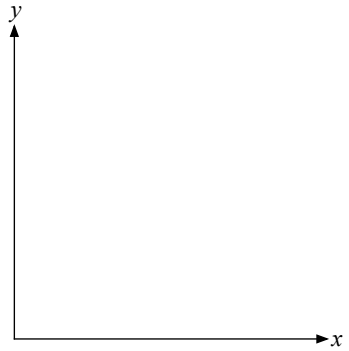
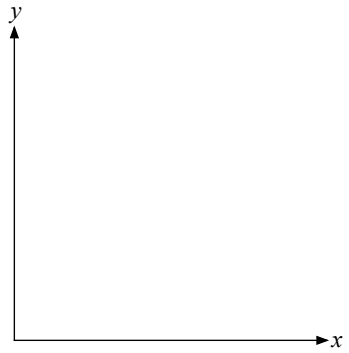
Another Four Situations

1. Sketch a graph to model each of the following situations.
Think about the shape of the graph and whether it should be a continuous line or not.

A: Photographer For each 'shoot' a photographer charges a fixed fee for expenses, then a fixed amount for each hour (or part of an hour.) x = the time a 'shoot' takes in hours. y = the total amount the photographer charges.	
B: Football In a football league, each team plays all other teams twice. x = the number of teams. y = the number of games played by one team.	
C: Cup of coffee A cup of coffee cools in a warm diner. x = the time that has elapsed in minutes. y = the temperature of the coffee in degrees Celsius.	
D: Saving up Tanya saves a fixed amount each week until she has enough money in the bank to buy a coat. x = the amount saved each week. y = the time that it takes Tanya to save enough for the coat.	

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2. The formulas below are models for the situations.

Which situation goes with each formula?

Write the correct letter (A, B, C or D) under each one.

$y = 20 + 70 \times (0.3)^x$	$y = \frac{100}{x}$	$y = 2x - 2$	$y = 50(1 + x)$
Situation ……	*Situation ……*	*Situation ……*	*Situation ……*

3. Answer the following questions using the formulas.
Under each answer show your reasoning.

a. How much will the photographer charge for a 7-hour 'shoot'?

---

b. If there are 20 teams in the league, how many games will each team play?

---

c. What will be the temperature of the coffee after 3 minutes?

---

d. If Tanya saves $5 a week, how long will it take her to save for the coat?

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Painting the Bridge

Painting the bridge

A group of workers are planning to paint a bridge.

x = the number of workers

y = the length of time it will take the workers to paint the bridge

Matching the Cards

1. Take turns to match a situation card to one of the sketch graphs.

2. If you place a card, explain why that situation matches the graph.

3. If you think the graph could be improved in any way, then say how it should be changed. (For example, you may think that it should be discrete points rather than a continuous line.)

4. Arrange cards side by side (not on top of one another) so I can see them as I walk round.

Everyone in your group should agree on
and be able to explain your choice.

Mathematics Assessment Project

CLASSROOM CHALLENGES

This lesson was designed and developed by the
Shell Center Team
at the
University of Nottingham
Malcolm Swan, Nichola Clarke, Clare Dawson, Sheila Evans
with
Hugh Burkhardt, Rita Crust, Andy Noyes, and Daniel Pead

It was refined on the basis of reports from teams of observers led by
David Foster, Mary Bouck, and Diane Schaefer
based on their observation of trials in US classrooms
along with comments from teachers and other users.

This project was conceived and directed for
MARS: Mathematics Assessment Resource Service
by
Alan Schoenfeld, Hugh Burkhardt, Daniel Pead, and Malcolm Swan
and based at the University of California, Berkeley

We are grateful to the many teachers, in the UK and the US, who trialed earlier versions
of these materials in their classrooms, to their students, and to
Judith Mills, Carol Hill, and Alvaro Villanueva who contributed to the design.

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We are particularly grateful to
Carina Wong, Melissa Chabran, and Jamie McKee

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Egg Launch Contest

NAME: _____ DATE: ___

Mr. Rhodes' class is holding an egg launching contest on the football field. Teams of students have built catapults that will hurl an egg down the field. Ms. Monroe's class will judge the contest. They have various tools and ideas for measuring each launch and how to determine which team wins.

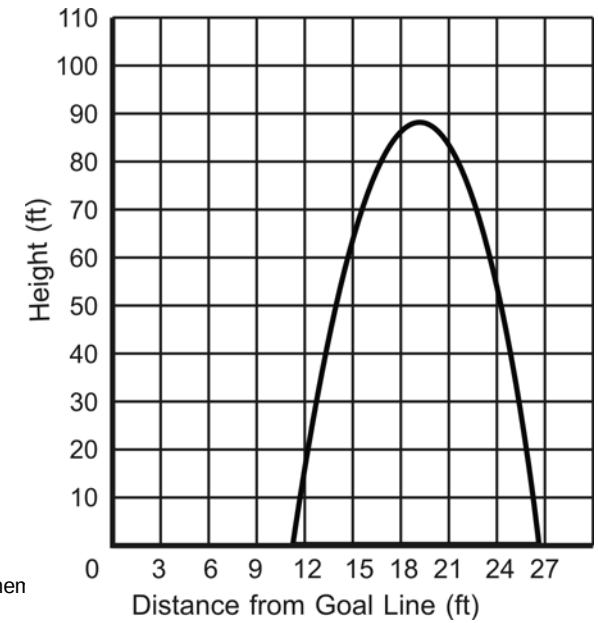
Team A used their catapult and hurled an egg down the football field. Students used a motion detector to collect data while the egg was in the air. They came up with the table of data below.

DISTANCE FROM THE GOAL LINE (IN FEET)	HEIGHT (IN FEET)
7	19
12	90
14	101
19	90
21	55
24	0

Team B's egg flew through the air and landed down the field. The group of students tracking the path of the egg determined that the equation $y = -0.8x^2 + 19x - 40$ represents the path the egg took through the air, where x is the distance from the goal line and y is the height of the egg from the ground. (Both measures are in feet.)

When **Team C** launched an egg with their catapult, some of the judges found that the graph to the right shows the path of the egg.

Which team do you think won the contest? Why?



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Team A

1. Using the data from Team A, determine an equation that describes the path of the egg. Describe how you found your equation.

2. On the graph below, graph the path of Team A's egg.

3. What is the maximum height that the egg reached? How far was the egg hurled?

Team B

4. Using the equation from Team B, generate a table of values that shows different locations of the egg as it flew through the air.

x												
y												

5. On the graph below, graph the path of Team B's egg.

6. What is the maximum height that the egg reached? How far was the egg hurled?

Team C

7. Using the data from Team C, generate a table of values that shows different locations of the egg as it flew through the air.

x												
y												

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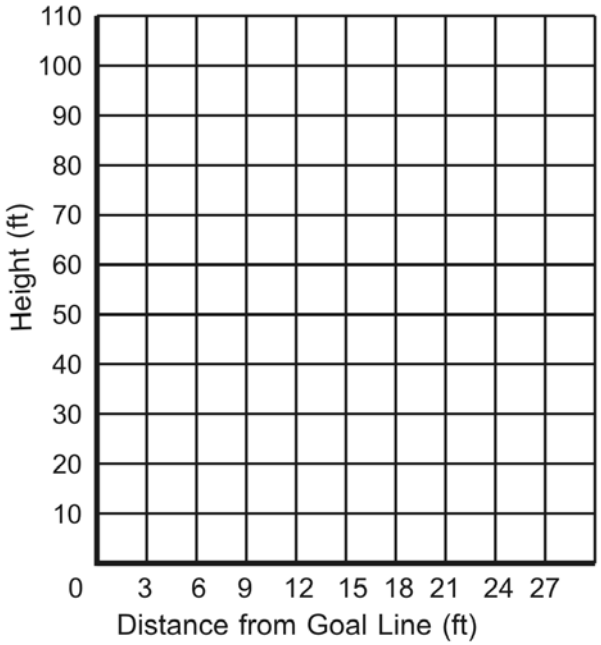
8. On the graph below, re-graph the path of Team C's egg.

9. What is the maximum height that the egg reached? How far was the egg hurled?



10. If it is a height contest, which team wins? How do you know?

11. If it is a distance contest, which team wins? How do you know?

12. Find a method of determining a winner so that the team that did not win in Question 10 or Question 11 would win using your method.

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Unit 3 – Modeling with Functions

Domain: Functions (F)
Essential Question: How do functions help to model, analyze and predict situations?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Synthesize properties of functions to compare model situations using various non-linear functions.** **F-IF** Interpret functions that arise in applications in terms of the context	**How…** **F.IF.5:** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* **F.IF.6:** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	**Students will…** **Match the Domain Exit Check** ▪ Compare and contrast graphs and functions to determine what the appropriate domain would include. **Function Experiments Activity:** ▪ Evaluate rates of change for functions that model experiments or simulations.	**F.IF.5:** Students may explain orally, or in written format, the existing relationships. **F.IF.6:** See Unit Addendum

Vocabulary: average rate of change, base, equivalent forms of expressions, exponent, exponential function, exponential model, logarithm, logarithmic function, step function, system of equations, system of inequalities, translation of function

Unit 3 – Modeling with Functions

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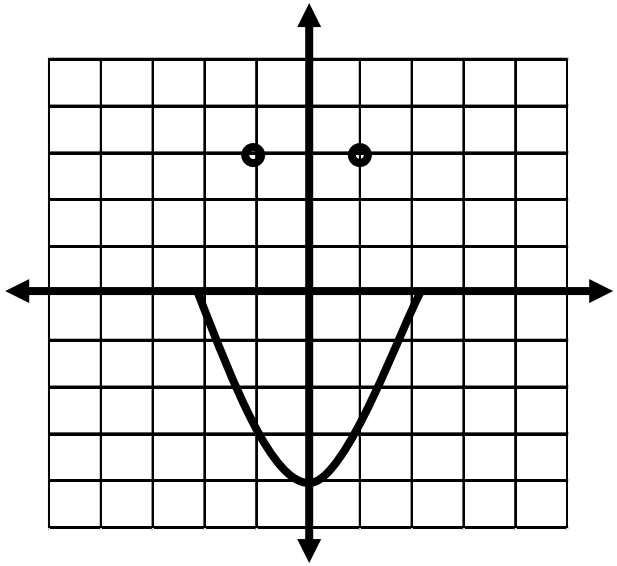
Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Synthesize properties of functions to compare model situations using various non-linear functions.** **F-IF** Analyze functions using different representations	**How…** **F.IF.7:** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude **F.IF.8:** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. **F.IF.9:** Compare properties of two functions each represented	**Students will…** **Family function logo design** ▪ Compare and contrast graphs using sketches, domain and range, and key characteristics of the functions. Function Family Logo Design Actvity.c **Functions and transformation project** functions and transformations proje **Function Activity** ▪ Evaluate equivalents forms of functions to identify properties of the function and use those properties to differentiate one function from another.	**F.IF.7:** See Unit Addendum **F.IF.9:** See Unit Addendum

Vocabulary: average rate of change, base, equivalent forms of expressions, exponent, exponential function, exponential model, logarithm, logarithmic function, step function, system of equations, system of inequalities, translation of function

Logo Design Activity

You have just landed your first job as a graphic designer. Your first project is to design a face as a logo for a local company. The design software used by your company requires that you provide the equations that would graph the different shapes on the logo. You drew the smile and eyes as shown below.



Your boss looks at the logo and asks you to do the following:

- Create a circle to border the face
- Create a nose and two eyebrows using both square root functions and absolute value functions

Name _____ Logo Design Activity

Required Elements for Submission:

Students will include:

- An appropriate display of data.
    - A graph that completely represents the logo including the smile, eyes, border of face, nose and brows. Label axes and label the brows as brow 1 and brow 2.

- A complete interpretation of the data. Complete the given table with the following information:
    - Equations that accurately represent the drawn graph of the logo and the ordered pairs for eyes.
    - Equations for nose and brows that use both square root and absolute value equations.
    - Accurate labels for families of functions for each element of the design
- A clear and complete conclusion.
    - This will be your written explanation describing what transformations you used to create the equations for the logo

Name _____ Logo Design Activity

Required Elements of Student Submission:

1. Display of Data

Name _____ Logo Design Activity

2. Interpretation of Data (Calculations/Formulas):

Elements of Logo	Family of Functions	Equation or Ordered Pairs
Eyes	Ordered Pairs	
Border of face		
Smile		
Brow 1		
Brow 2		
Nose		

3. Conclusion (Summary and Justification):

_____

_____

_____

_____

_____

_____

_____

Algebra II: Graphing Project
Due:

Task: Create an image of algebraic functions and relations that represents *you*.

Objective: To apply your understanding of families of functions and transformations.

Process: Create a drawing of a graphic that represents you. It can be symbolic. It can represent your hobbies or interests or family. It can represent anything about you that is appropriate for school. Be creative.

Use at least 6 functions or relations to represent the image. Equations must come from at least three different families of functions. Include an equation and if necessary a restricted domain for that equation. For example if you need a parabolic shape you may use $y = x^2$ but you probably will only need to use a piece of the graph of $y = x^2$. In order to do this you may need to restrict the domain from -2 to 5 by writing "$y = x^2,\ -2 \le x \le 5$."

Draft: A draft of your sketch and equations is due on Thursday April 29. On this day you will have the opportunity to check your equations for accuracy by a classmate and/or by me.

Final Product: Your final product must include three parts.

I: *Final Image on Graph Paper*

Requirements:

Equations must accurately represent the image.
Image must include at least 6 pieces of equations from at least three families.
Image should be aesthetically pleasing. Use color if appropriate.

II. *Individual pieces represented by each equation*

Requirements:

These pieces should be completed with each individual equation graphed accurately on each small grid in the packet enclosed. Each graph should be labeled and any domain restrictions should be clearly stated.

III. *Written paragraph*

Requirements:

Paragraph should describe what the image is and why it is representative of you.

MHS Algebra 2 Mathematics Curriculum

Unit 3 – Modeling with Functions

Domain: <u>Functions</u> (F)

Essential Question: How do functions help to model, analyze and predict situations?

Mathematical Practices:

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7. **Look for and make use of structure.**
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Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Synthesize properties of functions to compare model situations using various non-linear functions.** **F-BF** Build a function that models a relationship between two quantities	**How…** **F.BF.1:** Write a function that describes a relationship between two quantities. b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*	**Students will…** **Comparing Linear, Quadratic and Exponential Models** • Background Lesson Background Activity Comparing Patterns.c • • Group Activity Activity Comparing Patterns.docx • Follow-Up Assessment Comparing Patterns Assessment.docx **Graphing Change Worksheet:** ▪ Transform functions and evaluate the impact of those transformations on key aspects of the function.	**F.BF.1:** See Unit Addendum
F-BF Build new functions from existing functions *Use transformations of functions to find models as students consider increasingly more complex situations.* *For F.BF.3, note the effect of multiple transformations on a single graph and the common effect of each transformation across function types.*	**F.BF.3:** Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*		**F.BF.3:** See Unit Addendum

Vocabulary: average rate of change, base, equivalent forms of expressions, exponent, exponential function, exponential model, logarithm, logarithmic function, step function, system of equations, system of inequalities, translation of function

Background Activity: Comparing Patterns

In this activity you will examine patterns for four different functions: paper folding, diagonals of a polygon, interior angle sums and M&M's. Find the data pattern for each activity and complete the questions asked. There will be an individual follow-up assessment for this activity to check your understanding so please stay engaged and ask questions of your group members as needed! The information below will be necessary to understand all of the questions in the activity.

Background Vocabulary

Real Numbers

Integers

Whole Numbers

Domain

Recursive Formula

Explicit Formula

Discrete Function

Continuous Function

Background Skills for TI Data Analysis

1. *Enter data*
   Go to Stat
   Edit

   and then type data into chosen L1, L2, . . .

2. *Graph data on a stat plot*
   Set a stat plot by hitting 2nd, Stat Plot, Plot ___, and then follow the cues.

   Set an appropriate window based on the smallest and largest x and y values in your chosen lists.
   Turn off other equations and hit graph.

3. *Find regression equations*

After viewing the data on a graph choose an appropriate model for the regression equation. From the home screen go to Stat, Calc, and then choose the most appropriate regression from the given list. Then name the two lists and hit enter.

Example: LinReg (ax+b) L1,L2

You can re-graph your data with your regression equation to make sure that it is a good fit.

<u>Background Practice finding Recursive and Explicit Formulas</u>

Find a recursive formula for each of the sequences below.

a. 4,7,10,13, . . .

b. 4,7,11,16,22, . . .

Find an explicit formula to represent each table below.

a.

x	0	1	2	3
y	4	7	10	13

b.

x	0	1	2	3	4
y	4	7	11	16	22

Activity: Comparing Patterns

A. Paper Folding Activity

Begin with a sheet of white paper with the longer side lying horizontally. You will be folding it in half repeatedly with **vertical folds.** After each fold you will need to open the paper, draw a pencil line over the crease(s) created and count the number of regions. After counting the number of regions you will need to refold it before adding the new fold. Complete the table below and then answer the questions.

Number of Folds	Number of Regions
0	
1	
2	
3	
4	

1. What do you notice happening each time you fold the paper with the number of regions created?

2. How many regions would you have if you folded the paper in half a fifth time?

3. How many folds would you need to make in order to get more than one hundred regions?

4. What is the number of regions that you would get by folding the paper the number of times calculated in #3?

5. Find a recursive and an explicit equation for this data.

   Recursive: Explicit:

B. Interior Angle Sum Activity

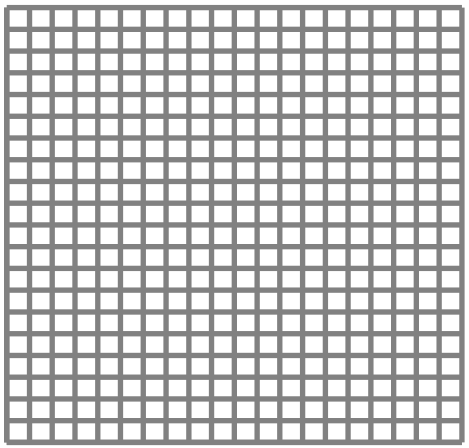
1. Extend the following pattern to find the next three terms in the table.

Name of polygon	Triangle	Quadrilateral	Pentagon	Hexagon	Heptagon	Octagon
Number of Sides (n)						
Interior Angle Sum						

2. Determine a *recursive* formula for the sum of the interior angles of any polygon.

3. Determine an *explicit* formula for the sum of the interior angles of any polygon.

4. Graph the explicit formula from problem 3. Input the data into a graphing calculator and compare your graph to the statplot given by the calculator.



5. Use the linear regression function on your calculator to determine the equation of the line. Does this match your answer in problem 3?

6. State the domain for the function that represents the sum of the interior angles of any polygon. Use this to explain what portion of the graph should be used when graphing this specific physical example.

7. Use the formulas you have found to determine the interior angle sum of a decagon.

C. Diagonals of a Polygon

1. Draw all of the diagonals and name each of the following polygons.

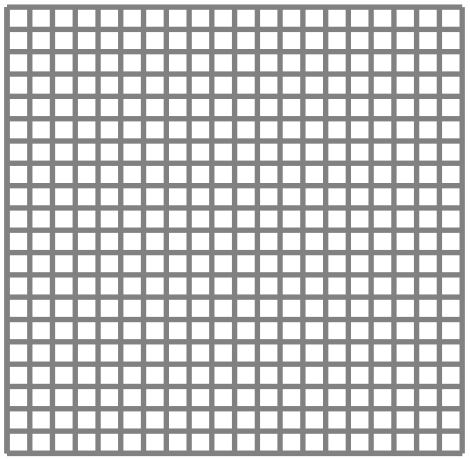
2. Use the given polygons to determine the number of diagonals that are in each polygon and complete the table.

Name of polygon	Triangle	Quadrilateral	Pentagon	Hexagon	Heptagon	Octagon
Number of Sides						
Number of Diagonals						

3. Determine a recursive formula for the number of diagonals in a polygon.

4. Determine an explicit formula for the number of diagonals in a polygon.

5. On the grid below, graph the equation for the explicit formula from #4. Input your data to a graphing calculator and create a statplot. How does the statplot compare to the graph below?



6. Use the quadratic regression function on your calculator to determine the equation of the parabola. Does this match your answer in problem 4?

7. State the domain for the number of diagonals in a polygon. Use this to explain what portion of the graph should be used when graphing this specific physical example.

8. Use the formulas you have found to determine the number of diagonals if the polygon has 20 sides.

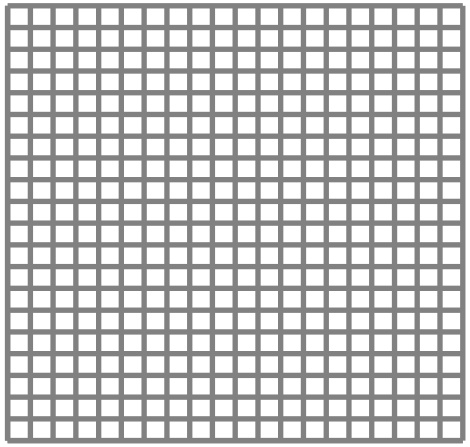
D. M&M's Activity

Materials: 2 paper plates, 1 cup, 50 M&M's, Graphing Calculator

Trial Number	Number of M & M's
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Process:

1. Count the number of M & M's that you have and record it in the first blank in the table above.
2. Shake the M & M's in the cup and then pour them onto the plate. Remove all those that have the "m" visible. (facing upwards)
3. Count the remaining M & M's and record that number in the table.
4. Place the M & M's from step 3 in the cup.
5. Repeat steps 2-4 until there are no M & M's left. You may need to add additional rows in the table.
6. Graph the data using the trial number as as the x-value and the number of M & M's as the y-value.



Questions:

7. Describe the shape of the graph.

8. Enter this data into 2 lists in the calculator, but do not enter the final data point from the table into the calculator list. Use the calculator to graph the function as a scatterplot. Use the calculator to find the curve of best fit based on your answer to question #7.

9. What is the equation of the curve of best fit?

10. Why is $y = 50(\frac{1}{2})^x$ not an appropriate model to represent this experiment?

11. Why do we disregard the trial when the number of M & M's is zero when we are using the calculator?

Name _____

Algebra 2: Comparing Patterns Assessment

A. Refer to the Paper Folding Activity to answer the following questions.

1. What type of pattern is being created with the number of regions column?

2. Graph the explicit function found in #5.

3. Explain the formula in terms of how the regions were created.

B. Refer to the Interior Angle Sum Activity to answer the following questions.

4. What is the interior angle sum of a 30-sided polygon.
5. How many sides does a polygon have if the interior angle sum is $4140°$?
6. What type of function is being created by this data?

C. Refer to the Diagonal Activity to answer the following questions.

7. Determine the number of diagonals if the polygon has 15 sides.
8. How many sides does a polygon have if it has 275 diagonals.
9. What type of function is being created by this data?

A. Refer to the M & M's activity to answer the following questions.

10. What effect will changing the initial number of M & M's have on the activity?
11. What type of function is created by this data?

B. Summary

12. Explain how to identify an equation as linear, quadratic, or exponential?
13. Explain how to identify a graph as linear, quadratic, or exponential?
14. Describe the difference between the domain of a discrete function versus a continuous function. .

Unit 3 – Modeling with Functions

Domain: Functions (F)

Essential Question: How do functions help to model, analyze and predict situations?

Mathematical Practices:

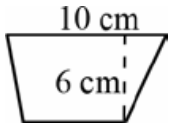
1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Synthesize properties of functions to compare model situations using various non-linear functions.** **F-BF** Build new functions from existing functions *Extend F.BF.4a to simple rational, simple radical, and simple exponential functions; connect F.BF.4a to F.LE.4.*	**How…** **F.BF.4:** Find inverse functions. a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, f(x) =2 x3 or f(x) = (x+1)/(x–1) for x ≠ 1.*	**Inverse Functions Worksheet:** ▪ Evaluate a function and its inverse to determine the limits of the function.	**F.BF.4:** See Unit Addendum
F-LE Construct and compare linear, quadratic, and exponential models and solve problems *Consider extending this unit to include the relationship between properties of logarithms and properties of exponents*	**F.LE.4:** For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology.	**Exponential Models Exit Check:** ▪ Analyze and evaluate exponential and logarithmic functions that model situations.	**F.LE.4:** See Unit Addendum

Vocabulary: average rate of change, base, equivalent forms of expressions, exponent, exponential function, exponential model, logarithm, logarithmic function, step function, system of equations, system of inequalities, translation of function

MHS Algebra 2 Mathematics Curriculum

Unit 3 Addendum

Performance Standard	Instructional Strategies
A.CED.1:	Equations can represent real world and mathematical problems. Include equations and inequalities that arise when comparing the values of two different functions, such as one describing linear growth and one describing exponential growth. Examples: • Given that the following trapezoid has area 54 cm^2, set up an equation to find the length of the base, and solve the equation.  • Lava coming from the eruption of a volcano follows a parabolic path. The height *h* in feet of a piece of lava *t* seconds after it is ejected from the volcano is given by $h(t)= -t^2 + 16t + 936$. After how many seconds does the lava reach its maximum height of 1000 feet?
A.CED.3:	Example: • A club is selling hats and jackets as a fundraiser. Their budget is $1500 and they want to order at least 250 items. They must buy at least as many hats as they buy jackets. Each hat costs $5 and each jacket costs $8.

A.CED.4:	Examples:
	• The Pythagorean Theorem expresses the relation between the legs a and b of a right triangle and its hypotenuse c with the equation $a^2 = b^2 = c^2$.
	  o Why might the theorem need to be solved for c?
	  o Solve the equation for c and write a problem situation where this form of the equation might be useful.
	• Solve $V = \frac{4}{3}\pi r^3$ for radius r.
	• Motion can be described by the formula below, where t = time elapsed, u = initial velocity, a = acceleration, and s = distance traveled. $s = ut + \frac{1}{2}at^2$
	  o Why might the equation need to be rewritten in terms of a?
	  o Rewrite the equation in terms of a.

MHS Algebra 2 Mathematics Curriculum

Unit 3 Addendum

<table>
<tr><th>Performance Standard</th><th>Instructional Strategies</th></tr>
<tr><td>F.IF.4:</td><td>
Students may be given graphs to interpret or produce graphs given an expression or table for the function, by hand or using technology.

Examples:
• A rocket is launched from 180 feet above the ground at time $t = 0$. The function that models this situation is given by $h = -16t^2 + 96t + 180$, where t is measured in seconds and h is height above the ground measured in feet.
o What is a reasonable domain restriction for t in this context?
o Determine the height of the rocket two seconds after it was launched.
o Determine the maximum height obtained by the rocket.
o Determine the time when the rocket is 100 feet above the ground.
o Determine the time at which the rocket hits the ground.
o How would you refine your answer to the first question based on your response to the second and fifth questions?

• Compare the graphs of $y = 3x^2$ and $y = 3x^3$.

• Let $R(x) = \frac{2}{\sqrt{x-2}}$. Find the domain, rnage, zeros, and asymptotes of R(x).

• Let $f(x) = 5x^3 - x^2 - 5x + 1$. Graph the function and identify end behavior and any intervals of constancy, increase, and decrease.

• It started raining lightly at 5am, then the rainfall became heavier at 7am. By 10am the storm was over, with a total rainfall of 3 inches. It didn't rain for the rest of the day. Sketch a possible graph for the number of inches of rain as a function of time, from midnight to midday.
</td></tr>
<tr><td>F.IF.6:</td><td>
The average rate of change of a function y = f(x) over an interval [a,b] is $\frac{4y}{4x} = \frac{f(b)-f(a)}{b-a}$.
In addition to finding average rates of change from functions given symbolically, graphically, or in a table, students may collect data from experiments or simulations (ex. Falling ball, velocity of a car, etc.) and find average rates of change for the function modeling the situation.

Examples:
• Use the following table to find the average rate of change of g over the intervals [-2,-1] and [0,2]:
<table>
<tr><th>x</th><th>g(x)</th></tr>
<tr><td>-2</td><td>2</td></tr>
<tr><td>-1</td><td>-1</td></tr>
<tr><td>0</td><td>-4</td></tr>
<tr><td>2</td><td>-10</td></tr>
</table>
</td></tr>
</table>

MHS Algebra 2 Mathematics Curriculum

Unit 3 Addendum

<table>
<tr><th>Performance Standard</th><th>Instructional Strategies</th></tr>
<tr><td></td><td>

- The table below shows the elapsed time when two different cars pass a 10, 20, 30, 40, and 50 meter mark on a test track.
  - For car 1, what is the average velocity (change in distance divided by change in time) between the 0 and 10 meter mark? Between the 0 and 50 meter mark? Between the 20 and 30 meter mark? Analyze the data to describe the motion of car 1.
  - How does the velocity of car 1 compare to that of car 2?

<table>
<tr><th></th><th>Car 1</th><th>Car 2</th></tr>
<tr><th>d</th><th>t</th><th>t</th></tr>
<tr><td>10</td><td>4.472</td><td>1.742</td></tr>
<tr><td>20</td><td>6.325</td><td>2.899</td></tr>
<tr><td>30</td><td>7.746</td><td>3.831</td></tr>
<tr><td>40</td><td>8.944</td><td>4.633</td></tr>
<tr><td>50</td><td>10</td><td>5.348</td></tr>
</table>

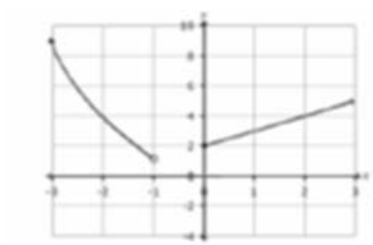
</td></tr>
<tr><td>F.IF.7:</td><td>

Key characteristics include but are not limited to maxima, minima, intercepts, symmetry, end behavior, and asymptotes. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to graph functions.

Examples:

- Describe key characteristics of the graph of f(x) = |x-3| + 5.
- Sketch the graph and identify the key characteristics of the function described below.

$$F(x) = \begin{cases} x = 2 \; for \; x \; \geq 0 \\ -x^2 \; for \; x \; < \; -1 \end{cases}$$

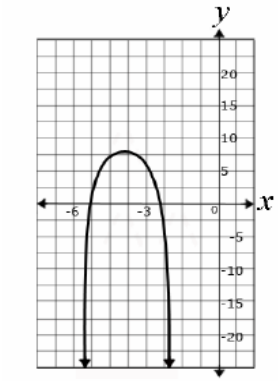


- Graph the function $f(x) = 2^x$ by creating a table of values. Identify the key characteristics of the graph.
- Graph f(x) = 2 tan x – 1. Describe its domain, range, intercepts, and asymptotes.
- Draw the graph of f(x) = sin x and f(x) = cos x. What are the similarities and differences between the two graphs?

</td></tr>
</table>

MHS Algebra 2 Mathematics Curriculum

Unit 3 Addendum

Performance Standard	Instructional Strategies
F.IF.9:	Example: ● Examine the functions below. Which function has the larger maximum? How do you know? $f(x) = -2x^2 - 8x + 20$ 
F.LE.4:	Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to analyze exponential models and evaluate logarithms. Example: ● Solve 200 e0.04t = 450 for t. Solution: We first isolate the exponential part by dividing both sides of the equation by 200. $e^{0.04t} = 2.25$ Now we take the natural logarithm of both sides. $ln\ e^{0.04t} = ln\ 2.25$ The left hand side simplifies to 0.04t, by logarithmic identity 1. $0.04t = ln\ 2.25$ Lastly, divide both sides by 0.04 $t = ln\ (2.25) / 0.04$ $t \approx 20.3$

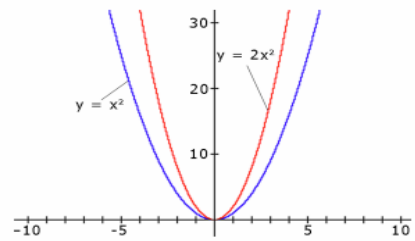
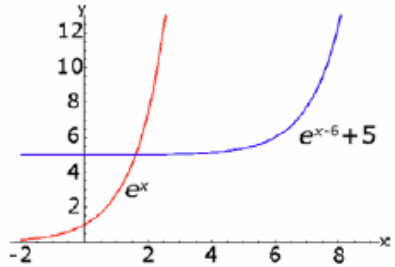
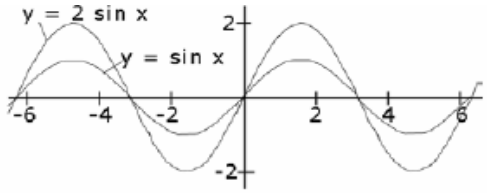
MHS Algebra 2 Mathematics Curriculum

Unit 3 Addendum

Performance Standard	Instructional Strategies
F.BF.1:	Students will analyze a given problem to determine the function expressed by identifying patterns in the function's rate of change. They will specify intervals of increase, decrease, constancy, and, if possible, relate them to the function's description in words or graphically. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model functions. Examples: • You buy a $10,000 car with an annual interest rate of 6 percent compounded annually and make monthly payments of $250. Express the amount remaining to be paid off as a function of the number of months, using a recursion equation. • A cup of coffee is initially at a temperature of 93º F. The difference between its temperature and the room temperature of 68º F decreases by 9% each minute. Write a function describing the temperature of the coffee as a function of time. • The radius of a circular oil slick after t hours is given in feet by $r = 10t^2 - 0.5t$, for $0 \leq t \leq 10$. Find the area of the oil slick as a function of time.
F.BF.4:	Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model functions. Examples: • For the function $h(x) = (x - 2)^3$, defined on the domain of all real numbers, find the inverse function if it exists or explain why it doesn't exist. • Graph $h(x)$ and $h^{-1}(x)$ and explain how they relate to each other graphically. • Find a domain for $f(x) = 3x^2 + 12x - 8$ on which it has an inverse. Explain why it is necessary to restrict the domain of the function.

MHS Algebra 2 Mathematics Curriculum

Unit 3 Addendum

Performance Standard	Instructional Strategies
F.BF.3:	Students will apply transformations to functions and recognize functions as even and odd. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to graph functions. Examples: • Is $f(x) = x^3 - 3x^2 + 2x + 1$ even, odd, or neither? Explain your answer orally or in written format. • Compare the shape and position of the graphs of $f(x) = x^2$ and $g(x) = 2x^2$, and explain the differences in terms of the algebraic expressions for the functions  • Describe effect of varying the parameters *a*, *h*, and *k* have on the shape and position of the graph of $f(x) = a(x-h)^2 + k$. • Compare the shape and position of the graphs of $f(x) = e^x$ to $g(x) = e^{x-6} + 5$, and explain the differences, orally or in written format, in terms of the algebraic expressions for the functions  • Describe the effect of varying the parameters *a*, *h*, and *k* on the shape and position of the graph $f(x) = ab^{(x+h)} + k$., orally or in written format. What effect do values between 0 and 1 have? What effect do negative values have? • Compare the shape and position of the graphs of $y = \sin x$ to $y = 2 \sin x$. 

Unit 4 – Inferences and Conclusions from Data

Domain: Statistics and Probability (S)
Essential Question: How do data interpretations help develop informed decisions and predictions?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Evaluate study design and analyze normal distributions to draw conclusions about populations.** **S-ID** Summarize, represent, and interpret data on a single count or measurement variable *While students may have heard of the normal distribution, it is unlikely that they will have prior experience using it to make specific estimates. Build on students' understanding of data distributions to help them see how the normal distribution uses area to make estimates of frequencies (which can be expressed as probabilities). Emphasize that only some data are well described by a normal distribution.*	**How…** **S.ID.4:** Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.	**Is This Normal? Worksheet:** ▪ Evaluate population data to determine if the data set is normal and draw appropriate conclusions about the population. **Normal Distribution Activity** S.ID.4 Fitting Normal Distributions.docx	**S.ID.4:** See Unit Addendum

Vocabulary: inference, mean, normal approximation, normal distribution, observational study, population parameter, population percentage, randomized experiment, sample survey, spreadsheet, standard deviation

Name _____ Date _____

Fitting Normal Distributions
Group Station Activity

EXAMPLE 1: The dressed weights of Excelsior Chickens are approximately normally distributed with mean 3.20 pounds and standard deviation 0.40 pound. About what proportion of the chickens have dressed weights greater than 3.60 pounds?

EXAMPLE 2: Suppose that the daily demand for change (meaning coins) in a particular store is approximately normally distributed with mean $800.00 and standard deviation $60.00. What is the probability that, on any particular day, the demand for change will be below $600?

EXAMPLE 3: Consider the situation of the previous problem. Find the amount M of change to keep on hand if one wishes, with certainty 99%, to have enough change. That is, find M so that $P[\, X \leq M \,] = 0.99$.

EXAMPLE 4: A machine that dispenses corn flakes into packages provides amounts that are approximately normally distributed with mean weight 20 ounces and standard deviation 0.6 ounce. Suppose that the weights and measures law under which you must operate allows you to have only 5% of your packages under the weight stated on the package. What weight should you print on the package?

EXAMPLE 5: A patient recently diagnosed with Alzheimer's disease takes a cognitve abilities test. The patient scores a 45 on the test (mean = 52, standard deviation of 5. What is this patient's percentile rank?

EXAMPLE 6: A fifth grader takes a standardized achievement test (mean = 125, S = 15) and scores a 148. What is this child's percentile rank?

Unit 4 – Inferences and Conclusions from Data

Domain: Statistics and Probability (S)

Essential Question: How do data interpretations help develop informed decisions and predictions?

Mathematical Practices:

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Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Evaluate study design and analyze normal distributions to draw conclusions about populations.** **S-IC** Understand and evaluate random processes underlying statistical experiments	**How…** **S.IC.1:** Understand statistics as a process for making inferences about population parameters based on a random sample from that population. **S.IC.2:** Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*	**Inference Exit Check:** ▪ Evaluate sample data to make inferences about population parameters. **Outcome Matching Worksheet:** ▪ Compare models and outcomes to determine the accuracy of the simulation. S.IC.2 - Penny & dice activity.doc	**S.IC.2:** Possible data-generating processes include (but are not limited to): flipping coins, spinning spinners, rolling a number cube, and simulations using the random number generators. Students may use graphing calculators, spreadsheet programs, or applets to conduct simulations and quickly perform large numbers of trials. The law of large numbers states that as the sample size increases, the experimental probability will approach the theoretical probability. Comparison of data from repetitions of the same experiment is part of the model building verification process. **Example:** Have multiple groups flip coins. One group flips a coin 5 times, one group flips a coin 20 times, and one group flips a coin 100 times. Which group's results will most likely approach the theoretical probability?

Vocabulary: inference, mean, normal approximation, normal distribution, observational study, population parameter, population percentage, randomized experiment, sample survey, spreadsheet, standard deviation

Name_____ Date_____ Block_____

College Statistics

Penny/Dice Activity

PART I – PENNY AGES

Each student will be given 20 pennies to work with. You will record the age of each penny and create a frequency distribution histogram for these ages.

1. AGES- record them here and store them in L_1.

2. HISTOGRAM- Note the frequency of each age and store this data in L_2. Plot the two lists and describe the histogram.

3. MEAN- Now find the mean of your distribution and write it below.

4. NEW HISTOGRAM- Note the frequency of each mean and store this data in L_3. Plot this with the data in L_1.

5. COMPARE- Compare this histogram with your original frequency distribution. What generalization can you make based on the results?

PART II – ROLLING DICE

Each student will be given 2 dice to work with. You will roll the dice 20 times and record the sum of the numbers on the dice each time.

1. SUMS- record them here and store them in L_1.

2. HISTOGRAM- Note the frequency of each sum and store this data in L_2. Plot the two lists and describe the histogram.

3. MEAN- Now find the mean of your distribution and write it below.

4. NEW HISTOGRAM- Note the frequency of each mean and store this data in L_3. Plot this with the data in L_1.

5. COMPARE- Compare this histogram with your original frequency distribution. What generalization can you make based on the results?

Unit 4 – Inferences and Conclusions from Data

Domain: Statistics and Probability (S)

Essential Question: How do data interpretations help develop informed decisions and predictions?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
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6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Evaluate study design and analyze normal distributions to draw conclusions about populations.** **S-IC** Make inferences and justify conclusions from sample surveys, experiments, and observational studies	**How…** **S.IC.3:** Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.	**That's So Random – Case Study Stations:** ▪ Evaluate experiments and observations for key characteristics and explain the role of randomization in each study. S.IC. 3,4,5,6 Muddying the waters.	**S.IC.3:** Students should be able to explain techniques/applications for randomly selecting subjects from a population and how those techniques/applications differ from those used to randomly assign existing subjects to control groups or experimental groups in a statistical experiment. In statistics, an observational study draws inferences about the characteristics of subjects (for example, observing data on academic achievement and socio-economic status to see if there is a relationship between them). This is in contrast to controlled experiments, such as randomized controlled trials, where each subject is randomly assigned to a treated group or a control group before the start of the treatment.

Vocabulary: inference, mean, normal approximation, normal distribution, observational study, population parameter, population percentage, randomized experiment, sample survey, spreadsheet, standard deviation

PROBLEM SOLVING

Mathematics Assessment Project

CLASSROOM CHALLENGES

A Formative Assessment Lesson

Interpreting Statistics: A Case of Muddying the Waters

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley

Beta Version

For more details, visit: http://map.mathshell.org
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Interpreting Statistics: *A Case of Muddying the Waters*

MATHEMATICAL GOALS

This lesson unit is intended to help you assess how well students are able to:

- Interpret data and evaluate statistical summaries.
- Critique someone else's interpretations of data and evaluations of statistical summaries.

The lesson also introduces students to the dangers of misapplying simple statistics in real-world contexts, and illustrates some of the common abuses of statistics and charts found in the media.

COMMON CORE STATE STANDARDS

This lesson relates to the following *Mathematical Practices* in the *Common Core State Standards for Mathematics*:

3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.

This lesson gives students the opportunity to apply their knowledge of the following *Standards for Mathematical Content* in the *Common Core State Standards for Mathematics:*

S-ID: Summarize, represent, and interpret data on two categorical and quantitative variables. Interpret linear models.

S-IC: Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

INTRODUCTION

This lesson unit is structured in the following way:

- Before the lesson, students work individually on a task that is designed to reveal their current understandings and difficulties. You review their work and create some questions that will help them to improve their solutions.
- At the start of the lesson, students role-play a scene that provides the background to the lesson task. Next, students work in small groups on a collaborative discussion task. They extract information from evidence and critically assess how it was gathered and presented.
- After a whole-class discussion students again work individually, on a new task. This has similar content to the original task, allowing them to demonstrate the progress they have made during the lesson.

MATERIALS REQUIRED

- Each student will need a copy of the initial assessment task *Muddying the Waters*, and a copy of the assessment task *Unhappy Campers*.
- Each small group of students will need a copy of the role-play script *A Case of Muddying the Waters,* and a copy of the worksheet *Case Notes*.
- There are some projector resources to support the role-play and whole-class discussion. Use of a computer and projector is recommended. You could print the images onto overhead projector slides instead. If you do not have access to a projector, print a copy of these slides for each small group of students: *Exhibits 1, 2, 3, 4, Scene 1: The Assistant DA's office,* and *Scene 2: At the Factory*.
- A teaspoon may be useful to help illustrate the concept of 'mg per cubic meter'.

TIME NEEDED

30 minutes of one lesson, a second 1-hour lesson, and 15 minutes in a third lesson (or for homework). All timings are approximate. Exact timings will depend on the needs of the students.

SUGGESTED LESSON OUTLINE: LESSON 1

Interactive whole-class introduction (10 minutes)

Have the students do this task in class a day or more before the formative assessment lesson. This will give you an opportunity to assess the work, and to find out the kinds of difficulties students have with it. You will then be able to target your help more effectively in the follow-up lesson.

Give each student a copy of the task *Muddying the Waters*. Ask students to read through the task carefully. Some teachers ask students to take turns to read parts of the task aloud.

Introduce the task, asking questions to help students to understand the problem and its context.

This task is concerned with the river pollution and its effect on the environment. What do I mean by river pollution?

What does it mean when someone says that the level of pollution in a river is illegal?

Does anybody know of a river that is polluted? What was the source of the pollution?

How can you tell that a river is polluted?

In particular, explain how chemical pollution in a river is measured. You could use a teaspoon to help illustrate this.

Chemical pollution is measured in milligrams per cubic meter of river water.

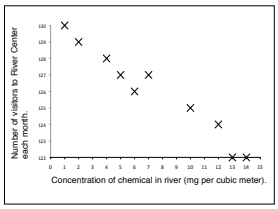
Does anyone know how much a milligram is? A cubic meter is?

A teaspoon of sugar is about 4,000 mg. This classroom is about 300 cubic meters. [Use figures for your own room.]

Muddying The Waters

The manager of the Riverside Center is concerned about visitor numbers.
He is certain the Center's popularity has been badly affected by an increase in river pollution.
He feels the local Environmental Agency should do something about it.

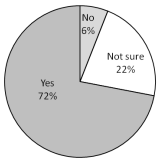
To support his argument he measured the chemical concentration in the river each month.
He also counted the number of people visiting the Center over several months.
He used the results to draw this chart.



Scatter chart: Chemical concentration and number of visitors

At the same time the manager asked 18 visitors this question:
"The odor you can smell originates from the pollution in the river"
Is it spoiling your enjoyment of the Center?'

He displayed the results as a pie chart.



Pie chart showing the percentage of visitors whose enjoyment was spoiled.

Assessment task: *Muddying the Waters* (15 minutes)

Ask students to work through the task.

Spend fifteen minutes working individually on this task.

Don't worry if you can't complete everything.

There will be a lesson [tomorrow] that should help you understand the math better.

Your goal is to be able to answer questions like these confidently by the end of the next lesson.

At this stage, do not help students with the task or comment on their work. Stop them after 15 minutes, whether or not they have finished. Collect in students' papers for analysis.

Preparation for the next lesson (5 minutes)

Explain the theme for the next lesson. You need three copies of the role-play script *A Case of Muddying the Waters*.

Next lesson we are going to continue the theme of river pollution with some role-play.

The role-play sets the scene for the task. An Assistant DA talks to a local Environmental officer and a Factory owner about the pollution of a river. Who wants to play these three roles?

Give each volunteer a copy of the role-play script *A Case of Muddying the Waters*.

Ask the volunteers to read over their parts before the next lesson. If you wish, enrich the role-play by asking the volunteers to bring in a few theatrical props.

Assessing students' responses

Collect students' responses to the task. Make some notes on what their work reveals about their current levels of understanding. The purpose of this is to forewarn you of issues that will arise during the lesson itself, so that you may prepare carefully.

We suggest that you do not score students' work. The research shows that this will be counterproductive as it will encourage students to compare their scores, and will distract their attention from what they can do to improve their mathematics.

Instead, help students to make further progress by summarizing their difficulties as a series of questions. Some suggestions for these are given in the *Common issues* table (below). These have been drawn from common difficulties observed in trials of this lesson unit.

We suggest that you write a list of your own questions, based on your students' work, using the ideas below. You may choose to write questions on each student's work. If you do not have time to do this, select a few questions that will be of help to the majority of students. These can be written on the board at the beginning of the lesson.

Common issues:	Suggested questions and prompts:
Student does not understand that there are alternative interpretations of data and statistics, some of which may be biased. For example: The student copies or paraphrases the Riverside Manager's interpretations.	• What does the chart show? What does the Riverside Manager say the chart shows? Is there a difference? • In what ways is what the Riverside Manager said misleading?
Student does not recognize that things can happen together without one causing the other For example: The student does not contradict the causal claim made about the scatter plot.	• If two things happen at the same time, does that mean one made the other happen? • Write down another reason that there might be a correlation. • What other interpretations of the correlation can you find?
Student does not understand that survey questions may push respondents towards a particular response For example: The student does not recognize that the phrasing of the statement biases respondents towards thinking of the river as polluted and smelly.	• Does the way this question is asked make a 'yes' response more likely than 'no' or 'maybe'? Why do you think that? Does it matter? • Try writing this question in a way that doesn't push the respondent towards a particular answer.
Student does not recognize that statistics may be compiled in ways that push readers towards a biased interpretation For example: The student does not recognize that on the scatter plot, starting the 'number of visitors' scale at 122 (rather than 0) distorts perceptions of the proportional change in the number of visitors. Or: The student does not recognize that it is inappropriate to draw conclusions about the whole population from such a small sample size.	• Notice that the scale on this graph starts at 122. How different would the graph look if you drew the axis showing the whole range? How might that affect your interpretation? • How many visitors were there overall? When was the survey conducted? Can you think of a more convincing way to set the survey? • How do the ways data was collected affect your interpretation of the results?

SUGGESTED LESSON OUTLINE: LESSON 2

Review individual solutions to *Muddying the Waters* (10 minutes)

Remind students of their work on the assessment task.

Recall the work you did in the last lesson on river pollution.

In this lesson you will build on that work.

Return the papers to the students. If you chose to write questions on the board rather than on individual papers, display them now.

I read your papers and I've some questions about them.

I'd like you to work on improving your answers for a few minutes, using my questions.

Ask students to work on their own for a few minutes, answering your questions.

Interactive role-play introduction (10 minutes)

To introduce the lesson task, use a projector and slides P-3 to P-9 from the projector resource: *Exhibits 1, 2, 3, 4; Scene 1: The assistant DA's office; Scene 2: At the Factory*. If you do not have a projector, hand out the printed copies of these slides. You also need four copies of the role-play script, *A Case of Muddying The Waters* (one copy for each actor and one copy for yourself.)

In the next section of this lesson you will be working on river pollution again.

The role-play sets the scene.

Ask the actors to read out the script. Advise them to talk slowly, and to pause at the end of each sentence, as the script contains a lot of information. Encourage the students to listen carefully to the facts being presented about the river pollution.

Collaborative small-group work (25 minutes)

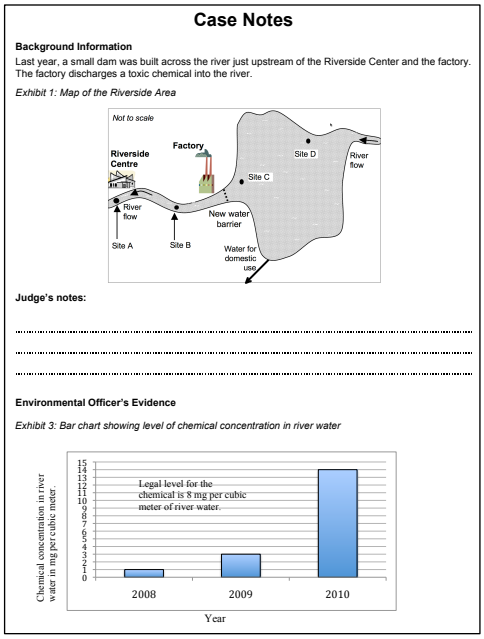
Once the students have acted out the scenes (and the applause has died down) turn to the class and say:

The case goes to court. The Assistant DA prosecutes the Factory Owner for polluting the river.

What does 'prosecution' mean?

Your task now is to be the judges. You have to reach a fair judgment about who wins the case.

Organize students into groups of two or three. Provide each small group with a copy of the worksheet *Case Notes*. *Case Notes* contains the map, information from the script, and arguments made in court by the Environmental Officer and the Factory Owner.



Case Notes

Background Information

Last year, a small dam was built across the river just upstream of the Riverside Center and the factory. The factory discharges a toxic chemical into the river.

Exhibit 1: Map of the Riverside Area

Judge's notes:

Environmental Officer's Evidence

Exhibit 3: Bar chart showing level of chemical concentration in river water

Help students to understand the task and its context.

I'm giving you a copy of the arguments presented in court.

Read through the information carefully. Write notes on what you think the data and statistics show.

In particular, ask them to focus on critical analysis of the information presented.

Explain why you agree or disagree with the arguments people made, using math.

The important thing is to look critically at all of the information. Do not just accept what people say as fact.

At the end of the lesson, you will use your work to decide together whether the factory owner is guilty, or not guilty, of polluting the river.

These instructions are reproduced on slide P-8, *Judge's Instructions*.

During small-group work you have two tasks, to notice strengths and weaknesses you see in students' work, and to support their thinking.

Notice strengths and weaknesses in students' work

Find out about students' current levels of understanding and the difficulties they encounter in the task. Students may be used to interpreting statistical diagrams, but may find it more difficult to critique someone else's biased reading of information. Students may fail to notice a bias in a question, or may struggle to understand the issue of small sample size. You can use the information about common difficulties to focus the whole-class discussion towards the end of the lesson.

Support student thinking

Try not to solve students' difficulties for them. Instead, ask them questions to help them move their thinking on.

You could strengthen your argument if you did some math on the data you've been given.

Is there another way to present this data?

Could you redraw that chart so it displays the important features of the data better?

Questions similar to those in the *Common issues* table on p. 3 were found to be useful in lesson trials.

For students who are struggling, it may help to ask some specific questions about aspects of the mathematics:

Describe this chart.

Is there another way to present this data?

The Environmental Officer/Factory Owner drew this conclusion. Can you draw any different conclusions from this evidence?

Encourage students to explain their reasoning to others in the group before writing it down. Other group members may question and refine the explanations.

Whole-class discussion: reaching a judgment (10 minutes)

Organize a whole-class discussion, focusing on the mathematical practice of critiquing the reasoning of others.

Choose a group to present their argument about one piece of evidence. Instruct the other groups to listen and write down questions about the group's argument.

Hani, does this evidence help the factory owner show he is not guilty? Tell us why you think that.

If you disagree with the group's interpretation of the evidence, write why, and challenge them at the end of their presentation.

Once the group has presented their case, other groups get a chance to challenge the details of their argument. If the challenge is not based on mathematics, you can rule it out of court.

That is not a mathematical argument. As there is no good evidence, it can't be accepted in court.

Once you have modeled this process for students, give them responsibility for deciding whether there is an evidential base for each claim and challenge.

Is that a mathematical argument? Is there good evidence for what [Shelley] has just said?

Does the evidence support her conclusion?

Once students have worked through the four pieces of evidence, ask them to come to a collective decision about the verdict.

Do you find the factory owner guilty or not guilty?

If there is disagreement, take arguments from both sides. You may find you cannot reach a collective decision. In that case, suggest students send the defendant for a retrial.

Whole-class discussion: Summing up (5 minutes)

Point out that an important message of this lesson is that it is easy to 'get it wrong' when interpreting statistics, especially in complicated real-world situations. In reality, most of the 'evidence' in this lesson is too vague to draw any firm scientific conclusions. A lot of questions are left unanswered.

How, exactly, were the wildlife surveys conducted? Why did the second survey look at so many more fish than the first? How do you 'count the number of invertebrates' at a site?

The dam has reduced the flow of the river by 80%. Even without the pollution, is it possible that this could affect the wildlife or the popularity of the Riverside Center?

Can you see any other problems with the data collection? With the statistics that have been calculated?

If you think it is appropriate to the class, you could mention that if they study statistics further they will learn how to calculate **significance**: the likelihood that a difference in two results is not just 'the luck of the draw'. Most serious scientific studies will do this, but you do not often find it in news reports!

Next lesson: assessment task *Unhappy Campers* (15 minutes)

Ask students to do this task in the next lesson, or for homework.

Give each student a copy of the assessment task, *Unhappy Campers*. Explain that this task uses very similar math to the lessons on river pollution, but in a new context.

Help students to read through the task sheet, and use questions to help them understand the context.

What is a wind turbine?

What are they used for?

What do decibels measure?

How loud is 50 decibels? 30 decibels? [0 decibels is the threshold for human hearing. A whisper in a quiet library is about 25 decibels. Normal conversation at about five feet is around 60 decibels.]

Ask students to work on their own on the assessment task, bearing in mind what they have learned during the previous lesson.

I want you to work on this task, using those same ideas about a fair, mathematical critique.

Remember not to believe all the arguments someone gives you using statistics.

After the assessment, you may find it useful to ask students to compare their responses to the first and second assessment tasks, so they can see the progress they have made.

SOLUTIONS

Assessment task: Muddying the Waters

Question 1.

Interpreting the scatter chart

- The water was tested on a monthly basis for 10 months and at the same time the number of visitors to the Center was recorded.
- There is a negative correlation.
- The number of visitors per month varies from 122 to 130. The range is 8 and the mean is 126.
- Chemical concentration varies from 1 to 14 mg per m^3. The range is 13 mg per m^3 and the mean is 7.4 mg per m^3.

Interpreting the pie chart

- Eighteen people are involved in the survey. 13 replied yes, 1 replied no and 4 were unsure.

Question 2.

The Riverside Manager's argument is misleading in various ways.

- The scatter plot has a misleading scale. It gives the impression that correlated with a rise in pollution there has been a massive drop in visitor numbers. In fact, there is a fall of only 8. Overall the decrease is 6%.
- There is a negative correlation on the scatter chart. This may not be causal as there are many other reasons why the visitor numbers fell, such as change in season. If the dam was reducing the amount of water in the river, this might have made it less attractive to visitors. The survey was over 10 months, not a year.
- The pie chart is based on a survey that uses a biased question: people may not have noticed a smell until they were asked about it.
- The sample size for the pie chart is small. The results of the survey are unlikely to be a true representation of all the visitors to the Center. Providing the number of people as well as the percentages in each response category would be helpful.

Lesson task: *Case Notes*

The concentration of the chemical in the river has risen above the legal limit.

The bar chart is appropriate, and is clearly shows that three distinct tests have been carried out. The concentration of the chemical in the river **is** now above the legal limit.

The levels were within the limit in the previous two years. The factory is discharging the same amount of chemical, but the flow rate of the water has reduced, meaning that the **concentration** is now above the legal limit.

Students may have calculated the concentration of the chemical in the water:

Last year:

$$\frac{60}{20} = 3mg/m^3.$$

This year:

$$\frac{60}{4} = 15mg/m^3.$$

Arguing that the mean concentration is within the legal limit is a misuse of statistics, because the low measurements in the first two years disguise the much higher figure in the last year:

$$\frac{1+3+14}{4} = 6mg/m^3.$$

A more sympathetic judge might argue that there are not enough test sites to prove that the pollution was not caused by another source: it might have been useful to test the water upstream of the factory, to find out whether that water was polluted to start with.

There has been an increase in the number of diseased fish due to the rise in chemical pollution.

The survey is misleading because the sample sizes are different. Arguing that there are now ten times more diseased fish is incorrect because it ignores the sample sizes. If students have calculated proportions or percentages, they will get a better sense of the data than if they rely on numbers:

Two years ago:

$$\frac{6}{300} \times 100 = 2\%$$

This year:

$$\frac{64}{1600} \times 100 = 4\%$$

Using this to argue that the number of diseased fish has doubled is still a misleading use of statistics: finding slightly more or fewer diseased fish in either survey (due to weather, the way the fish were caught, or just 'the luck of the draw') would make a big difference to the percentages.

The judge could argue that the survey is poor evidence because the sample sizes are too small to detect a difference in such a small percentage of diseased fish. Or the judge might argue that there are not enough survey sites to show whether being downstream of the factory makes a difference. The reduced flow rate of the river might have affected the health of the fish regardless of the pollution. Why did the second survey look at so many more fish than the first: were the fish harder to find the first time?

The number of invertebrates has not changed.

There has been hardly any change in the mean number of invertebrates. Two years ago the mean across four sites was 21, and now it is 19.

However, two years ago the range was 4. Now the range is 20. This is a big increase.

The sites most likely to have been affected by the pollution are A and B, downstream from the factory. Two years ago the mean number of invertebrates at these sites was 21, now it is 12. This is quite a large decrease. In contrast, the mean at sites C and D has increased.

Arguing that the mean number of invertebrates has hardly changed is a misuse of statistics: taking the mean of all four sites (including two which would not have been affected by pollution from the factory) hides the possibly significant reduction at the polluted sites.

The number of birds has increased.

Using a line graph to represent this data is inappropriate because it gives the impression that the birds were continually monitored. A bar chart with two bars would be more appropriate.

The scale on the line graph is misleading because it gives the impression that there has been a dramatic increase in the number of birds. The data show that there has only been an increase of 6 birds (about 7%). This is insignificant, especially without more details of how the birds were counted or at what time of year.

Arguing that the chart shows that the number of birds has increased dramatically in the last two years is a misleading use of statistics.

Assessment task: *Unhappy Campers*

Question 1.

Interpreting the scatter chart:

- There are fourteen data points on the scatter chart; the survey took place over a two-week period. There is a negative correlation.
- The number of visitors ranges from 70 to 78, a range of 8 with a mean average of 75. The noise level ranges from 10 to 60 decibels, with a range of 50 decibels and a mean average of 35 decibels[1].

Interpreting the pie chart:

- The number of campers surveyed was 50.
- 80% of the sample responded yes, 16% unsure, and 4% no.
- The numbers of respondents are 40, 8, and 2 respectively.

Question 2.

The camp manager's argument is biased in several ways.

Her choice of math introduces bias:

- The scatter plot has a misleading scale. The scale on the 'number of campers' axis starts at 40 rather than 0. It gives the impression that correlated with the rise in noise level there has been a large drop in visitor numbers. The number of campers only varies by 8 across the fourteen-day period, decreasing by about 10% between the quietest and noisiest day.
- The survey statement and question is biased. Stating that the noise is 'loud' and assuming that the respondent can hear the noise pushes the respondent towards a positive response; the use of 'spoiling enjoyment' in the question also introduces potential bias.
- The pie chart is based on a relatively small sample (50 campers). The survey took place on one day. On only 2 days on the scatter chart were there 50 campers. Both days were particularly noisy. Surveying only on a noisy day produces potential bias in the survey responses. It would have been helpful to show the number of respondents, not just percentages, on the pie chart, to aid interpretation of the results.

Her interpretations of her data and statistics are incorrect.

Peggy claims that the noise from the turbines has **caused** a drop in camper numbers. The correlation between the noise level in decibels and the number of campers does not show there is a causal relationship between the two variables. There may be other explanations of why the number of campers and the noise level correlate. For example, the turbine noise increases with the wind level, so you would expect fewer people to want to camp at noisy times, because it is windier then.

Although most of the people surveyed did state that the wind turbines spoilt their enjoyment, the questionnaire was biased, the sample was small, and the survey took place on a noisy day. As the results of the survey are dubious, there is no evidence to support Peggy's interpretation that most people coming to the camp would agree with the results of the survey. She cannot generalize from a small, biased sample, and she cannot rely on responses to a biased question.

[1]From the US Environmental Protection Agency website:

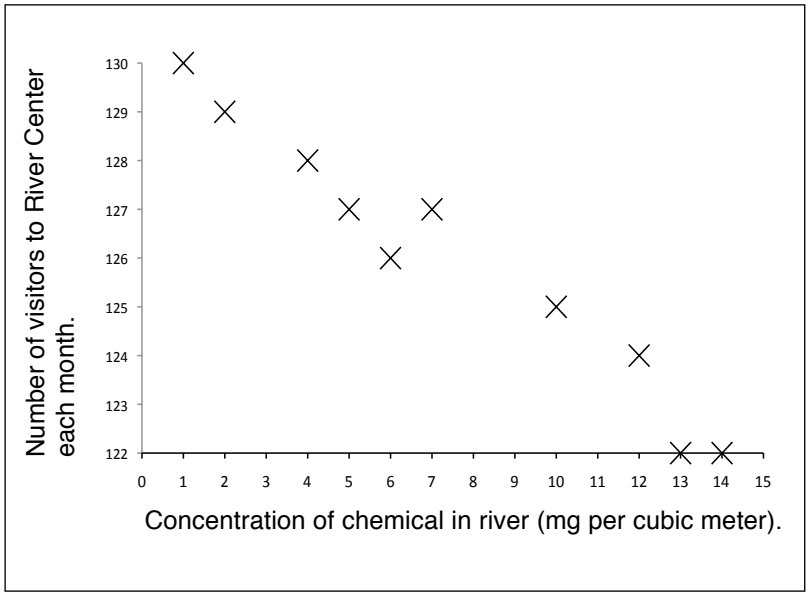
The document identifies a 24-hour exposure level of 70 decibels as the level of environmental noise which will prevent any measurable hearing loss over a lifetime. Likewise, levels of 55 decibels outdoors and 45 decibels indoors are identified as preventing activity interference and annoyance. These levels of noise are considered those which will permit spoken conversation and other activities such as sleeping, working and recreation, which are part of the daily human condition.

http://www.epa.gov/history/topics/noise/01.htm

Muddying The Waters

The manager of the Riverside Center is concerned about visitor numbers.
He is certain the Center's popularity has been badly affected by an increase in river pollution.
He feels the local Environmental Agency should do something about it.

To support his argument he measured the chemical concentration in the river each month.
He also counted the number of people visiting the Center over several months.
He used the results to draw this chart.

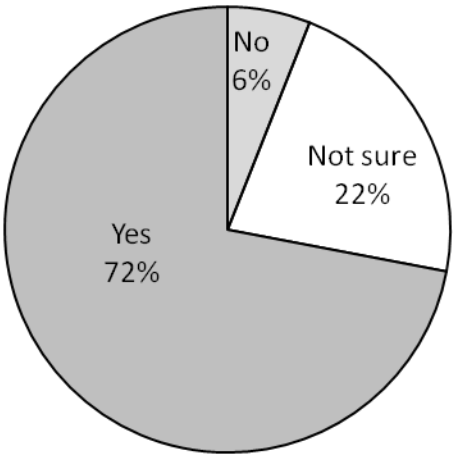


Scatter chart: Chemical concentration and number of visitors

At the same time the manager asked 18 visitors this question:

'The odor you can smell originates from the pollution in the river.
Is it spoiling your enjoyment of the Center?'

He displayed the results as a pie chart.



Pie chart showing the percentage of visitors whose enjoyment was spoiled.

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The centre manager writes to the Environmental Officer to try to get something done about the river pollution.

Dear Environmental Officer,

Please find enclosed two charts.

The scatter plot clearly shows that the increase in the concentration of the chemical in the river has caused a real drop-off in visitor numbers to the Center over the last year.

The pie chart proves that people (not surprisingly) don't like the acrid smell of pollution wafting up from the river.

The river needs to be cleaned up; it's not good for the environment and it's certainly not good for my business. Please let me know what action you intend to take.

Yours faithfully,

Manager, Riverside Center

Tasks

1. Describe in detail what you think the two charts show.

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2. Do you think the Riverside Center Manager's argument is fair?
Explain your reasoning.

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A Case Of Muddying The Waters: Role-Play Scripts

Scene 1: Environmental Officer talking to the Assistant DA in the DA's office

Teacher	A year ago a small dam was built across the river to provide extra water for local domestic use. It was positioned just upstream of the Riverside Center and a factory [*show class **Exhibit 1** and point out the landmarks*]. This factory continually discharges a toxic chemical into the river. Prompted by the letter [*show class **Exhibit 2***] from the manager of the Riverside Center, the local Environmental Officer [*indicate the student playing this role*] checks the concentration of the chemical in the river. Unhappy with the result he arranges to meet the Assistant DA at his office [*indicate the student playing this role*]. Here is how the meeting went *[show class the photograph of the DAs office*]:
Environmental Officer	Each year I test the river water for this toxic chemical, and up until this year the concentration levels have been fine.
Assistant DA	But there's a problem now?
Environmental Officer	That's right. As you can see from this chart, [*show class **Exhibit 3***] the concentration of the chemical in the river is now disturbingly high.
Assistant DA	What can account for this change?
Environmental Officer	Well, downstream of the barrier the flow of the river has decreased from 20 to 4 cubic meters per second.
Assistant DA	Go on.
Environmental Officer	This decrease does affect the concentration of the chemical in the river. To calculate this concentration you divide the chemical discharge from the factory by the rate of flow of the river.
Assistant DA	Oh yeah, right [*not understanding a word*]. Have you noticed any other changes?
Environmental Officer	Yes, the number of diseased fish in the river. Two years ago we only found 6 diseased fish in the waters near the factory, but just last week we found 64. I'm sure you'll agree that's a massive increase.
Assistant DA	Mmm…. I think I better take a ride out to the factory and see what the owner has to say about all this.

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Scene 2: Assistant DA interviewing the Factory Owner in the Factory Owner's office

Show the photograph of the factory.

Teacher	So the assistant DA then drove out to the factory to interview its owner [*indicate the student playing this role to the class*].
Assistant DA	Do you know you are discharging a toxic chemical into the river?
Factory owner	Sure we know that, but we're only discharging it at the rate of 60 milligrams per second. When I first opened the factory some years ago, my manager checked that this was within the legal limits, and this rate has not changed since then.
Assistant DA	I see. But the fact is, the concentration of the chemical in the river is now above the legal limit, and you are the only factory in the area producing the stuff.
Factory owner	Well, it's a mystery to me. Something else must have caused the increase.
Assistant DA	I hear what you're saying, but I still think you're responsible.
Factory owner	Do you reckon – because I don't. What is important, though, is the health of the river. As you probably know, a good sign of a healthy river is the variety of the invertebrates in it. I've been keeping an eye them in our river.
Assistant DA	What, you mean you've actually been counting the number of snails and water insects in the river?
Factory owner	Well yes, my people have counted all animals without a backbone. They've been counted at four different sites and the average number has barely changed in the last two years [*show class Exhibit 1 and point to the 4 sites*]. That's a good sign, don't you think?
Assistant DA	That is interesting. Have you checked anything else?
Factory owner	I've also kept a close watch on the number of birds around the factory, and, as you can see from the chart, there has been a dramatic increase [*show class Exhibit 4*]. Another sure sign of a healthy river.
Assistant DA	Well, I will get my experts to check all this. But from what I've heard, there is a case to answer. So I'll see you in court!

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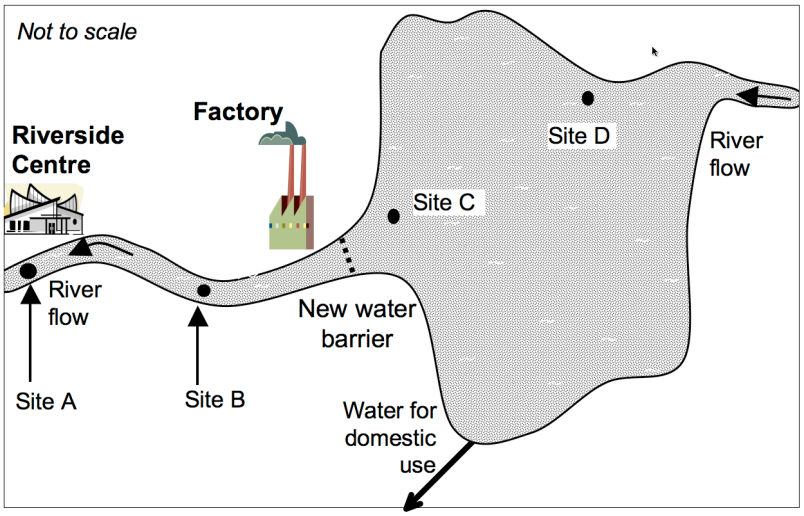
Case Notes

Background Information

Last year, a small dam was built across the river just upstream of the Riverside Center and the factory. The factory discharges a toxic chemical into the river.

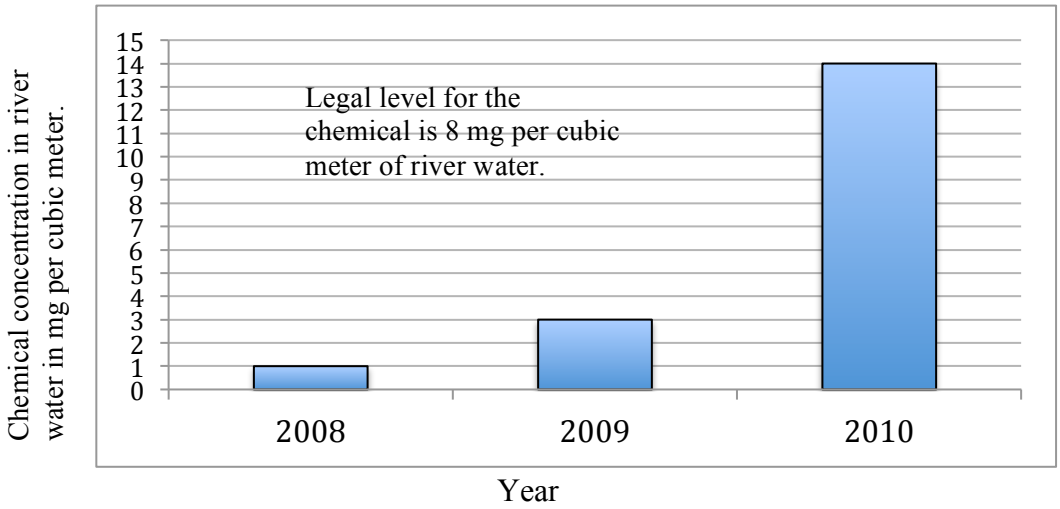
Exhibit 1: Map of the Riverside Area



Judge's notes:

Environmental Officer's Evidence

Exhibit 3: Bar chart showing level of chemical concentration in river water



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"Each year, I test the river water for this toxic chemical, and up until this year the concentration levels have been fine.

The chart shows that the concentration of the chemical in the river is now disturbingly high.

Downstream of the barrier, the flow of the river has decreased from 20 to 4 cubic meters per second. This affects the concentration of the chemical in the river.

To calculate this concentration you divide the chemical discharge from the factory by the rate of flow of the river."

Judge's notes:

Factory Owner's Evidence

"We are discharging a toxic chemical, but only at the rate of 60 milligrams per second.

When I first opened my factory the manager checked that this is within the legal limits.

The rate has not changed since then.

If the increase is now outside the legal limits, someone else must have caused the change."

Judge's notes:

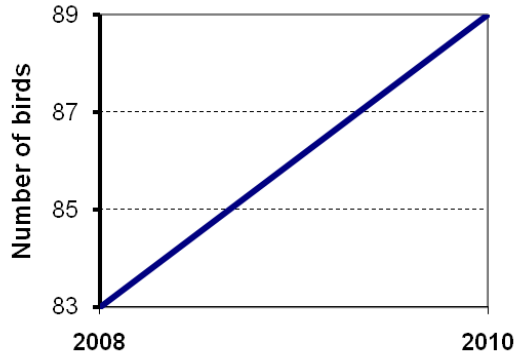
Factory Owner's Evidence

"I've kept a close watch on the number of birds around the factory.

You can see from the chart that there has been a dramatic increase in the number of birds around here.

This is another sure sign of a healthy river."

Exhibit 4: Survey of the number of birds seen next to the factory in one afternoon



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Judge's notes:

Environmental Officer's Evidence

"There has been an increase in the number of diseased fish in the river."

Exhibit 5: Survey of diseased fish near site A

Both surveys covered a period of five days.

- Two years ago 6 fish out of 300 were diseased.
- Last week 64 fish out of 1,600 were diseased.

Judge's notes:

The Factory Owner's Evidence

"It's important to think about the health of the river.
A sign of a good healthy river is the variety of invertebrates in it.
I've been counting the number of animals without a backbone – things like snails and water insects."

Exhibit 6: Survey of numbers of invertebrates

	Two years ago	Now
Site A	20	15
Site B	22	9
Site C	19	23
Site D	23	29

"Invertebrates were counted at four different sites.
On the map (Exhibit 1), you can see where the sites are.
The average number of invertebrates has barely changed in the last two years - a good sign."

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Judge's notes:

Judge's summing up:

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Unhappy Campers

Peggy is the manager of a campground.

Some new wind turbines are built next to the camp.

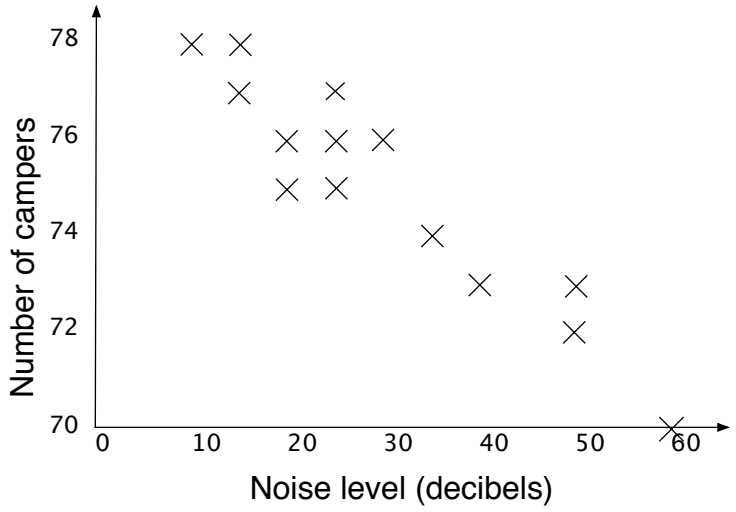
Peggy feels sure fewer people are visiting her site because of the turbines.

Peggy wants the Environmental Agency to get rid of the turbines.

To make her case she does some math.

For two weeks, Peggy records the number of visitors to the campsite each day. She also records the highest level of noise the turbines make each day.

She uses the information to draw a scatter chart.



Peggy also conducts a survey of the 50 campers who visit one day.

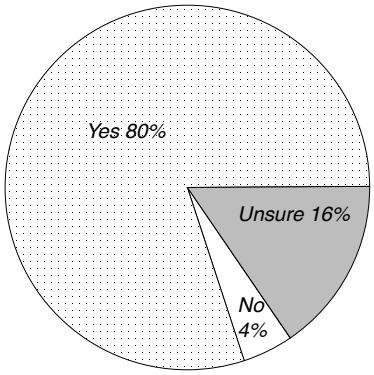
She asks the campers this question:

The loud noise you can hear comes from turbines on the wind farm.

Is the wind farm spoiling your enjoyment of the campsite? *Yes* *No* *Unsure*

Peggy uses the survey results to draw a pie chart.

Pie chart showing percentage of campers whose enjoyment was spoiled by turbine noise.



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Peggy writes to the Environmental Officer.

Dear Environmental Officer,

Please find enclosed two charts.

The new wind farm is noisy and ugly, and I have evidence to prove it.

The scatter chart shows that the wind farm drives campers away. The noise causes fewer people to come to camp.

The pie chart shows that most people don't like the wind farm. It's affecting their enjoyment of the camp.

You should close the wind farm because it is spoiling the local environment.

Yours faithfully,

Camp Manager

1. Describe what you think is shown by the scatter graph and pie chart.

2. Do you think the camp manager makes a fair argument? Explain your answer.

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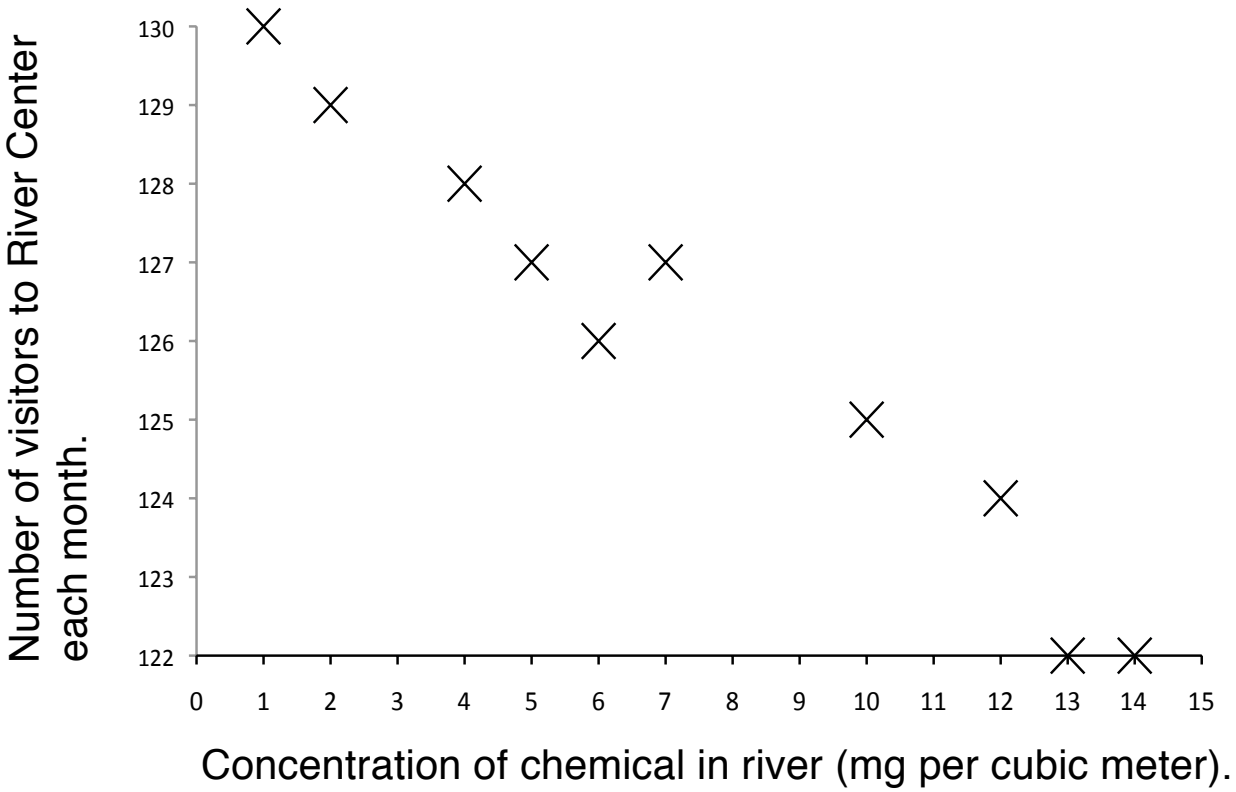
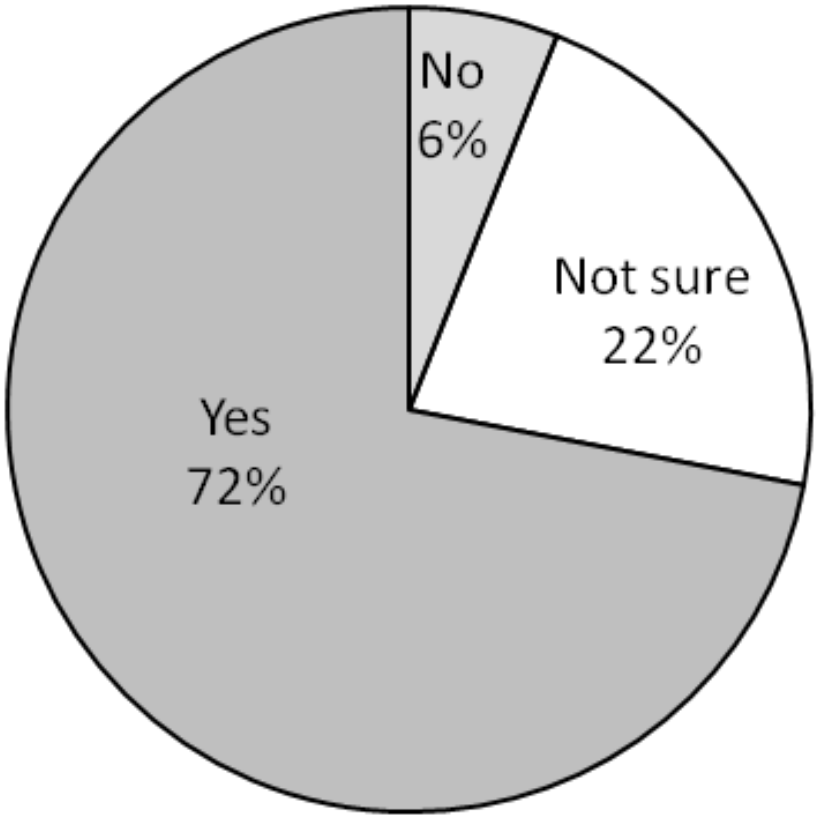
Muddying the Waters: Scatter Chart



Scatter chart: Chemical concentration and number of visitors.

Muddying the Waters: Pie Chart



Pie chart showing the percentage of visitors whose enjoyment was spoiled.

Exhibit 1: Map of Riverside Area

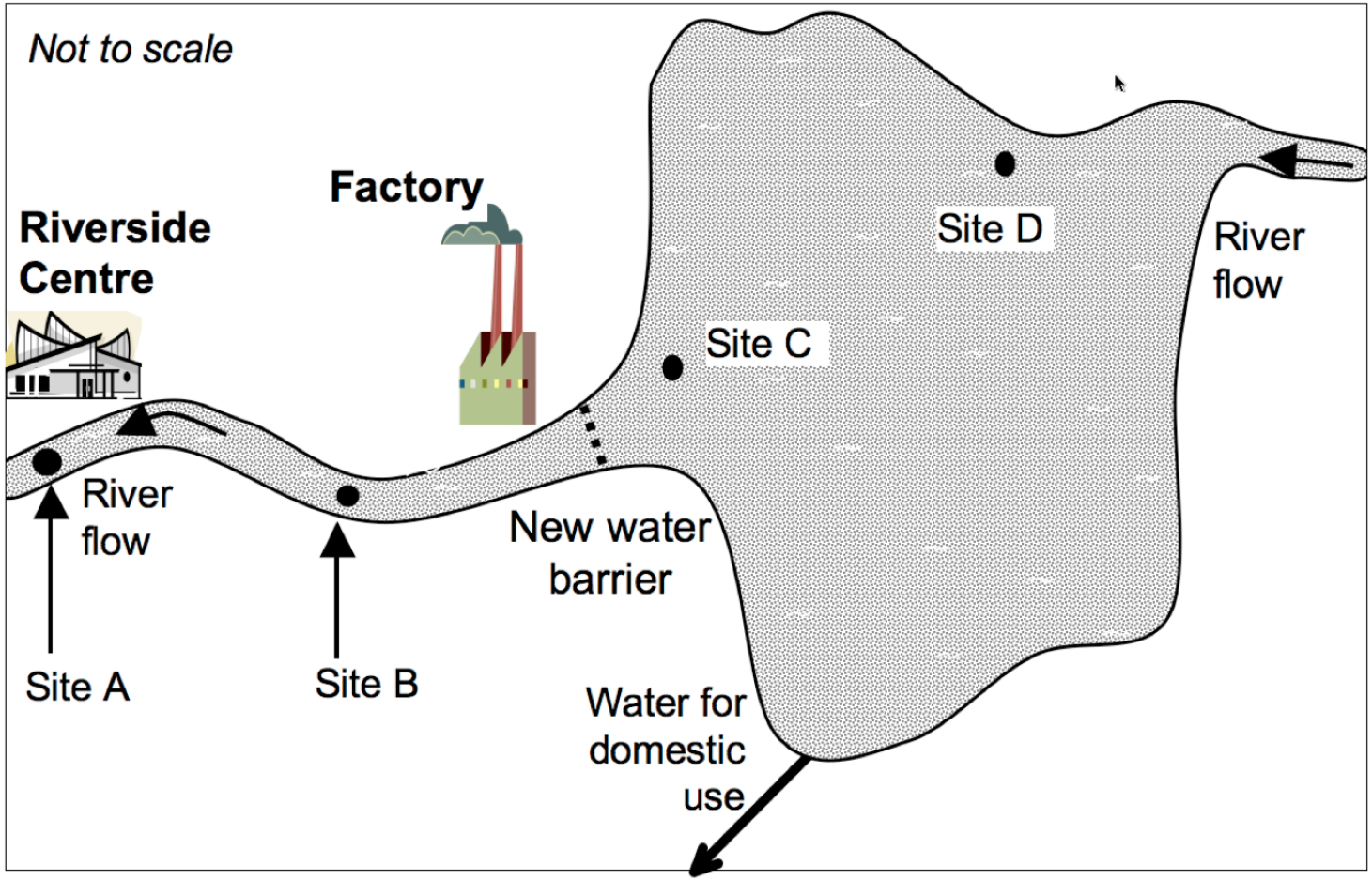


Exhibit 2: Riverside Manager's Letter

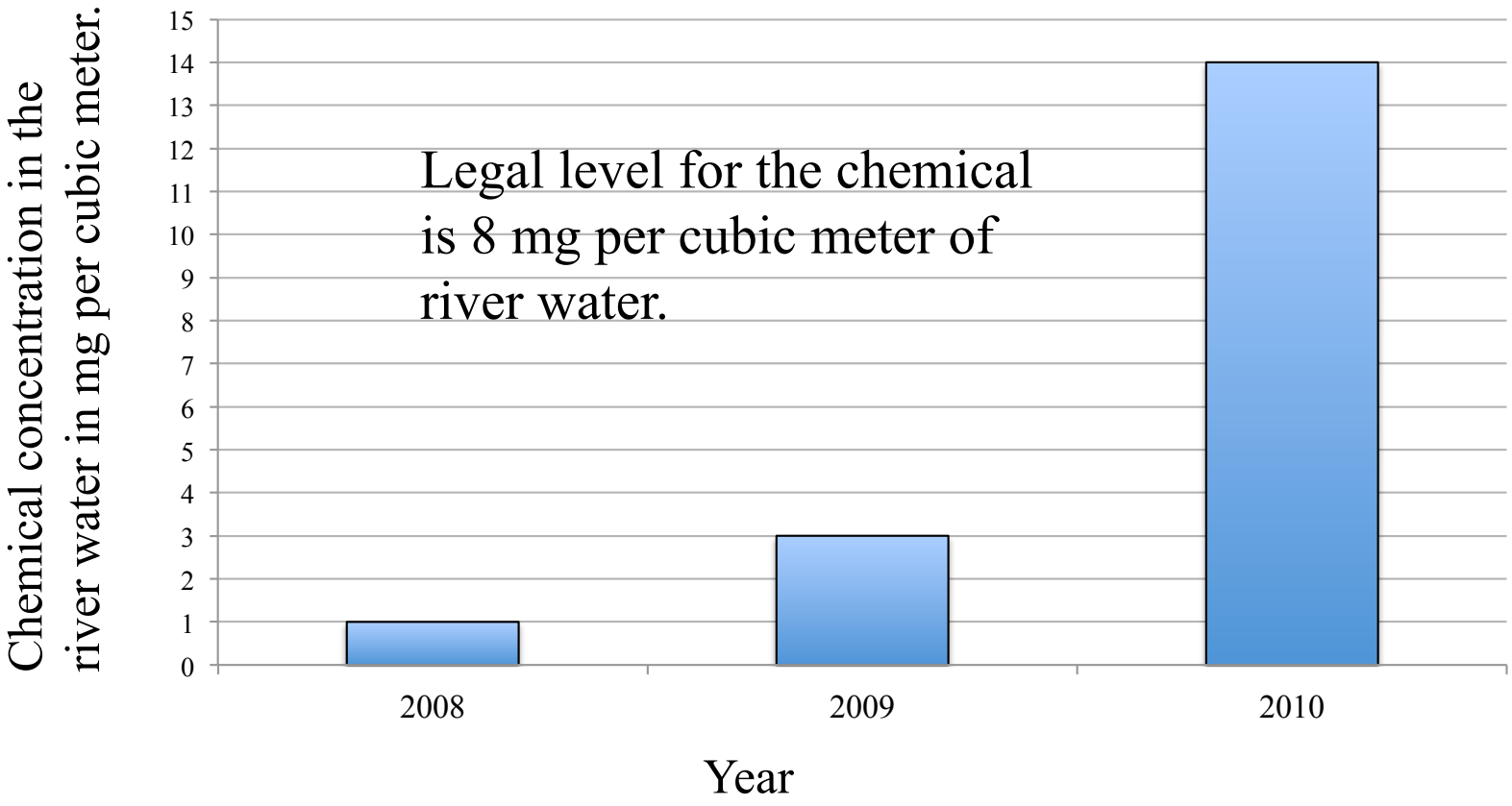
Dear Environmental Officer,
Please find enclosed two charts.
The scatter plot clearly shows that the increase in the concentration of the chemical in the river has caused a real drop-off in visitor numbers to the Center over the last year.
The pie chart proves that people (not surprisingly) don't like the acrid smell of pollution wafting up from the river.
The river needs to be cleaned up; it's not good for the environment and it's certainly not good for my business.
Please let me know what action you intend to take.
Yours faithfully,
Manager, Riverside Center

Scene 1: The Assistant DA's Office

Scene 2: At the factory

Exhibit 3

Bar chart showing level of chemical concentration in the river water.



Judge's Instructions

You have to reach a fair judgment about who wins the case.

- Read through the information carefully.
- Write notes on what you think the data and statistics show.
- Explain why you agree or disagree with the arguments, using math.

- Look critically at all of the information.
- Do not just accept what people say as *fact*.

Exhibit 4

Survey of the number of birds seen next to the factory in one afternoon.

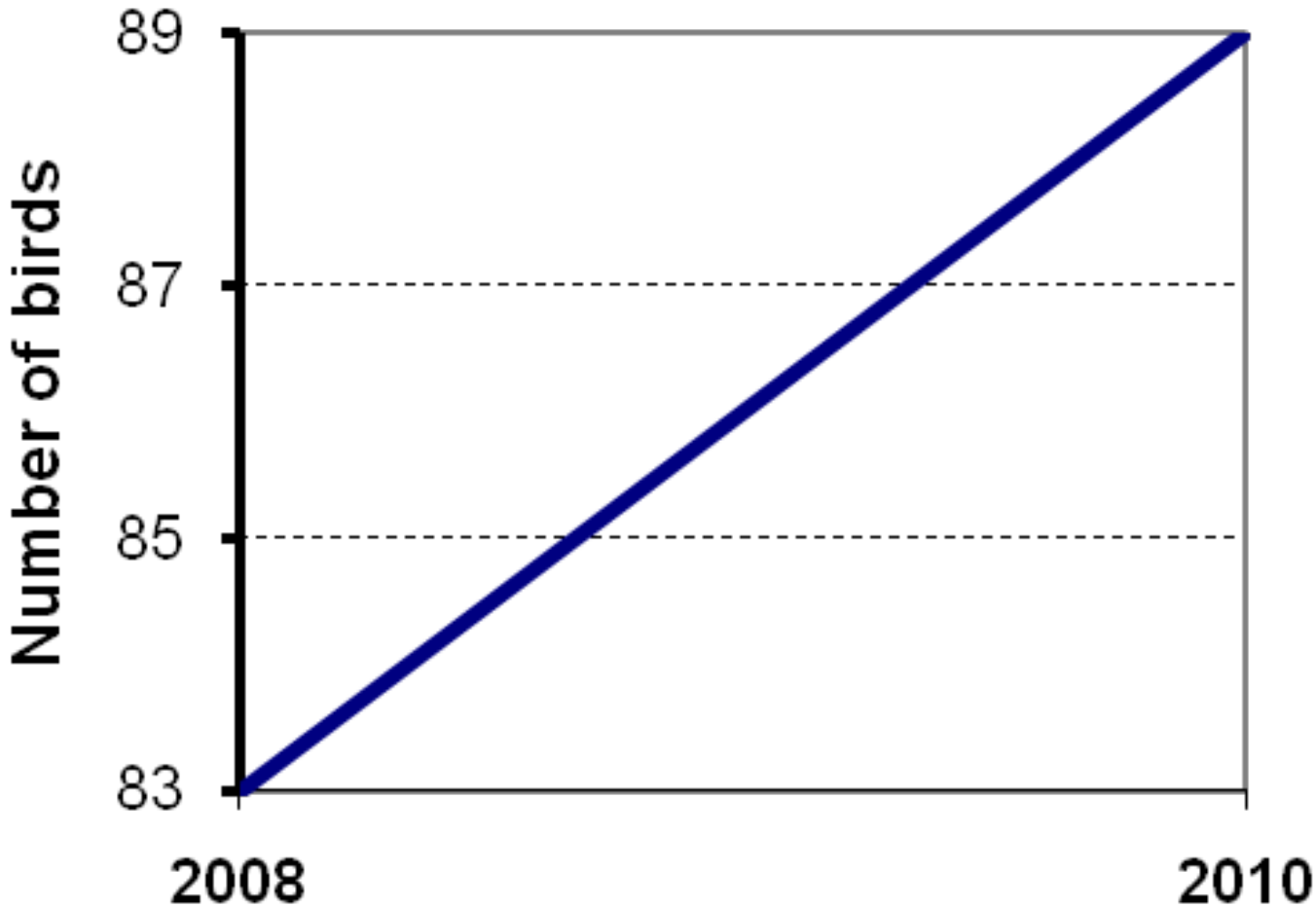


Exhibit 5

Survey of diseased fish near site A

Both surveys covered a period of five days.

- Two years ago 6 fish out of 300 were diseased.
- Last week 64 fish out of 1,600 were diseased.

Exhibit 6

Survey of numbers of invertebrates

	Two years ago	Now
Site A	20	15
Site B	22	9
Site C	19	23
Site D	23	29

Mathematics Assessment Project

CLASSROOM CHALLENGES

This lesson was designed and developed by the
Shell Center Team
at the
University of Nottingham
Malcolm Swan, Nichola Clarke, Clare Dawson, Sheila Evans
with
Hugh Burkhardt, Rita Crust, Andy Noyes, and Daniel Pead

It was refined on the basis of reports from teams of observers led by
David Foster, Mary Bouck, and Diane Schaefer
based on their observation of trials in US classrooms
along with comments from teachers and other users.

This project was conceived and directed for
MARS: Mathematics Assessment Resource Service
by
Alan Schoenfeld, Hugh Burkhardt, Daniel Pead, and Malcolm Swan
and based at the University of California, Berkeley

We are grateful to the many teachers, in the UK and the US, who trialed earlier versions
of these materials in their classrooms, to their students, and to
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We are particularly grateful to
Carina Wong, Melissa Chabran, and Jamie McKee

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Unit 4 – Inferences and Conclusions from Data

Domain: Statistics and Probability (S)
Essential Question: How do data interpretations help develop informed decisions and predictions?

Mathematical Practices:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with Mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Evaluate study design and analyze normal distributions to draw conclusions about populations.** **S-IC** Make inferences and justify conclusions from sample surveys, experiments, and observational studies	**How…** **S.IC.4:** Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. **S.IC.5:** Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.	**Sample Survey Simulation:** ▪ Analyze population parameters based on data collected from a sample simulation done with a graphing calculator.	**S.IC.4:** Students may use computer generated simulation models based upon sample surveys results to estimate population statistics and margins of error. **S.IC.5:** Students may use computer generated simulation models to decide how likely it is that observed differences in a randomized experiment are due to chance. Treatment is a term used in the context of an experimental design to refer to any prescribed combination of values of explanatory variables. For example, one wants to determine the effectiveness of weed killer. Two equal parcels of land in a neighborhood are treated; one with a placebo and one with weed killer to determine whether there is a significant difference in effectiveness in eliminating weeds.

Vocabulary: inference, mean, normal approximation, normal distribution, observational study, population parameter, population percentage, randomized experiment, sample survey, spreadsheet, standard deviation

Unit 4 – Inferences and Conclusions from Data

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Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Evaluate study design and analyze normal distributions to draw conclusions about populations.** **S-IC** Make inferences and justify conclusions from sample surveys, experiments, and observational studies	**How…** **S.IC.6:** Evaluate reports based on data.	**What's Wrong With This? Worksheet** ▪ Compare and contrast data sets to determine if interpretations of the data are accurate and if appropriate calculations were completed.	**S.IC.6:** Explanations can include but are not limited to sample size, biased survey sample, interval scale, unlabeled scale, uneven scale, and outliers that distort the line-of-best-fit. In a pictogram the symbol scale used can also be a source of distortion. As a strategy, collect reports published in the media and ask students to consider the source of the data, the design of the study, and the way the data are analyzed and displayed. Example: • A reporter used the two data sets below to calculate the mean housing price in Arizona as $629,000. Why is this calculation not representative of the typical housing price in Arizona? • King River area {1.2 million, 242000, 265500, 140000, 281000, 265000, 211000} • Toby Ranch homes {5million, 154000, 250000, 250000, 200000, 160000, 190000}

***Vocabulary*:** inference, mean, normal approximation, normal distribution, observational study, population parameter, population percentage, randomized experiment, sample survey, spreadsheet, standard deviation

Unit 4 – Inferences and Conclusions from Data

Domain: Statistics and Probability (S)

Essential Question: How do data interpretations help develop informed decisions and predictions?

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1. **Make sense of problems and persevere in solving them.**
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4. **Model with Mathematics.**
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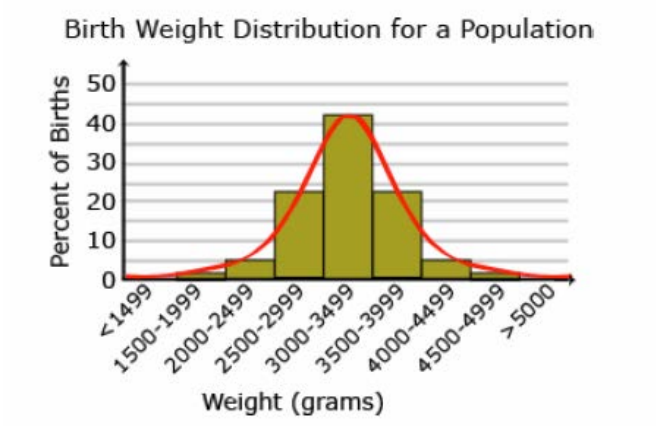
Unit Goal	*Performance Standards*	*Expected Performance Assessment*	*Instructional Strategies*
Students will… **Evaluate study design and analyze normal distributions to draw conclusions about populations.** **S-MD** Use probability to evaluate outcomes of decisions	**How…** **S.MD.6:** (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). **S.MD.7:** (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).	**Probability Based Decision Making Exit Check** ▪ Evaluate probabilities to make determinations about a population or sample. **Does This Make Sense Case Study Analysis:** ▪ Review case studies in which probability models were used to make decisions and evaluate the accuracy of those models.	**S.MD.6:** Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to midland interpret parameters in linear, quadratic or exponential functions. **S.MD.7:** Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to midland interpret parameters in linear, quadratic or exponential functions.

Vocabulary: inference, mean, normal approximation, normal distribution, observational study, population parameter, population percentage, randomized experiment, sample survey, spreadsheet, standard deviation

MHS Algebra 2 Mathematics Curriculum

Unit 4 Addendum

Performance Standard	Instructional Strategies
S.ID.4:	Students may use spreadsheets, graphing calculators, statistical software and tables to analyze the fit between a data set and normal distributions and estimate areas under the curve. Examples: ● The bar graph below gives the birth weight of a population of 100 chimpanzees. The line shows how the weights are normally distributed about the mean, 3250 grams. Estimate the percent of baby chimps weighing 3000-3999 grams.  ● Determine which situation(s) is best modeled by a normal distribution. Explain your reasoning. o Annual income of a household in the U.S. o Weight of babies born in one year in the U.S.

Mathematics Common Core Glossary

Mathematics Glossary

Addition and subtraction within 5, 10, 20, 100, or 1000. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: 8 + 2 = 10 is an addition within 10, 14 – 5 = 9 is a subtraction within 20, and 55 – 18 = 37 is a subtraction within 100.

Additive inverses. Two numbers whose sum is 0 are additive inverses of one another. Example: 3/4 and – 3/4 are additive inverses of one another because 3/4 + (– 3/4) = (– 3/4) + 3/4 = 0.

Associative property of addition. See Table 3 in this Glossary.

Associative property of multiplication. See Table 3 in this Glossary.

Bivariate data. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team. Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.[1]

Commutative property. See Table 3 in this Glossary.

Complex fraction. A fraction A/B where A and/or B are fractions (B nonzero).

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. *See also:* computation strategy.

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. *See also:* computation algorithm.

Congruent. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

Counting on. A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by counting on—pointing to the top book and saying “eight,” following this with “nine, ten, eleven. There are eleven books now.”

Dot plot. *See*: line plot.

Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.

Expanded form. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten. For example, 643 = 600 + 40 + 3.

Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

First quartile. For a data set with median M, the first quartile is the median of the data values less than M. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.[2] *See also:* median, third quartile, interquartile range.

Fraction. A number expressible in the form a/b where a is a whole number and b is a positive whole number. (The word fraction in these standards always refers to a non-negative number.) *See also:* rational number.

Identity property of 0. See Table 3 in this Glossary.

Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.

Integer. A number expressible in the form a or –a for some whole number a.

Interquartile Range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 – 6 = 9. *See also:* first quartile, third quartile.

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. Also known as a dot plot.[3]

Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.4 Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

Midline. In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values. Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: 72 ÷ 8 = 9.

Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. Example: 3/4 and 4/3 are multiplicative inverses of one another because 3/4 × 4/3 = 4/3 × 3/4 = 1.

Number line diagram. A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

Percent rate of change. A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by 5/50 = 10% per year.

Probability distribution. The set of possible values of a random variable with a probability assigned to each.

Properties of operations. See Table 3 in this Glossary.

Properties of equality. See Table 4 in this Glossary.

Properties of inequality. See Table 5 in this Glossary.

Properties of operations. See Table 3 in this Glossary.

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

Probability model. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. *See also:* uniform probability model.

Random variable. An assignment of a numerical value to each outcome in a sample space. Rational expression. A quotient of two polynomials with a non-zero denominator.

Rational number. A number expressible in the form a/b or – a/b for some fraction a/b. The rational numbers include the integers.

Rectilinear figure. A polygon all angles of which are right angles.

Rigid motion. A transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

Repeating decimal. The decimal form of a rational number. *See also:* terminating decimal.

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

Scatter plot. A graph in the coordinate plane representing a set of bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.[5]

Similarity transformation. A rigid motion followed by a dilation.

Tape diagram. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model.

Terminating decimal. A decimal is called terminating if its repeating digit is 0.

Third quartile. For a data set with median M, the third quartile is the median of the data values greater than *M*. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. *See also:* median, first quartile, interquartile range.

Transitivity principle for indirect measurement. If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C. This principle applies to measurement of other quantities as well.

Uniform probability model. A probability model which assigns equal probability to all outcomes. *See also:* probability model.

Vector. A quantity with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers.

Visual fraction model. A tape diagram, number line diagram, or area model.

Whole numbers. The numbers 0, 1, 2, 3,....

Table 1 Common addition and subtraction situations[1]

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ? **(K)**	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5 **(1st)**	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5 **(2nd)**
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ? **(K)**	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 - ? = 3 **(1st)**	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3 **(2nd)**
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[2]**
Put Together/ Take Apart[3]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ? **(K)**	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ? **(K)**	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2 **(1st)**
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[4]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? **(1st)** ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 + ? **(1st)**	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? **(1st)** (Version with "fewer") Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ? **(1st)**	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? **(1st)** (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 + ?, ? + 3 = 5 **(2nd)**

K: Problem types to be mastered by the end of the Kindergarten year.

1st: Problem types to be mastered by the end of the First Grade year, including problems types from the previous year(s). However, First Grade students should have experiences with all 12 problem types.

2nd: Problem types to be mastered by the end of the Second Grade year, including problem types from the previous year(s).

[1]Adapted from box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

[2]These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.

[3]Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

[4]For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

Table 2 Common multiplication and division situations[1]

	Unknown Product 3 x 6 = ?	Group Size Unknown ("How many in each group?" Division) 3 x ? = 18 and 18 ÷ 3 = ?	Number of Groups Unknown ("How many groups?" Division) ? x 6 = 18 and 18 ÷ 6 = ?
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag?	If 18 plumbs are to be packed 6 to a bag, then how many bags are needed?
	Measurement example. You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	*Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	*Measurement example.* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays[2], Area[3]	There are 3 rows of apples with 6 apples in each row. How many apples are there?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be?
	Area example. What is the area of a 3 cm by 6 cm rectangle?	*Area example.* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	*Area example.* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs $5. A red hat costs 3 times as much as the blue hat. How much does the red hat cost?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat?
	Measurement example. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	*Measurement example.* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	*Measurement example.* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	*a x b = ?*	*A x ? = p, and p ÷ a + ?*	*? x b = p, and p ÷ b = ?*

[1]The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

[2]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[3]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

Table 3 The properties of operations

Here *a, b* and *c* stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Associative property of multiplication	$(a \times b) \times c = a \times (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Distributive property of multiplication over addition	$a \times (b + c) = a \times b$

8 Mathematical Practices Presentation

Transitioning to the Common Core State Standards for Mathematics

KRISTIN SMITH
MANCHESTER PUBLIC SCHOOLS
APRIL 19, 2012

Session Goals

- Provide a close look at the 8 Mathematical Practices.
- Next steps….reflection.

Common Core Mission Statement

- The Common Core State Standards provide a consistent, clear understanding of what students are expected to learn, so teachers and parents know what they need to do to help them. The standards are designed to be robust and relevant to the real world, reflecting the knowledge and skills that our young people need for success in college and careers. With American students fully prepared for the future, our communities will be best positioned to compete successfully in the global economy.

Common Core State Standards for Mathematics

- Math content standards describe what students should understand and be able to do.

- Math practices describe ways in which students should interact with mathematics.

(CSS p.8)

The 8 Mathematical Practices

- The 8 mathematical practices are located in the front within the "nature of mathematics" section of the Common Core document. This is the foundation for the pedagogical shift in mathematics.

- Specifically, the 8 mathematical practices illustrate the connection between 21st century skills and mathematical content and instruction.

8 Standards of Mathematical Practice

1. **Make sense of problems and persevere in solving them**
2. **Reason abstractly and quantitatively**
3. **Construct viable arguments and critique the reasoning of others**
4. **Model with mathematics**
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

Mathematically Proficient Students Will…

1 Make sense of problems and persevere in solving them.

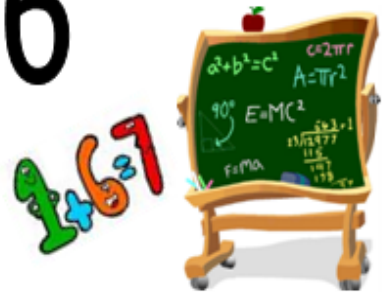
2 Reason abstractly and quantitatively.

3 Construct viable arguments and critique the reasoning of others.

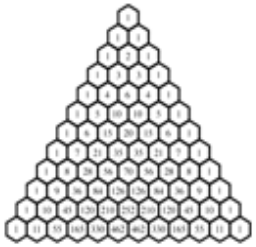
4 Model with mathematics.

5 Use appropriate tools strategically.

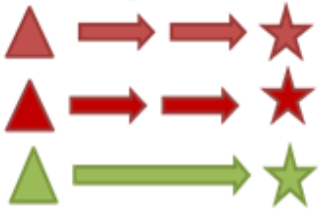
6 Attend to precision.



7 Look for and make use of structure.



8 Look for and express regularity in repeated reasoning.



The Rigor…….

Mathematically Proficient Students…

To really understand the rigor involved with the mathematical practices, consider the verbs that illustrate the student actions….

The Standards for [Student] Mathematical Practice

For example, examine Practice #3: Construct viable arguments and critique the reasoning of others.

Highlight the verbs.

What jumps out at you?

Mathematical Practice #3: Construct viable arguments and critique the reasoning of others

Mathematically proficient students:

• understand and use stated assumptions, definitions, and previously established results in constructing arguments.

• make conjectures and build a logical progression of statements to explore the truth of their conjectures.

• analyze situations by breaking them into cases, and can recognize and use counterexamples.

• justify their conclusions, communicate them to others, and respond to the arguments of others.

• reason inductively about data, making plausible arguments that take into account the context from which the data arose.

• compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and-if there is a flaw in an argument-explain what it is.

• construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades.

• determine domains to which an argument applies.

• listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Mathematical Practice #3: Construct viable arguments and critique the reasoning of others

Mathematically proficient students:

- understand and use stated assumptions, definitions, and previously established results in constructing arguments.
- make conjectures and build a logical progression of statements to explore the truth of their conjectures.
- analyze situations by breaking them into cases, and can recognize and use counter examples.
- justify their conclusions, communicate them to others, and respond to the arguments of others.
- reason inductively about data, making plausible arguments that take into account the context from which the data arose.
- compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and-if there is a flaw in an argument-explain what it is.
- construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades.
- determine domains to which an argument applies.
- listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Mathematical Practice #3: Construct viable arguments and critique the reasoning of others

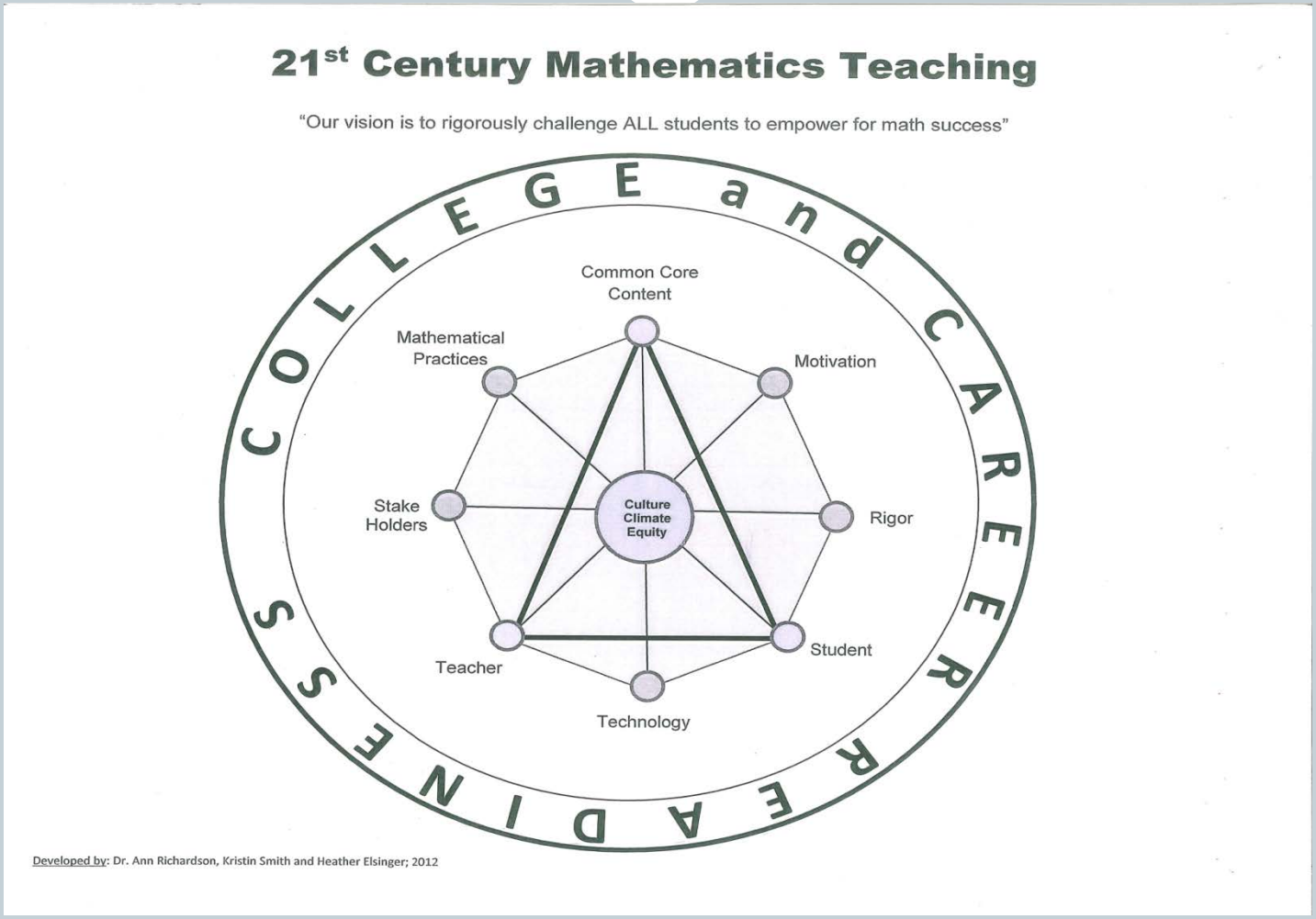
On a scale of 1 (low) to 6 (high), to what extent are you promoting students' proficiency in Practice 3?

- *<u>District expectations</u>*

Students will:

- *Will construct arguments and justifications using our resources.*
- *Will provide rigor so that students will be college and career ready.*

Our Vision


21st Century Mathematics Teaching
"Our vision is to rigorously challenge ALL students to empower for math success"
COLLEGE and CAREER READINESS
Common Core Content
Motivation
Rigor
Student
Technology
Teacher
Stake Holders
Mathematical Practices
Culture Climate Equity
Developed by: Dr. Ann Richardson, Kristin Smith and Heather Elsinger; 2012

Make Sense of Problems and Persevere in Solving Them

- Frame the class environment to encourage student interaction and conversation – math discourse
- Allow students to "struggle" with the mathematical tasks – ***avoid rescuing too soon to diminish the cognitive load***
- Emphasize equivalent representations of a given situation or mathematical relationship

1: Make sense of problems and persevere in solving them.

Mathematically Proficient Students:

- Explain the meaning of the problem to themselves
- Look for entry points
- Analyze givens, constraints, relationships, goals
- Make conjectures about the solution
- Plan a solution pathway
- Consider analogous problems
- Monitor and evaluate progress, and change course if necessary
- Check their answer to problems using a different method
- Continually ask themselves "Does this make sense?"

Gather Information → Make a plan → Anticipate possible solutions → Continuously evaluate progress → Check results → Question sense of solutions

2. Reason Abstractly and Quantitatively

- Base instruction on making sense and select practice that involves the application of concepts being learned
- Emphasize reasoning as opposed to only learning procedures
- Allow students to develop a representation of mathematical problems on a regular basis

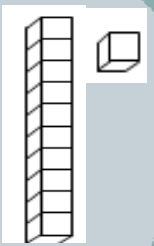
2: Reason abstractly and quantitatively

Mathematical Problem

Decontextualize

Represent as symbols, abstraction

4



x x x x

5 + 2 =?

Refer back to the situation

Contextualize

3. Construct Viable Arguments and Critique the Reasoning of Others

- Use problem-based activities – rich tasks
- Practice the language of “argument,” conjecture, and discourse while students are engaged in mathematical tasks.
- Encourage taking risks, defending solutions

4. Model with Mathematics

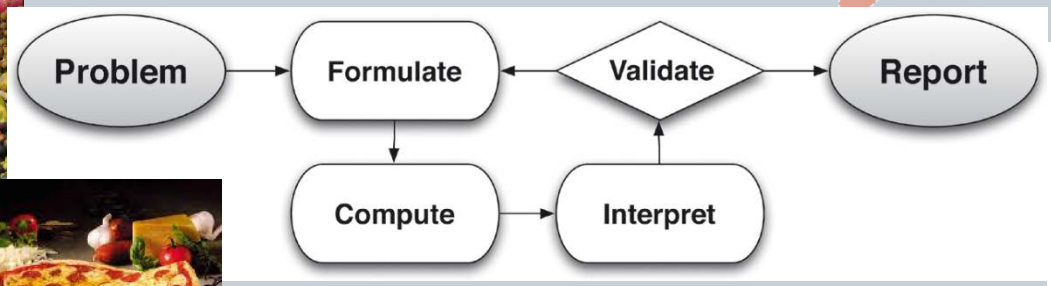
Problems in everyday life…

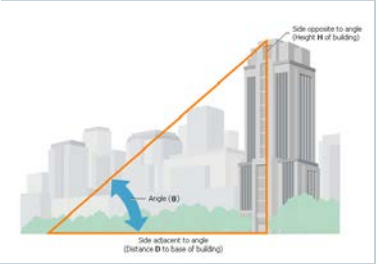
- Frequently use manipulatives and other representations to represent math situations found in real world scenarios.
- Representing the same situation in different ways
- Guide students to see similarities in different ways to represent the same situations

4: Model with mathematics

Problems in everyday life…

…reasoned using mathematical methods





Mathematically proficient students:

- make assumptions and approximations to simplify a situation, realizing these may need revision later
- interpret mathematical results in the context of the situation and reflect on whether they make sense

Images: http://tandrageemaths.wordpress.com, asiabcs.com, ehow.com, judsonmagnet.org, life123.com, teamuptutors.com, enwikipedia.org, glennsasscer.com

If we focus on the first 4 Mathematical Practices, will we also take care of the last 4?

1. **Make sense of problems and persevere in solving them**
2. **Reason abstractly and quantitatively**
3. **Construct viable arguments and critique the reasoning of others**
4. **Model with mathematics**
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

A Balanced Combination

- **"The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the (8)practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices."**

(http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf)

Activity: Rich Tasks

Pick **one** task in your packet.

- Read it.
- Talk about what students would need to be able to do to solve it.
- What Standards of Mathematical Practice will this activity cover?

Focus on Specific Examples

- Compare and contrast selected current state standards with related standards from the Common Core for Grade 5.

Comparing Standards

CT- GLE's 5th Grade

- **2.2.12**Develop and use strategies involving place value relationships, inverse operations and algebraic properties (commutative, associative and distributive) to simplify addition, subtraction and multiplication problems with three-, four- and five-digit numbers and money amounts and division by one-digit factors.
- (**Grade 6, 2.2.9 and Grade 7, 2.2**)

New York –5th grade

- Use a variety of strategies to divide three-digit numbers by one-and two-digit numbers.

CCSS- 5th Grade

- **Perform operations with multi-digit whole numbers and with decimals to hundredths.**
- **5.NBT.6** Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division.
- Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Highlights From Content

Specificity in content standards:

- *whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors*
- use strategies based on properties of operations
- use strategies based on place value
- use strategies based on the relationships between multiplication and division.
- *Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.*

CT Standard in Grade 5 is limited to one-digit divisors. CCSS includes two-digit divisors. CT standard does not include "using equations, rectangular arrays, and/or area models." (Grade 5 Mathematics Crosswalk-CCSS to CT Standards. pg 10)

In the Classroom…

What do the 8 Mathematical Practices look like, sound like, and feel like?

Focus for Viewing Video

1. **Make sense of problems and persevere in solving them**
2. **Reason abstractly and quantitatively**
3. **Construct viable arguments and critique the reasoning of others**
4. **Model with mathematics**
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

Watch Video Clip
Grade 5 Number Talk: 496 ÷ 8

Watch the following video clip for an example of instruction that addresses the 8 mathematical practices previously outlined.

Grade 5 class
South Shades Crest Elementary Hoover, AL

Number Talk about: 496 ÷ 8

http://www.mathsolutions.com/index.cfm?page=wp8&crid=494

Focus for Discussing Practices

1. **Using your notes, reflect upon the video and identify the 8 mathematical practices that were demonstrated in the classroom. Label your notes 1-8 to indicate the practices.**

2. **Share points out loud with group.**

Conclusion

The promise of standards:

These Standards are not intended to be new names for old ways of doing business. They are a call to take the next step. It is time for states to work together to build on lessons learned from two decades of standards based reforms. It is time to recognize that standards are not just promises to our children, but promises we intend to keep.

Common Core State Standards for Mathematics (p. 5). © 2010 Common Core State Standards Initiative.

Next Steps

- Continue to deepen your understanding of the content and goals of the CCSS for mathematics.

- Collaborate with others to learn more, ask questions, and investigate.

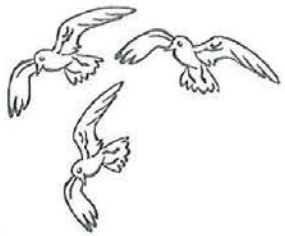
- Resources:

www.corestandards.org
www.edexcellence.net
www.achieve.org
www.mathsolutions.com/commoncore
www.atomicmath.org

Culturally Responsive Teaching

The Iceberg Concept of Culture

Like an iceberg, the majority of culture is below the surface.



Surface Culture

Above sea level
Emotional load: relatively low

food ▪ dress ▪ music ▪
visual arts ▪ drama ▪ crafts
dance ▪ literature ▪ language
celebrations ▪ games

Deep Culture

Unspoken Rules
Partially below sea level
Emotional load: very high

courtesy ▪ contextual conversational patterns ▪ concept of time
personal space ▪ rules of conduct ▪ facial expressions
nonverbal communication ▪ body language ▪ touching ▪ eye contact
patterns of handling emotions ▪ notions of modesty ▪ concept of beauty
courtship practices ▪ relationships to animals ▪ notions of leadership

Unconscious Rules
Completely below sea level
Emotional load: intense

tempo of work ▪ concepts of food ▪ ideals of childrearing
theory of disease ▪ social interaction rate ▪ nature of friendships
tone of voice ▪ attitudes toward elders ▪ concept of cleanliness
notions of adolescence ▪ patterns of group decision-making
definition of insanity ▪ preference for competition or cooperation
tolerance of physical pain ▪ concept of "self" ▪ concept of past and future
definition of obscenity ▪ attitudes toward dependents ▪ problem-solving
roles in relation to age, sex, class, occupation, kinship, and so forth

The differences between……

Multicultural Education:

Multicultural Education helps students to understand and appreciate cultural differences and similarities and to recognize the accomplishments of diverse ethnic, racial, and socioeconomic groups. Classroom materials portray these diverse groups realistically and from a variety of perspectives. It is a philosophical concept built on the ideals of freedom, justice, equality, equity, and human dignity as acknowledged in various documents, such as the U.S. Declaration of Independence, constitutions of South Africa and the United States, and the Universal Declaration of Human Rights adopted by the United Nations. It affirms our need to prepare students for their responsibilities in an interdependent world. It recognizes the role schools can play in developing the attitudes and values necessary for a democratic society. It values cultural differences and affirms the pluralism that students, their communities, and teachers reflect. It challenges all forms of discrimination in schools and society through the promotion of democratic principles of social justice. *(National Association for Multicultural Education, 2003)*

Differentiated Instruction:

Differentiated instruction is a broad term that refers to a variety of classroom practices that accommodate differences in students' learning styles, interests, prior knowledge, socialization needs, and comfort zones. On the secondary level, it involves a balance between the content and competencies expected on the mandated assessments and various pedagogical options to maximize durable learning. The standards tell us what students need to know and to be able to do. Differentiated instruction practices help to get students there, while at the same time teaching them *how* to learn in a meaningful way. A differentiated classroom is widely heterogeneous, dynamic purposeful, and intense. The pedagogical theory that guides differentiation is constructivism: *the belief that learning happens when the learner makes meaning out of information.* *(A. Tomlinson)*

Culturally Responsive Teaching:

Culturally responsive classrooms specifically acknowledge the presence of culturally diverse students and the need for these students to find relevant connections among themselves and with the subject matter and the tasks teachers ask

them to perform. In such programs teachers recognize the different learning styles of their students and develop instructional approaches that will accommodate these styles.

- It acknowledges the legitimacy of the cultural heritages of different ethnic groups.
- It builds bridges of meaningfulness between home and school experiences.
- It uses a wide variety of instructional strategies.
- It teaches students to know and praise their own and each others' cultural heritages.
- It incorporates multicultural information, resources, and materials in all subjects and skills routinely taught in schools.

"Culturally responsive teaching is not about one lesson on Martin Luther King, Jr., during Black History Month. It is not serving tacos in the cafeteria on Cinco de Mayo. Beyond heroes and holidays, it is about understanding students' home life, their language, music, dress, behavior, jokes, ideas about success, the role of religion in their community and more. It is brining the experiences of their 24-hour day into the seven-hour school day to give them information in a familiar context. Like the teacher in Atlanta who conducts a geometry lesson by using geometric patterns in Mexican pottery and in an African kente cloth….that students have brought into class."

"Race and poverty don't need to be the elephants in the classroom. As culturally responsive teaching takes root, these issues can actually help your students learn."

Cynthia Kopkowski, NEA Today November 2006

Creating an *Aesthetically* Culturally Responsive Classroom

Culturally Responsive Classroom defined:

A culturally responsive classroom is premised on the idea that culture is central to student learning. This classroom specifically acknowledges the presence of culturally diverse students and the need for these students to find relevant connections among themselves and with the subject matter and the tasks teachers ask them to perform. In such programs, teachers recognize the different learning styles of their students and develop instructional approaches that will accommodate these styles.

1. Determine who your audience is within the school community. Create an environment that looks like that community.

2. Make sure that the classroom reflects the exact student population. For example, if you have Latino students, make sure that what is in your classroom reflects the Puerto Rican, Mexican, and Columbian cultures as they are the three largest groups of Latino residing in Manchester.

3. Display a variety of cultures throughout and around the classroom:

    a. Current and relevant bulletin boards that display positive and meaningful activities and events involving cultural diverse people.
    b. Newspaper articles (local and national)
    c. Display artwork of diverse artists
    d. Create a book corner that appeals to all children with a variety and range of culturally diverse literature, fiction and nonfiction. The characters should be integrated naturally into the story and not depicted as anomalies or peculiarities in society
    e. Display student family pictures with cultural connections attached
    f. Display student biographies with pictures
    g. Display student cultural knowledge through writing, art, current events
    h. Display learning activities that are reflective of students' backgrounds

i. Ask students to bring in and share artifacts from their home that reflect their culture
j. Display students writing about traditions shared by their families
k. Artifacts on display by theme: Hanukkah, Kwanzaa, Three Kings' day, Women's History Month
l. Include Dr. Martin Luther King, Jr., but move beyond Martin Luther King, Jr. and include leaders like Malcolm X, Marcus Garvey, Luis Llorens Torres, Munoz Rivera, Queen Sondok, Trung Truc, Trung Nhi, Ahmed Deedat, Jabir Ibn Haiyan
m. Form book clubs or literature circles highlighting a variety of books that reflect the student population.
n. Create centers that have a multicultural theme.
o. Establish Morning Meetings to establish relationships with students as you get to know your students' cultural background.
p. Participate in interactive and authentic activities within the classroom.
q. Visual portrayal of male and female representatives of various races, cultures, and physical disabilities in both traditional and non-traditional roles, and in pictorial displays.
r. Structured environment (e.g., seating arrangements and physical placement of furniture) to facilitate integration of all members of the class.
s. Insure that adaptations are made in equipment and room organization so that the effect is a barrier-free environment.
t. The physical environment of the classroom reflects student population including displays on all bulletin boards, posters, and presentations used to decorate the room. This also includes greetings and messages posted on the walls.

Sources:

1. Educating Culturally Responsive Teachers – Ana Maria Villegas and Tamara Lucas, State University of New York Press
2. Culturally Responsive Teaching: Theory, Research, & Practice – Geneva Gay, Teacher's College Press
3. Creating Culturally Responsive Classrooms – Barbara J. Shade, Cynthia Kelly, and Mary Oberg – American Psychological Association
4. *"Creating Culturally Responsive, Inclusive Classrooms*" – Winifred Montgomery, The Council for Exceptional Children

Creating Culturally Responsive, Inclusive Classrooms

Winifred Montgomery

Culturally responsive classrooms specifically acknowledge the presence of culturally diverse students and the need for these students to find relevant connections among themselves and with the subject matter and the tasks teachers ask them to perform.

Let's repeat that: ***Culturally responsive classrooms specifically acknowledge the presence of culturally diverse students and the need for these students to find relevant connections among themselves and with the subject matter and the tasks teachers ask them to perform.*** In such programs teachers recognize the differing learning styles of their students and develop instructional approaches that will accommodate these styles. In light of the value of culturally responsive instructional practices, schools and districts need to support teachers in their quest to learn about the use of these strategies (see box, "Our Increasingly Diverse Classrooms"). This article provides guidelines for creating culturally responsive, inclusive classrooms. Teachers can use these guidelines with students from culturally and linguistically diverse backgrounds in all kinds of classrooms, but particularly in inclusive settings where general and special educators work together to promote the academic, social, and behavioral skills of all students. First, teachers need to take an honest look at their own attitudes and current practice.

Many teachers are faced with limited understanding of cultures other than their own and the possibility that this limitation will negatively affect their students' ability to become successful learners.

Conduct a Self-Assessment

Many teachers are faced with limited understanding of cultures other than their own and the possibility that this limitation will negatively affect their students' ability to become successful learners. Hence, teachers must critically assess their relationships with their students and their understanding of students' cultures (Bromley, 1998; Patton, 1998). The self-assessment in Figure 1, based on the work of Bromley, 1998), is one tool teachers can use to examine their assumptions and biases in a thoughtful and potentially productive way.

Following self-assessment, teachers need to take time to reflect on their responses (what they have learned about themselves) and make some critical decisions regarding ways to constructively embrace diversity and, thus, create learning environments that respond to the needs of their students.

Figure 1. Diversity Self-Assessment

- What is my definition of diversity?
- Do the children in my classroom and school come from diverse cultural backgrounds?
- What are my perceptions of students from different racial or ethnic groups? With language or dialects different from mine? With special needs?
- What are the sources of these perceptions (e.g., friends, relatives, television, movies)?
- How do I respond to my students, based on these perceptions?
- Have I experienced others' making assumptions about me based on my membership in a specific group? How did I feel?
- What steps do I need to take to learn about the students from diverse backgrounds in my school and classroom?
- How often do social relationships develop among students from different racial or ethnic backgrounds in my classroom and in the school? What is the nature of these relationships?
- In what ways do I make my instructional program responsive to the needs of the diverse groups in my classroom?
- What kinds of information, skills, and resources do I need to acquire to effectively teach from a multicultural perspective?
- In what ways do I collaborate with other educators, family members, and community groups to address the needs of all my students?

Source: Adapted from Bromley (1998).

Use a Range of Culturally Sensitive Instructional Methods and Materials

In addition to self-assessment, an important component of effective culturally responsive classrooms is the use of a range of instructional methods and materials (Bromley, 1998). Teachers need to use instructional methods that are tailored to suit the setting, the students, and the subject. By varying and adapting these methods and materials, teachers can increase the chances that their students will succeed. The following are effective culturally sensitive instructional methods.

Our Increasingly Diverse Classrooms

For many reasons, U.S. schools are serving a growing number of students from culturally and linguistically diverse backgrounds (Obiakor & Utley, 1997; Salend, 2001). In fact, the student population in the United States is growing fastest in those segments with which American education has traditionally been least successful—African Americans and Hispanics.

- ***Special Education Overrepresentation.*** A disproportionate number of students from culturally and linguistically diverse backgrounds are inappropriately referred to and placed in special education (Yates, 1998). Data from the Office of Civil Rights reveal that African-American and Hispanic-American students, particularly males, are overrepresented in terms of their identification in the disability categories of serious emotional disturbance and mental retardation (Oswald, Coutinho, Best, & Singh, 1999). These data also indicate that students from culturally and linguistically diverse backgrounds identified as needing special education services are more likely to be provided these services in more restrictive settings than their Caucasian counterparts.
- ***The Negative Effects of Tracking.*** The overrepresentation of students from culturally and linguistically diverse backgrounds in special education can have a negative effect on students and their school performance because it places them in a separate and unequal track that denies them access to the general education curriculum. In addition, once placed in special education classes, these students often encounter lowered teacher expectations, a watered down curriculum, and less effective instruction that can have deleterious effects on their school performance, self-esteem, behavior, education and career goals, and motivation to achieve (Nieto, 1996). As a result, these students often do not return to general education placements and frequently leave school before graduating.
- ***Need for Culturally Responsive Instruction.*** Though several factors contribute to the disproportional representation of students from culturally and linguistically diverse backgrounds in special education (Artiles & Zamora- Duran, 1997), one important factor is the failure of general education teachers to use culturally responsive instructional practices that address their educational, social, and cultural needs (Smith, Finn, & Dowdy, 1993).

Explicit, Strategic Instruction

Explicit, strategic instruction shows students what to do, why, how, and when. An effective strategy is the think-aloud method, a procedure that takes advantage of the benefits of modeling. In a "think-aloud," the teacher reads a passage and talks through the thought processes for students. The objective is to show students how to ask themselves questions as they comprehend text.

Another important strategy is reciprocal questioning where teachers and students engage in shared reading, discussion, and questioning (Leu & Kinzer,1999). The primary goal of this strategy is to help students learn to ask questions of themselves about the meaning they are constructing as they read.

Interdisciplinary Units

Interdisciplinary units include and connect content area learning with language arts and culturally diverse literature (Cooper, 2000; Leu & Kinzer, 1999). Many effective classrooms are organized around an interdisciplinary, or cross-curricular, theme with students participating in meaningful reading, writing, listening, and speaking tasks as they explore the theme through a variety of activities and books. The topic can be drawn from children's lives and interests and sometimes from the curriculum. Teachers can help their students successfully engage in cross-curricular activities by demonstrating how to make connections across the curriculum through literature, by making explicit connections among books, and by helping them recall how previous

activities and experiences relate to current studies.

Instructional Scaffolding

Instructional scaffolding involves the use of teacher demonstration and the modeling of strategies that students need to be successful with content area texts (Galda, Cullinan, & Strickland, 1997; Leu & Kinzer, 1999). In scaffolded instruction, teachers determine the difference between what students can accomplish independently and what they can accomplish with instructional support. Teachers then design instruction that provides just enough scaffolding for students to be able to participate in tasks that currently are beyond their reach.

Teachers need to use instructional methods that are tailored to suit the setting, the students, and the subject.

Over time, as the tasks become more under the control of the learner, the teacher can introduce more difficult tasks.

Journal Writing

Journal writing provides opportunities for students to share their personal understanding regarding a range of literature in various cultural contexts that inform, clarify, explain, or educate them about our culturally diverse society (Montgomery, in press). For example, character study journals permit students to make their own personal connections with a specific character as they read the story. Students develop their own insight into the characters and the events in the story, and they are given the independence to write what they want about the character. The teacher provides time for students to share their journal writings in small cooperative learning groups, with their teachers, with their tutor(s), or with a reading buddy.

In reciprocal questioning, teachers and students engage in shared reading, discussion, and questioning.

Open-Ended Projects

Open-ended projects allow students to contribute at their varying levels of ability. Such projects work well with diverse learners because they need not start or finish at the same time. Students can explore a topic of interest drawn from their readings of culturally rich literature or a content area topic they are currently studying. They may choose to write reports or prepare oral presentations and create artwork to illustrate some of the major concepts embedded in their topic. Goforth (1998) suggests a project in which interested students make artifacts such as dolls or "story cloths" representing an ethnic or cultural group. They may also want to write stories or poems about their artifacts.

Establish a Classroom Atmosphere That Respects Individuals and Their Cultures

Teachers can enhance students' self-esteem when they construct learning environments that reflect the cultural membership in the class. This strategy goes beyond wall decoration to atmosphere: Teachers must attend to all students and try to involve them equally in all class activities. This recognition gives students a positive feeling about

their worth as individuals and as productive members in their classroom. Some strategies to accomplish a positive classroom atmosphere include:

- *Current and relevant bulletin boards* that display positive and purposeful activities and events involving culturally diverse people. Include, for example, newspaper articles (local and national) reporting newsworthy events or accomplishments that involve people of color, photographs of community leaders from culturally diverse backgrounds, student-made posters depicting culturally relevant historical events, and original (student-written) stories and poems with culturally diverse themes.
- *A book corner* with a variety and range of culturally diverse literature, fiction and nonfiction (see box, "Culturally Complex Atmosphere"). The books that are chosen must also deal fairly with disabilities and special needs. The characters should be integrated naturally into the story and not depicted as anomalies or peculiarities in society (Russell, 1994).
- *Cross-cultural literature discussion groups* in which students discuss quality fiction and nonfiction literature that authentically depicts members of diverse cultural groups. Discussion groups help all students feel pride in themselves and in their culture when they see their backgrounds valued in classroom reading and study activities. In small groups, students can read a single work of literature on their own, follow the experiences of a particular character and his or her problems, form opinions about a specific issue put forward in the text, or respond to a significant event that occurred during the character's life (Montgomery, 2000). For example, the content and characterizations in culturally diverse books such as *Amazing Grace* (Hoffman, 1991), *Local News* (Soto, 1993), *Smoky Night* (Bunting, 1994), *The Story of Ruby Bridges* (Coles, 1995) and *Black Cowboys, Wild Horses* (Lester & Pinkney, 1998) can stimulate greater interest in reading *and* in reading to learn.
- *Language arts and social studies programs* provide opportunities for students to share written and oral reports pertaining to their heritage and cultural traditions. Teachers can introduce thematic units that offer excellent opportunities for children to explore a range (in terms of readability) of different forms of literature that look intensively into a single cultural or ethnic experience (Leu & Kinzer, 1999). If

Explicit, strategic instruction shows students what to do, why, how, and when.

Culturally Complex Atmosphere

Creating a *book corner* that appeals to all children can be a challenge for the teacher. The Internet has become an excellent resource for the kind of quality literature that will introduce children to other cultural contexts. Teachers will find valuable links to appropriate children's literature that will help their students appreciate and begin to understand the range of human experiences and cultural backgrounds.

- The Web site *Multicultural Resources* provides articles, reviews, and literature selections organized around specific cultural groups (http://falcon.jmu.edu/~ramseyil/multipub.html).
- An excellent Web resource for children's literature that addresses cultural differences is *The Children's Literature Web Guide* (http://www.acs.ucalgary.ca/~dkbrown/lists.html)
- *The Reading Zone of the Internet Public Library* (http://www.ipl.org/youth/lapage.html) is a central site that is useful for teachers and students.

Keypals

The Internet expands the appeal of pen pal activities in the classroom. A great site for Keypal contacts is:

http://www.stolaf.edu/network/iecc

At this site, intercultural *E-Mail Classroom Connections*, teachers will find a good source for developing keypals from different countries. There are several mailing lists for teachers looking for partner classrooms. Teachers can subscribe directly from this Web page.

learners are to be successful in understanding cultural traditions, trade books must be available in the classroom and in the school library to support these strategies.

Foster an Interactive Classroom Learning Environment

Students must have opportunities to interact with each other—to engage in shared inquiry and discovery—in their efforts to solve problems and complete tasks. The following are suggested activities for interactive engagement in the learning process:

- *Cooperative learning groups.* Cooperative groups bring students together within a variety of supportive and collaborative learning activities. The use of this kind of learning group allows all children to see the benefits of bringing together people with diverse backgrounds for problem-solving tasks. They use listening, speaking, reading, and writing together to achieve common goals and in the process become accountable since their performance affects group outcomes. They become active language users and learn to respect each other's opinions (Bromley, 1998). For example, the I-Search Strategy (Leu & Kinzer, 1999) is

Many effective classrooms are organized around an interdisciplinary, or cross-curricular theme.

an interdisciplinary, student-centered inquiry process that emphasizes participation and sharing of research findings in small cooperative learning groups, as well as in whole-group settings. To implement this strategy, children choose a motivating theme; with the teacher's assistance, they formulate their own research plans; next, they follow and revise their plans as they gather information, and then they prepare papers, posters, or presentations using computer software, or they prepare oral reports.

Teachers can design instruction that provides just enough scaffolding, or support, for students to be able to participate in tasks that currently are beyond their reach.

- *Guided and informal group discussions.* Informal discussions provide opportunities for able students and less able students to collaborate in constructing meaning from text and enable them to learn from each other by sharing their reflections, opinions, interpretations, and questions. The teacher models discussion techniques and guides the students through early discussion sessions. As students develop their discussion skills and begin to feel comfortable talking about story content and their opinions, they will begin to try out ideas without worrying about being wrong or sounding as if they do not understand the story.
- *The Internet.* On the Web, children can experience exciting cultural exchanges.

Keypals (see box) is the online equivalent of pen pals. It is an e-mail activity that may be particularly beneficial to second-language learners because the students are able to communicate in their native language with children from similar cultural and linguistic backgrounds. Moreover, important friendships can develop among *all* students as they find out about life in another part of the world, share useful Web sites, and even help one another with homework (Leu & Kinzer, 1999).

Send newsletters to all families providing an overview of culturally responsive curriculum goals, classroom activities, and selected student-written stories and poems.

Employ Ongoing and Culturally Aware Assessments

In culturally responsive classrooms, teachers employ ongoing and systematic assessment of student abilities, interests, attitudes, and social skills. This information provides a basis for instructional decision making and offers insights into what to teach and how to teach. In addition, there is an emphasis on student involvement in the assessment process. When students are permitted to participate in their assessment, they are able to reflect on their own progress and offer insights that adults may not have. Examples of culturally sensitive assessment include the following:

- *Daily observation of students' social and learning behaviors in all classroom situations.* Observations can be recorded on checklists, in notebooks, on file cards, or in any way that permits the teacher to summarize observations in a consistent and meaningful way. For example, the class roster can be used as a convenient recording form for observations. The teacher lists the names of the students in the class and then heads subsequent columns across the top of the roster to identify the project, activity, or behavior that is observed.
- *Portfolio assessment.* Student and teacher select samples of work that reveal the diverse needs and abilities of the student. Teachers, students, and family members reflect on what students have done over time, how well they are doing, and what areas need to be improved.
- *Teacher-made tests that are closely tied to the instructional program.* Special attention is given to the cognitive styles of all the students and their evolving academic skills. For example, teachers can design a test to assess students' knowledge or performance within a particular content area lesson.
- *Student self-assessment.* Students can respond to questions about their learning during periodic teacher/student conferences. Portfolios can be used during these conferences. For example, students can be shown their work, discuss it with their teachers, and then assess their own progress.
- *Teacher self-evaluation.* Self-evaluation is an integral part of teaching effectiveness. The kinds of questions teachers ask themselves about their choices of teaching behaviors and strategies, the effectiveness and cultural relevance of their lessons, and their reactions and responses to the cultural diversity in their classrooms can greatly contribute to continuing growth in teaching and learning.

Through the Internet, second-language learners may communicate in their native language with children from similar cultural and linguistic backgrounds.

Collaborate with Other Professionals and Families

Collaboration and communication with culturally diverse families and with other professionals are essential elements of culturally responsive classrooms. Families are a critical component of a strong instructional program and should be regularly informed about students' progress and encouraged to participate in class and school activities whenever possible. It is also important to establish strong collaborative relationships with colleagues to develop instructional programs that broaden the learning opportunities of all students. The following are specific collaborative activities that teachers and families might use:

- *Consult and share ideas regularly with other teachers with whom students work.* Meet with teachers to discuss students' academic and social progress, as well as specific learning needs.
- *Communicate regularly with families.* For example, send newsletters to all families providing an overview of culturally responsive curriculum goals, classroom activities, and selected student-written stories and poems.
- *Invite families to participate in classroom cultural celebrations and to assist in planning such events.* Encourage culturally diverse families to visit the classroom to learn what occurs in the learning environment and to see how well their children are doing—academically and socially.
- *Initiate a parent volunteer tutorial program.*
- *Use culturally diverse community resources.* Invite to your classroom culturally diverse civic leaders, business leaders, artists and writers, members of the police and fire department, college professors, and academically successful high school students.
- *Attend culturally diverse community or neighborhood events.*

Final Thoughts

Of primary importance in any culturally responsive classroom is the teacher's belief that children from culturally diverse backgrounds want to learn. Second, instructional strategies and specific teaching behaviors can encourage all students to engage in learning activities that will lead to improved academic achievement. Third, the development of instructional programs that prevent failure and increase opportunities for success should be the goal of every teacher. The strategies delineated in this article can become important ways of helping all children find purpose, pride, and success in their daily efforts to learn.

Learn More About It

The following resources can help teachers evaluate the results of self-assessment.

Books

Au, K. (1993). *Literacy instruction in multicultural settings*. New York: Harcourt Brace.

Garcia, E. (1994). *Understanding and meeting the challenge of student cultural diversity*. Boston: Houghton Mifflin.

Journal Articles

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Web Sites

Cultural Diversity in the Classroom (http://education.indiana.edu/cas/tt/v2i2/cultural.html)

ERIC Digests on Cultural Diversity (http://www.uncg.edu/edu/ericcass/diverse/digests/tableoc.htm)

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Patton, J. M. (1998). The disproportionate representation of African Americans in special education. *The Journal of Special Education, 32*(1), 25-31.

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Smith, T. E. C., Finn, D. M., & Dowdy, C. A. (1993). *Teaching students with mild disabilities.* Fort Worth: Harcourt Brace Jovanovich.*

Soto, G. (1993). *Local news.* Orlando, FL: Harcourt Brace.

Yates, J. R. (1998, April). *The state of practice in the education of CLD students.* Presentation at the annual meeting of the Council for Exceptional Children, Minneapolis, MN.

BooksNow

**To order the book marked by an asterisk (*), please call 24 hrs/365 days: 1-800-BOOKS- NOW (266-5766) or (732) 728-1040; or visit them on the Web at http://www.BooksNow. com/TeachingExceptional.htm. Use VISA, M/C, AMEX, or Discover or send check or money order + $4.95 S&H ($2.50 each add'l item) to: Clicksmart, 400 Morris Avenue, Long Branch, NJ 07740; (732) 728-1040 or FAX (732) 728-7080.*

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Six Goals of a Culturally Relevant Approach to Teaching

1. **Developing Multiple Historical Perspectives**
   Multiple historical perspectives are the knowledge and understanding of the heritage and contributions of diverse nations and ethnic groups, including one's own.
2. **Developing Cultural Consciousness**
   Cultural consciousness is the recognition or awareness on the part of an individual that he or she has a view of the world that is not universally shared and differs profoundly from that held by many members of different nations and ethnic groups.
3. **Increasing Intercultural Competence**
   Intercultural competence is the ability to interpret international communications (language, signs, gestures), some unconscious cue (such as body language), and customs in culture different from one's own.
4. **Combating Racism, Prejudice and Discrimination**
   Reducing racism, prejudice and discrimination mean revising negative attitudes and behaviors that are based upon misconceptions about the inferiority of races and cultures different from one's own.
5. **Developing Awareness of the State of the planet and Global Dynamics**
   Awareness of the State of the Planet and global dynamics is the knowledge about the prevailing world conditions, trends and developments.
6. **Developing Social Action Skills**
   Social action skills include the knowledge, attitudes and behavior needed to help resolve major problems that threaten the future of the planet and well being of humanity.

Steps to Writing a Culturally Responsive Lesson Plan

1. **Start with what you normally teach.**
    - Use your current curriculum.
    - Make changes one lesson at a time.
    - Lessons should be natural and flowing.
    - Integrate other subject matter.
    - All curricula should be multicultural.

2. **Look at current curriculum content and processes.**
    - Examine for bias.
    - Look for ways to infuse ethnic content.
    - Use other resources.
    - Consider varied teaching strategies.

3. **Consider other forms of evaluation.**
    - How else can students demonstrate learning?
    - How does the lesson improve reading, writing and speaking?
    - How does the lesson increase cultural competence?